The Question Bank of

Probability and Statistics 1

for CAIE 9709 paper 5.

v1.0

Edited by Thoridal

Instructions for Use

- This question bank is organized by chapter for systematic revision.
- This question bank is complied based on the 26-27 CAIE Probability and Statistics 1 syllabus, which is included as appendix.
- Each question includes its source for reference.
- Mark schemes are provided in the separate answer booklet.
- The formula sheet (MF19) is included as appendix.
- Use this resource for targeted practice and exam preparation.

Contents

Chapter 1

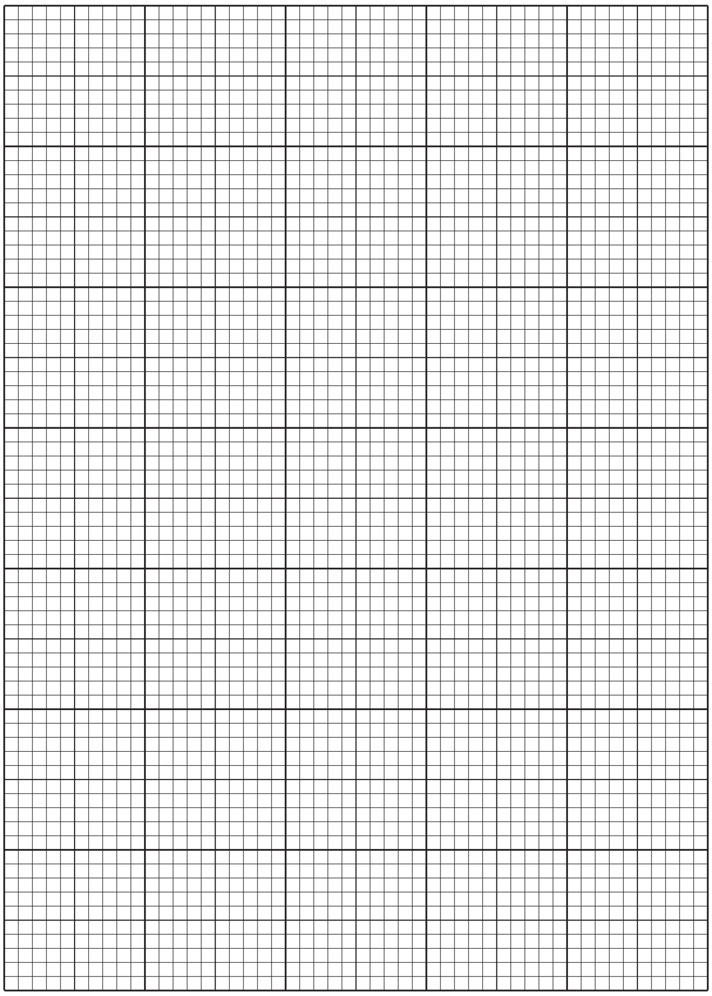
Representation of data

1. [9709/m25/52/q3]

The lengths of 250 leaves of a certain type of plant are measured, correct to the nearest centimetre. The results are summarised in the table below.

Length (cm)	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 39
Frequency	18	28	60	72	48	24

(a) On the grid below, draw a cumulative frequency graph to illustrate this information. [4]



(b) 38% of these leaves are of length k cm or more.

Use your graph to find an estimate for k. [2]

(c) Calculate an estimate for the mean length of these 250 leaves. [3]

2. [9709/s25/51/q3]

Last Sunday, teams of runners took part in a charity event. The time taken, in seconds, to run 50 m was recorded, correct to 1 decimal place, for each runner. The times recorded for 11 runners from each of the Gulls and the Herons are shown in the table.

Gulls	7.9	8.2	8.3	8.6	8.6	8.8	9.2	9.7	9.8	10.0	10.4
Herons	9.5	9.9	8.5	8.1	9.2	10.8	8.3	9.7	9.3	9.9	8.7

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Gulls on the left-hand side. [4]

(b) Find the median and the interquartile range of the times of the runners from the Gulls. [3]

Two other teams of runners, the Eagles and the Swifts, also took part in the event. The recorded times in seconds for 20 runners from the Eagles and 30 runners from the Swifts are denoted by x and y respectively.

It is given that $\sum x = 175.0$ and that the mean of y is 8.4.

(c) Find the mean of the times taken by all 50 runners. [2]

It is given that $\sum x^2 = 1823.0$.

It is also known that the standard deviation of the times taken by all 50 runners is 1.38 seconds.

(d) Find the value of $\sum y^2$, correct to 1 decimal place. [3]

3. [9709/s25/52/q5]

The times taken, t minutes, by 300 students to travel to Hollowton College are recorded. The results are summarised in the table below.

Time (t minutes)	$t \leqslant 10$	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 40$	$t \leqslant 60$	$t \leqslant 90$
Cumulative frequency	34	86	142	208	265	300

(a) On the grid, draw a cumulative frequency graph to illustrate this information. [2]

(b) 120 students take more than k minutes to travel to college. Use your graph to estimate the value of k. [2]

(c) Calculate estimates of the mean and standard deviation of the times taken to travel to college by the 300 students. [6]

4. [9709/s25/53/q1]

For a set of 40 values of x, it is found that

$$\Sigma(x-k) = 836.0, \qquad \Sigma(x-k)^2 = 25410.8,$$

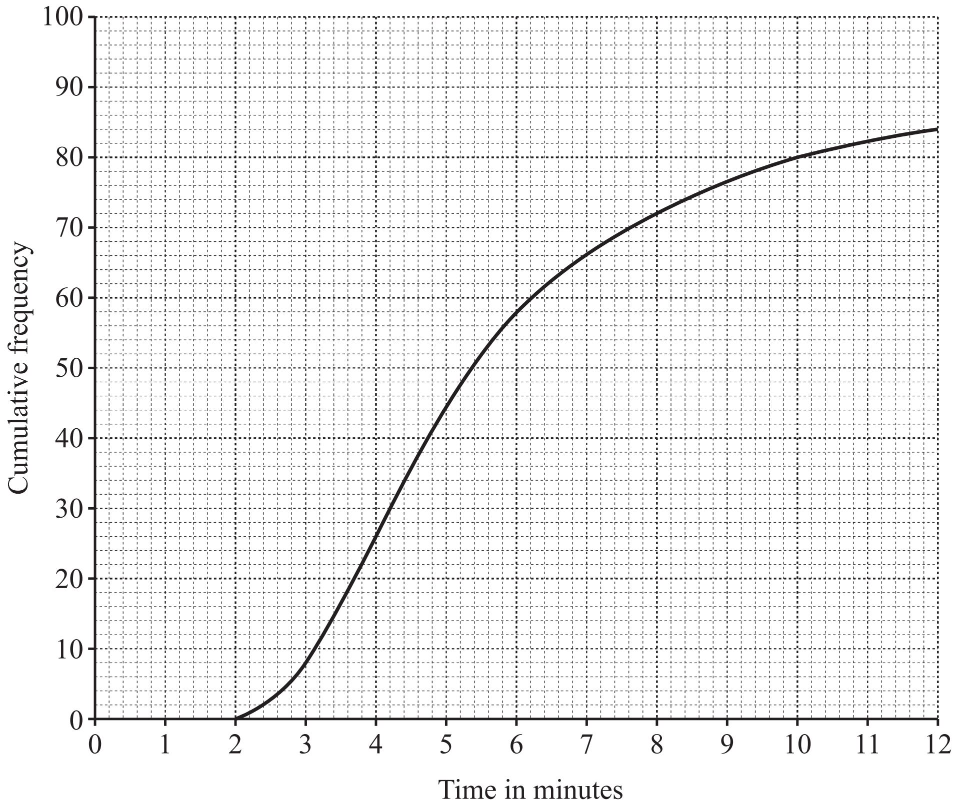
where k is a constant.

(a) Given that the mean of these 40 values is 124.0, find the value of k. [2]

(b) Find the standard deviation of these 40 values of x. [2]

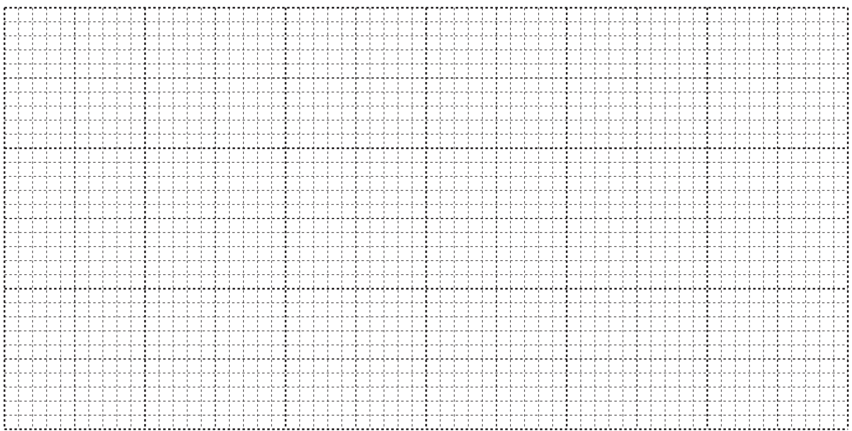
5. [9709/s25/53/q4]

84 people attempt a particular puzzle. The times taken, in minutes, to complete the puzzle are recorded. These times are represented in the cumulative frequency graph below.



(a) Use the graph to estimate how many people took between 4 and 7.5 minutes to complete the puzzle. [1]

(b) On the grid below, draw a box-and-whisker plot to summarise the information in the cumulative frequency graph. [4]



6. [9709/s25/55/q5]

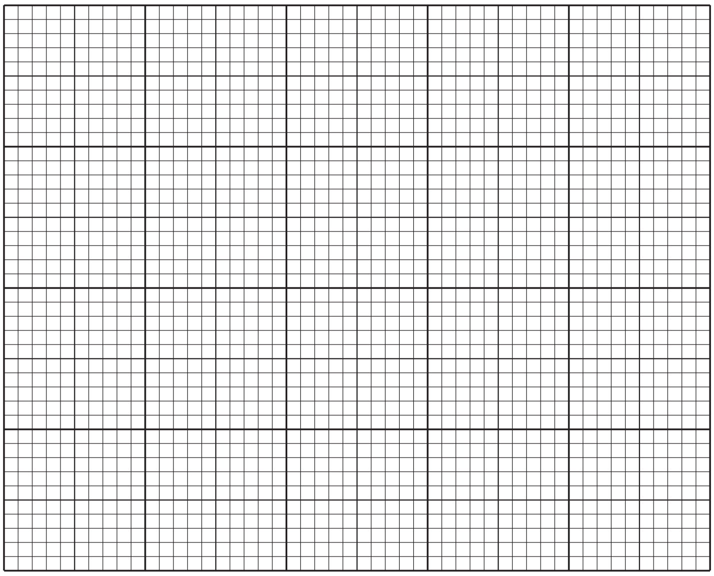
The Smarts and the Teasers are two quiz teams that each contain 11 members. Both complete a puzzle and the following table gives the times taken, in minutes, by the members of each team.

Smarts	38	30	13	29	18	22	28	18	11	9	41
Teasers	39	37	18	36	25	25	32	21	15	12	39

(a) Represent this information in a back-to-back stem-and-leaf diagram with Smarts on the left-hand side. [4]

For the Teasers, the values of the lower quartile, median and upper quartile are 18, 25 and 37 minutes respectively.

(b) On a single diagram draw box-and-whisker plots for the two teams. [4]



(c) Make two comparisons between the times for the two teams. [2]

7. [9709/w25/51/q3]

The back-to-back stem-and-leaf diagram shows the annual salaries, in dollars, of 27 employees at each of two companies, Browns and Greens.

	Browns		Greens	
(3)	9 8 4	30	6 8	(2)
(7)	8 8 5 3 3 1 0	31	2 4 5 8	(4)
(7)	9 7 6 4 2 2 0	32	3 6 6 7 9	(5)
(6)	8 7 5 5 3 1	33	1 1 3 7 8 8 9 9	(8)
(3)	4 2 2	34	0 2 3 5 6 9	(6)
(1)	7	35	3 7	(2)

Key: 6|32|7 means \$32 600 for Browns and \$32 700 for Greens.

(a) Find the median and interquartile range for the annual salaries of employees at Browns. [3]

The annual salary of an employee at Browns is denoted by x thousand dollars and the annual salary of an employee at Greens is denoted by y thousand dollars. It is given that, for the 27 employees at each of the companies,

$$\Sigma x = 878.3, \quad \Sigma x^2 = 28\,616.09, \quad \Sigma y = 896.5, \quad \Sigma y^2 = 29\,815.63.$$

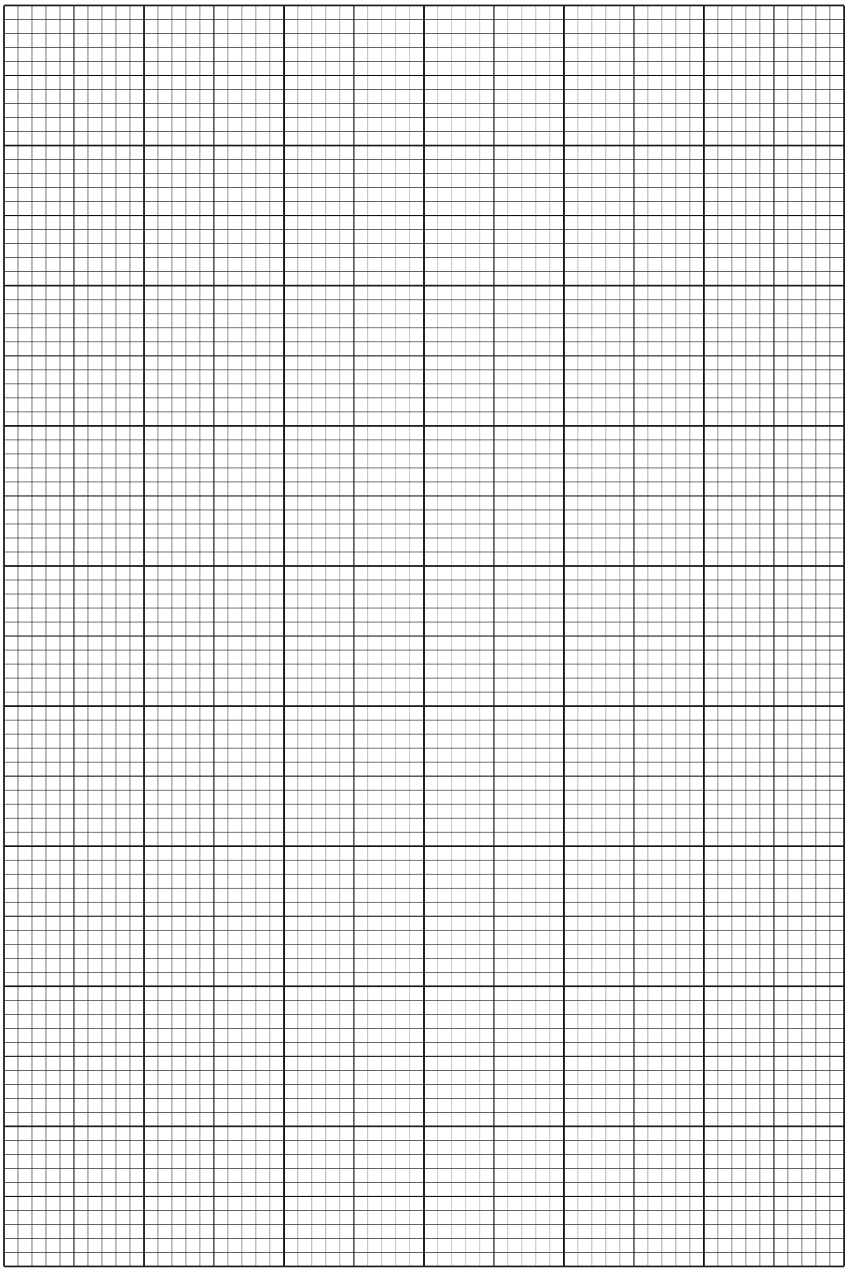
(b) Find the mean and standard deviation of the annual salaries of these 54 employees. [5]

8. [9709/w25/52/q5]

The times of 240 competitors taking part in an event are recorded correct to the nearest minute. The results are summarised in the table.

Time (minutes)	1 – 10	11 – 20	21 – 25	26 – 30	31 – 50
Frequency	12	38	68	76	46

(a) Draw a histogram to represent this information. [4]



(b) Calculate an estimate of the mean time taken by the 240 competitors. [3]

9. [9709/w25/53/q6]

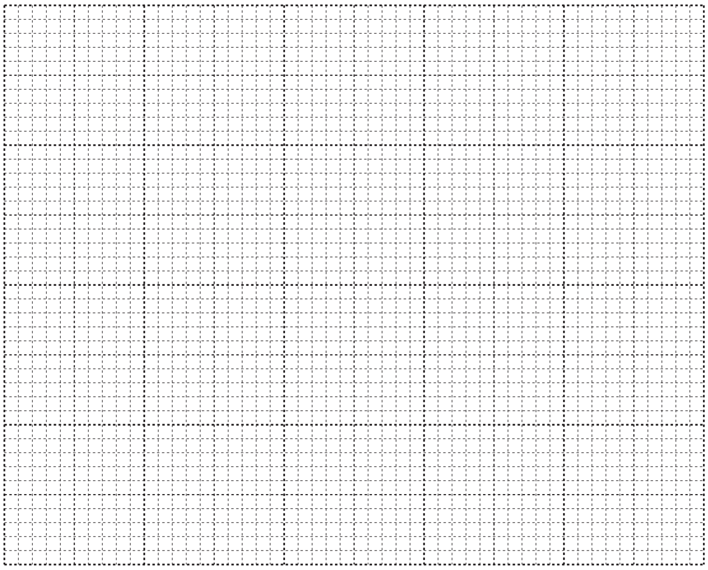
Last Saturday, a cycling competition for teams of 11 cyclists took place. For each cyclist, the time taken to complete the course was recorded to the nearest minute. The times taken by the cyclists from two teams, the Linnets and the Puffins, are shown in the following table.

Linnets	48	51	54	57	59	60	64	64	65	68	70
Puffins	45	49	51	55	55	58	59	62	64	64	74

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Linnets on the left-hand side. [4]

(b) Find the interquartile range of the times taken by the Linnets. [2]

(c) On the grid below, draw a box-and-whisker plot to represent the information for the Linnets and the Puffins. [3]



(d) Make one comparison between the times taken by the Linnets and the times taken by the Puffins. [1]

10. [9709/w25/55/q4]

The heights, in cm, of 15 players from each of two sports teams, Pelicans and Swans, are given in the table.

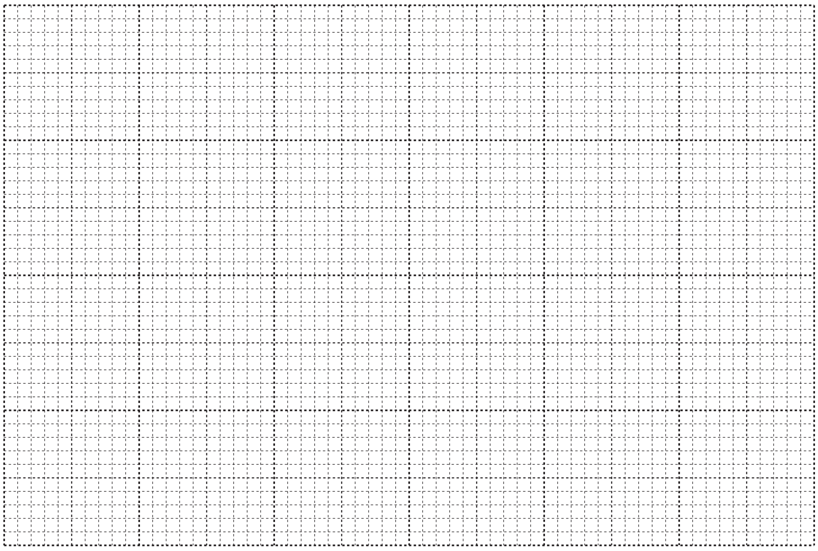
Pelicans	156	160	164	165	167	170	171	173	178	182	182	184	185	186	1
Swans	170	180	183	165	174	158	170	181	162	178	174	163	191	182	1

(a) Draw a back-to-back stem-and-leaf diagram to represent the heights of the players from Pelicans and Swans, with Pelicans on the left-hand side. [4]

(b) Find the median and the interquartile range of the heights of the Pelicans. [3]

For the Swans, the lower quartile of the heights is 165 cm, the median is 174 cm and the upper quartile is 181 cm.

(c)
Represent the data shown in the back-to-back stem-and-leaf diagram by a pair of box-and-whisker plots in a single diagram. [3]



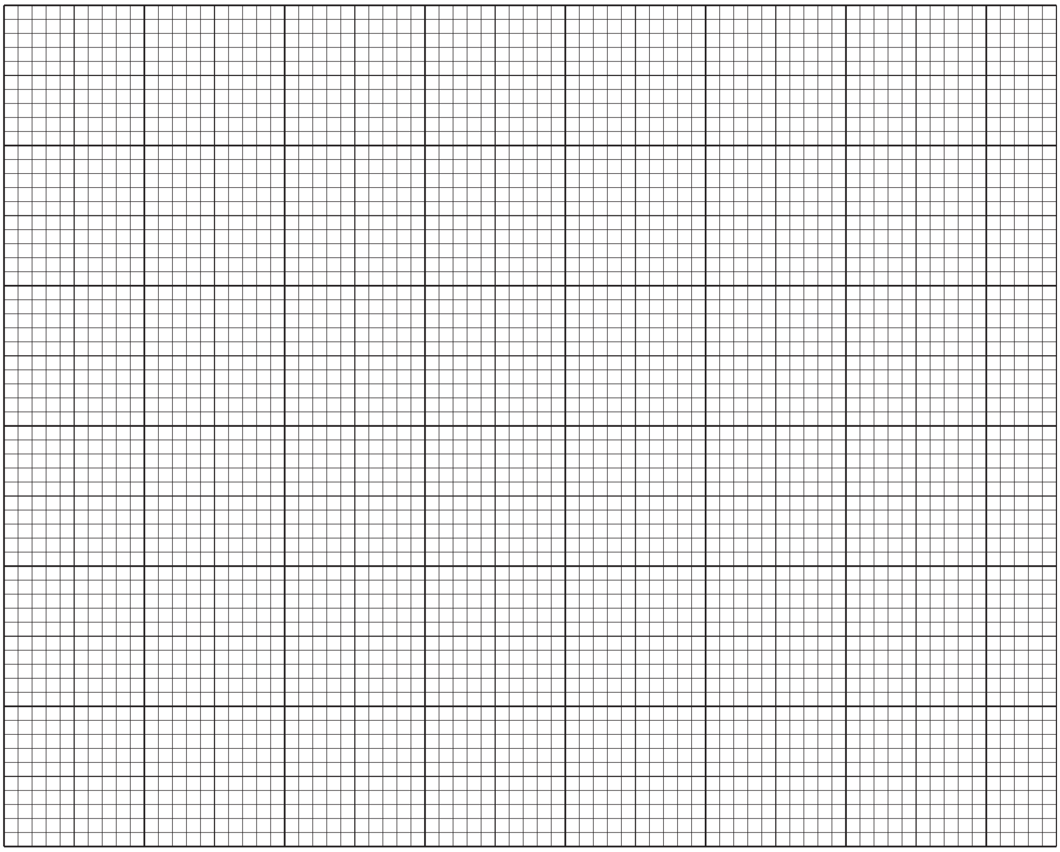
(d) Make one comparison between the heights of the Pelicans players and the heights of the Swans players. [1]

11. [9709/m24/52/q3]

The times taken, in minutes, by 150 students to complete a puzzle are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 70$
Frequency	8	23	35	52	20	12

(a) Draw a histogram to represent this information. [4]



(b) Calculate an estimate for the mean time for these students to complete the puzzle. [3]

(c) In which class interval does the lower quartile of the times lie? [1]

12. [9709/s24/51/q1]

A summary of 20 values of x gives

$$\sum(x-30) = 439, \qquad \sum(x-30)^2 = 12\,405.$$

A summary of another 25 values of x gives

$$\sum(x-30) = 470, \qquad \sum(x-30)^2 = 11\,346.$$

(a) Find the mean of all 45 values of x. [2]

(b) Find the standard deviation of all 45 values of x. [2]

13. [9709/s24/51/q3]

The heights, in cm, of 200 adults in Barimba are summarised in the following table.

Height (h cm)	$130 \leqslant h < 150$	$150 \leqslant h < 160$	$160 \leqslant h < 170$	$170 \leqslant h < 175$	$175 \leqslant h < 195$
Frequency	16	32	76	64	12

(a) Draw a histogram to represent this information. [4]

(b) The interquartile range is R cm. Show that R is **not** greater than 15. [2]

14. [9709/s24/52/q4]

The back-to-back stem-and-leaf diagram shows the annual salaries of 19 employees at each of two companies, Petral and Ravon.

		Petral							Ravon		
			3	0	0	30	2	6			
9	9	8	2	2	1	31	1	5			
		5	5	4	0	32	0	0	2		
			7	5	3	33	0	4	8	9	
				1	0	34	1	1	3	4	6
						35	3				
					8	36	7	9			

Key: 2 | 31 | 5 means \$31 200 for a Petral employee and \$31 500 for a Ravon employee.

(a) Find the median and the interquartile range of the salaries of the Petral employees. [3]

The median salary of the Ravon employees is \$33 800, the lower quartile is \$32 000 and the upper quartile is \$34 400.

(b) Represent the data shown in the back-to-back stem-and-leaf diagram by a pair of box-and-whisker plots in a single diagram. [3]

(c) Comment on whether the mean or the median would be a better representation of the data for the employees at Petral. [1]

15. [9709/s24/53/q4]

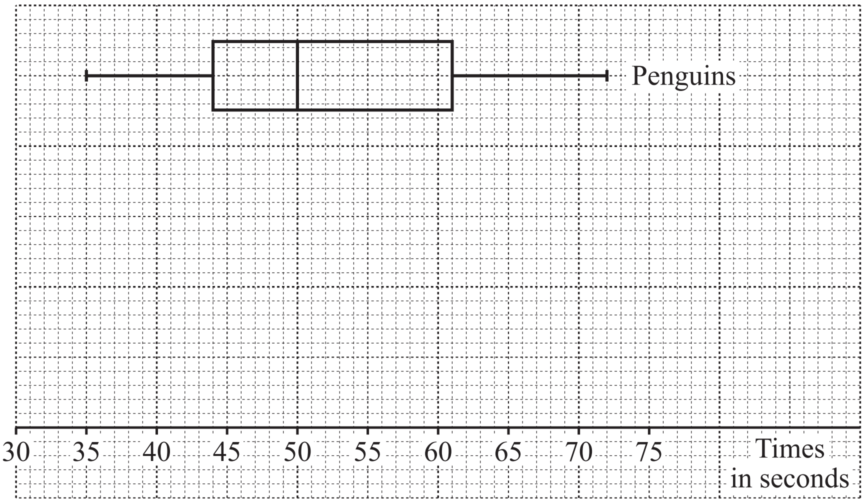
The times taken, in seconds, by 15 members of each of two swimming clubs, the Penguins and the Dolphins, to swim 50 metres are shown in the following table.

Penguins	35	39	42	44	45	45	48	50	56	58	59	61	66	68	72
Dolphins	36	41	43	48	49	49	50	51	54	56	56	60	61	64	71

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Penguins on the left-hand side. [4]

The diagram shows a box-and-whisker plot representing the times for the Penguins.

(b) On the same diagram, draw a box-and-whisker plot to represent the times for the Dolphins. [3]



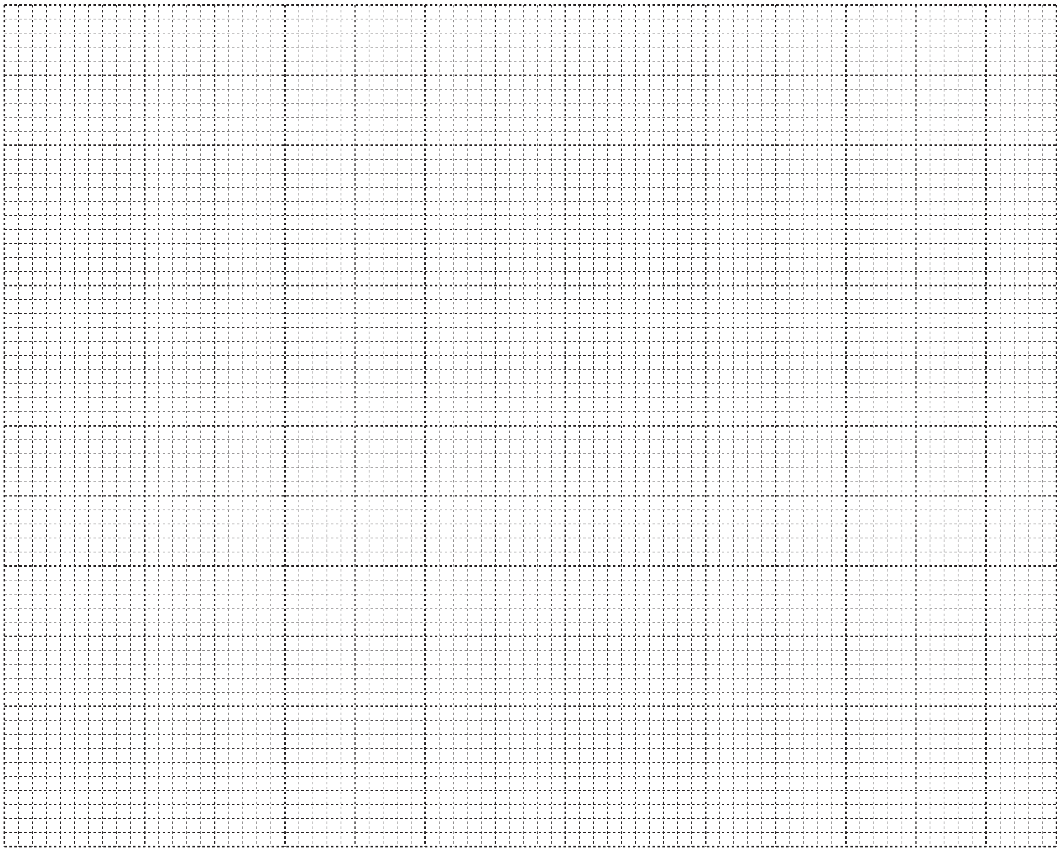
(c) Hence state **one** difference between the distributions of the times for the Penguins and the Dolphins. [1]

16. [9709/w24/51/q3]

The time taken, in minutes, to walk to school was recorded for 200 pupils at a certain school. These times are summarised in the following table.

Time taken (t minutes)	$t \leqslant 15$	$t \leqslant 25$	$t \leqslant 30$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 70$
Cumulative frequency	18	46	88	140	176	200

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the median and the interquartile range of the data. [3]

(c) Calculate an estimate for the mean value of the times taken by the 200 pupils to walk to school. [3]

17. [9709/w24/52/q6]

Teams of 15 runners took part in a charity run last Saturday. The times taken, in minutes, to complete the course by the runners from the Falcons and the runners from the Kites are shown in the table.

Falcons	38	39	42	44	46	48	50	51	52	56	58	59	64	69	76
Kites	32	40	40	45	47	48	52	54	58	59	59	60	61	63	65

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with the Falcons on the left-hand side. [4]

(b) Find the median and the interquartile range of the times for the Falcons. [3]

Let x and y denote the times, in minutes, of a runner from the Falcons and a runner from the Kites respectively.

It is given that

$$\sum x = 792, \qquad \sum x^2 = 43\,504, \qquad \sum y = 783, \qquad \sum y^2 = 42\,223.$$

(c) Find the mean and the standard deviation of the times taken by all 30 runners from the two teams. [3]

18. [9709/w24/53/q4]

On a certain day, the heights of 150 sunflower plants grown by children at a local school are measured, correct to the nearest cm. These heights are summarised in the following table.

Height (cm)	10–19	20–29	30–39	40–44	45–49	50–54	55–59
Frequency	10	18	32	42	28	14	6

(a) Draw a cumulative frequency graph to illustrate the data. [4]

(b) Use your graph to estimate the 30th percentile of the heights of the sunflower plants. [2]

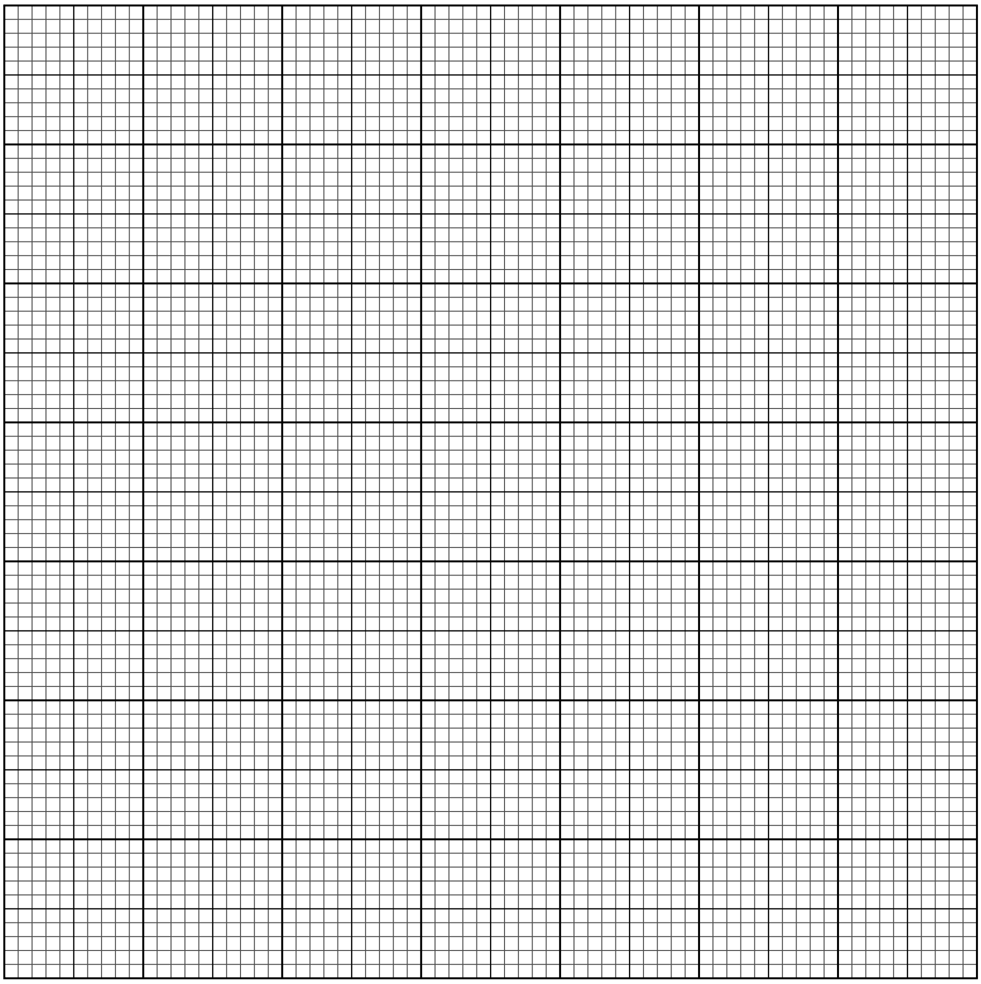
(c) Calculate estimates for the mean and the standard deviation of the heights of the 150 sunflower plants. [5]

19. [9709/m23/52/q1]

Each year the total number of hours, x, of sunshine in Kintoo is recorded during the month of June. The results for the last 60 years are summarised in the table.

x	$30 \leqslant x < 60$	$60 \leqslant x < 90$	$90 \leqslant x < 110$	$110 \leqslant x < 140$	$140 \leqslant x < 180$	$180 \leqslant x \leqslant 240$
Number of years	4	8	14	25	7	2

(a) Draw a cumulative frequency graph to illustrate the data. [3]



(b) Use your graph to estimate the 70th percentile of the data. [2]

(c) Calculate an estimate for the mean number of hours of sunshine in Kintoo during June over the last 60 years. [3]

20. [9709/s23/51/q1]

A summary of 50 values of x gives

$$\Sigma(x-q) = 700, \qquad \Sigma(x-q)^2 = 14\,235,$$

where q is a constant.

(a) Find the standard deviation of these values of x. [2]

(b) Given that $\Sigma x = 2865$, find the value of q. [2]

21. [9709/s23/51/q5]

The populations of 150 villages in the UK, to the nearest hundred, are summarised in the table.

Population	100 – 800	900 – 1200	1300 – 2000	2100 – 3200	3300 – 4800
Number of villages	8	12	50	48	32

(a) Draw a histogram to represent this information. [4]

(b) Write down the class interval which contains the median for this information. [1]

(c) Find the greatest possible value of the interquartile range for the populations of the 150 villages. [2]

22. [9709/s23/52/q3]

The following back-to-back stem-and-leaf diagram represents the monthly salaries, in dollars, of 27 employees at each of two companies, A and B.

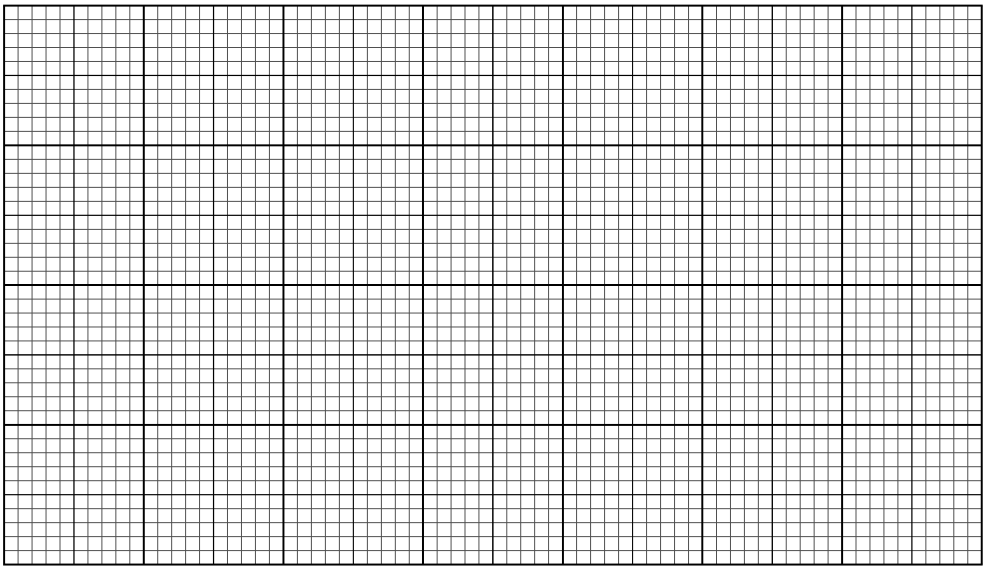
Company A		Company B
5 4 1 1 0	25	4 4 5 6 6 7
9 9 8 7 2 1 0	26	0 1 3 5 5 7 9 9
8 6 4 2 1 0	27	1 3 4 6 6 8 8
6 5 4 2 0	28	0 1 2 2 2
9 8 5	29	
1	30	9

Key: 1 | 27 | 6 means \$2710 for company A and \$2760 for company B

(a) Find the median and the interquartile range of the monthly salaries of employees in company A. [3]

The lower quartile, median and upper quartile for company B are \$2600, \$2690 and \$2780 respectively.

(b) Draw two box-and-whisker plots in a single diagram to represent the information for the salaries of employees at companies A and B. [3]



(c) Comment on whether the mean would be a more appropriate measure than the median for comparing the given information for the two companies. [1]

23. [9709/s23/53/q4]

The times taken, in minutes, to complete a cycle race by 19 cyclists from each of two clubs, the Cheetahs and the Panthers, are represented in the following back-to-back stem-and-leaf diagram.

Cheetahs		Panthers
9 8	7	4
8 7 3 2 0	8	6 8
9 8 7	9	1 7 8 9 9
6 5 3 3 1	10	2 3 4 4 5 6
9 8 2	11	1 2 8
4	12	0 6

Key: 7 | 9 | 1 means 97 minutes for Cheetahs and 91 minutes for Panthers

(a) Find the median and the interquartile range of the times of the Cheetahs. [3]

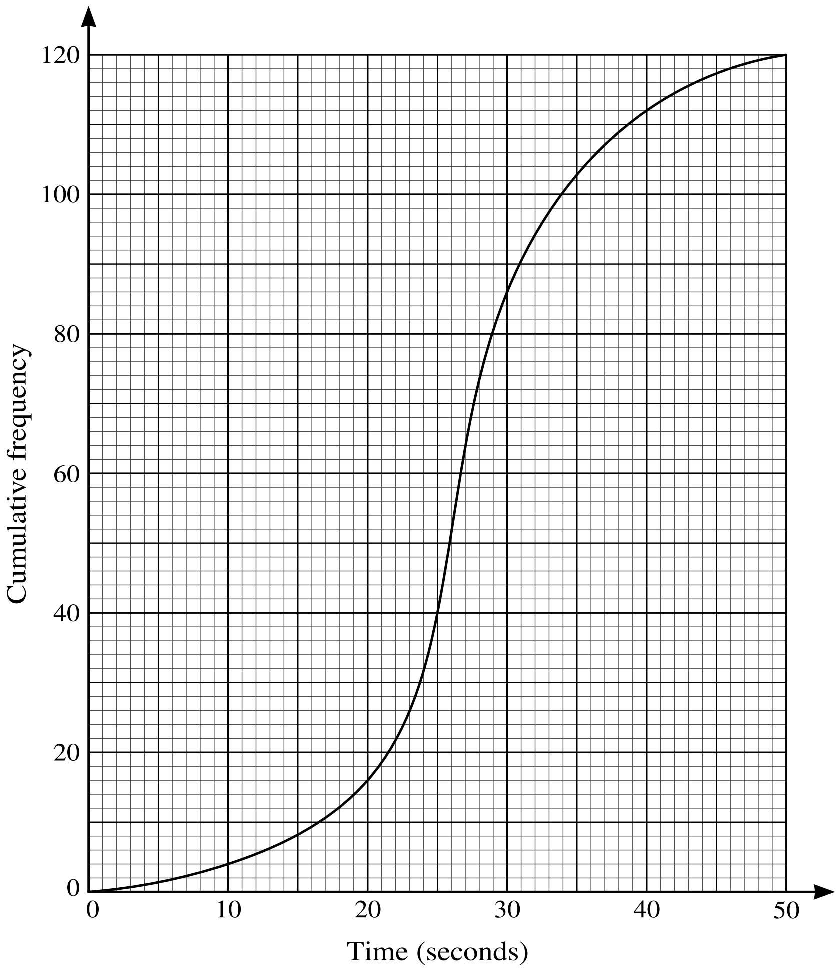
The median and interquartile range for the Panthers are 103 minutes and 14 minutes respectively.

(b) Make two comparisons between the times taken by the Cheetahs and the times taken by the Panthers. [2]

Another cyclist, Kenny, from the Cheetahs also took part in the race. The mean time taken by the 20 cyclists from the Cheetahs was 99 minutes.

(c) Find the time taken by Kenny to complete the race. [3]

24. [9709/w23/51/q1]



The times taken by 120 children to complete a particular puzzle are represented in the cumulative frequency graph.

(a) Use the graph to estimate the interquartile range of the data. [2]

35% of the children took longer than T seconds to complete the puzzle.

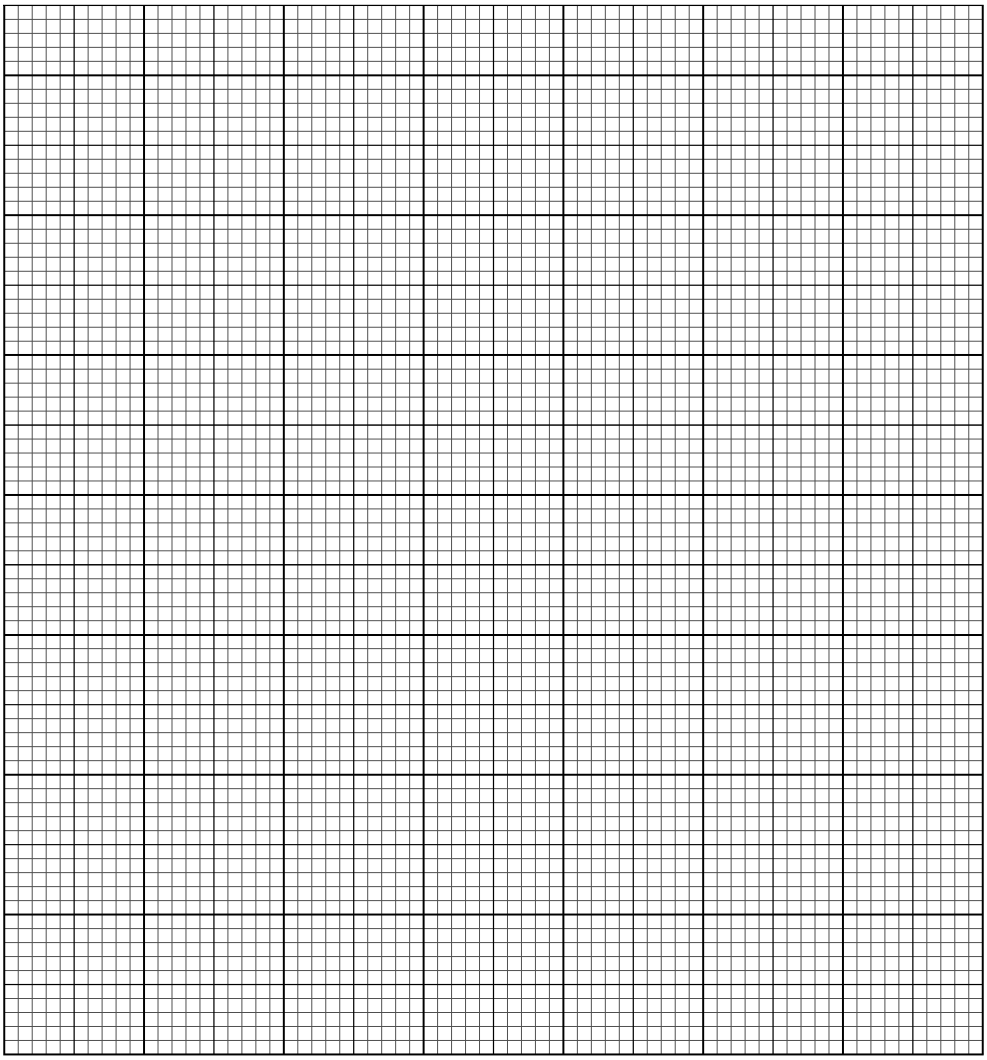
(b) Use the graph to estimate the value of T. [2]

25. [9709/w23/51/q4]

The times, to the nearest minute, of 150 athletes taking part in a charity run are recorded. The results are summarised in the table.

Time in minutes	101 – 120	121 – 130	131 – 135	136 – 145	146 – 160
Frequency	18	48	34	32	18

(a) Draw a histogram to represent this information. [4]



(b) Calculate estimates for the mean and standard deviation of the times taken by the athletes. [5]

26. [9709/w23/52/q4]

The heights, in cm, of the 11 players in each of two teams, the Aces and the Jets, are shown in the following table.

Aces	180	174	169	182	181	166	173	182	168	171	164
Jets	175	174	188	168	166	174	181	181	170	188	190

(a) Draw a back-to-back stem-and-leaf diagram to represent this information with the Aces on the left-hand side of the diagram. [4]

(b) Find the median and the interquartile range of the heights of the players in the Aces. [3]

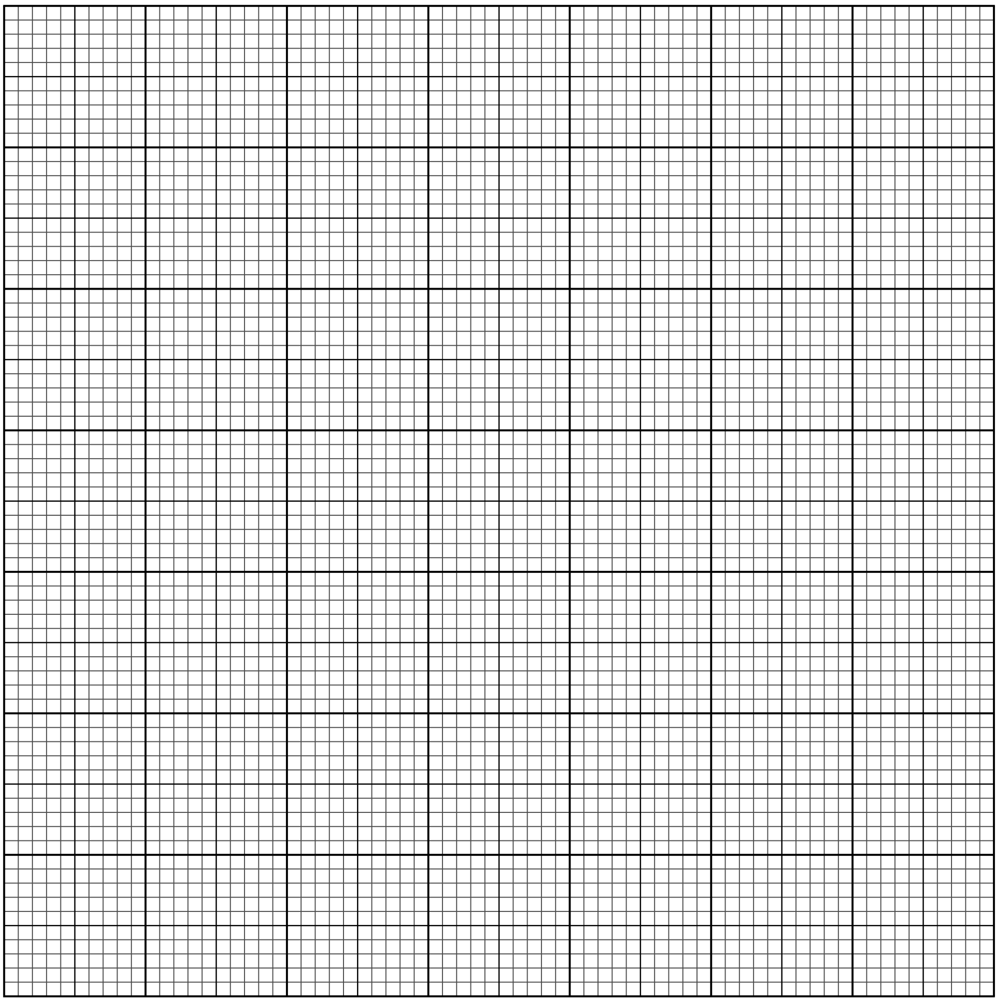
(c) Give one comment comparing the spread of the heights of the Aces with the spread of the heights of the Jets. [1]

27. [9709/w23/53/q4]

The weights, x kg, of 120 students in a sports college are recorded. The results are summarised in the following table.

Weight (x kg)	$x \leqslant 40$	$x \leqslant 60$	$x \leqslant 65$	$x \leqslant 70$	$x \leqslant 85$	$x \leqslant 100$
Cumulative frequency	0	14	38	60	106	120

(a) Draw a cumulative frequency graph to represent this information. [2]



(b) It is found that 35% of the students weigh more than W kg.

Use your graph to estimate the value of W. [2]

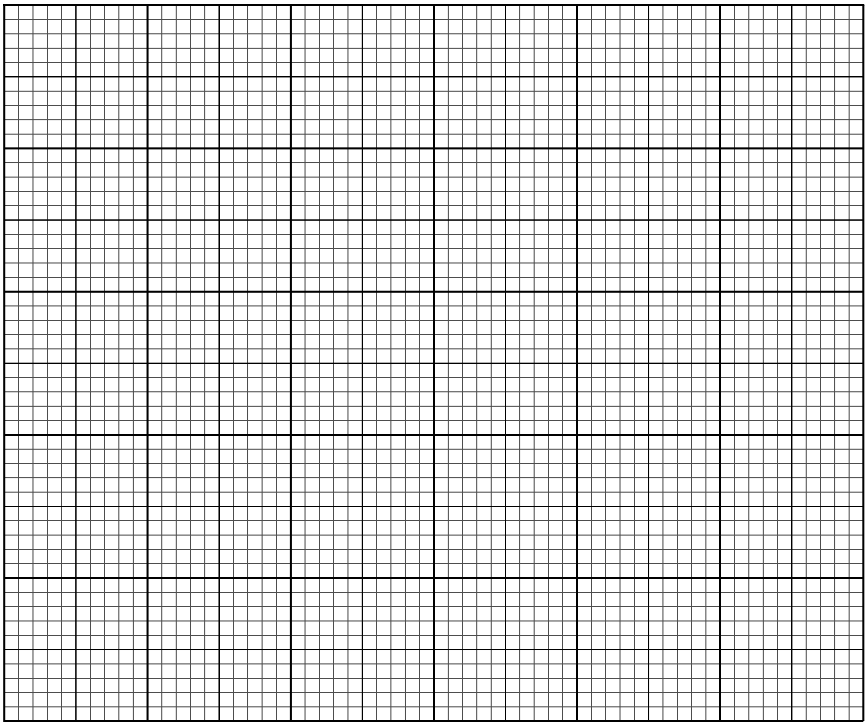
(c) Calculate estimates for the mean and standard deviation of the weights of the 120 students. [6]

28. [9709/m22/52/q3]

At a summer camp an arithmetic test is taken by 250 children. The times taken, to the nearest minute, to complete the test were recorded. The results are summarised in the table.

Time taken, in minutes	1 – 30	31 – 45	46 – 65	66 – 75	76 – 100
Frequency	21	30	68	86	45

(a) Draw a histogram to represent this information. [4]



(b) State which class interval contains the median. [1]

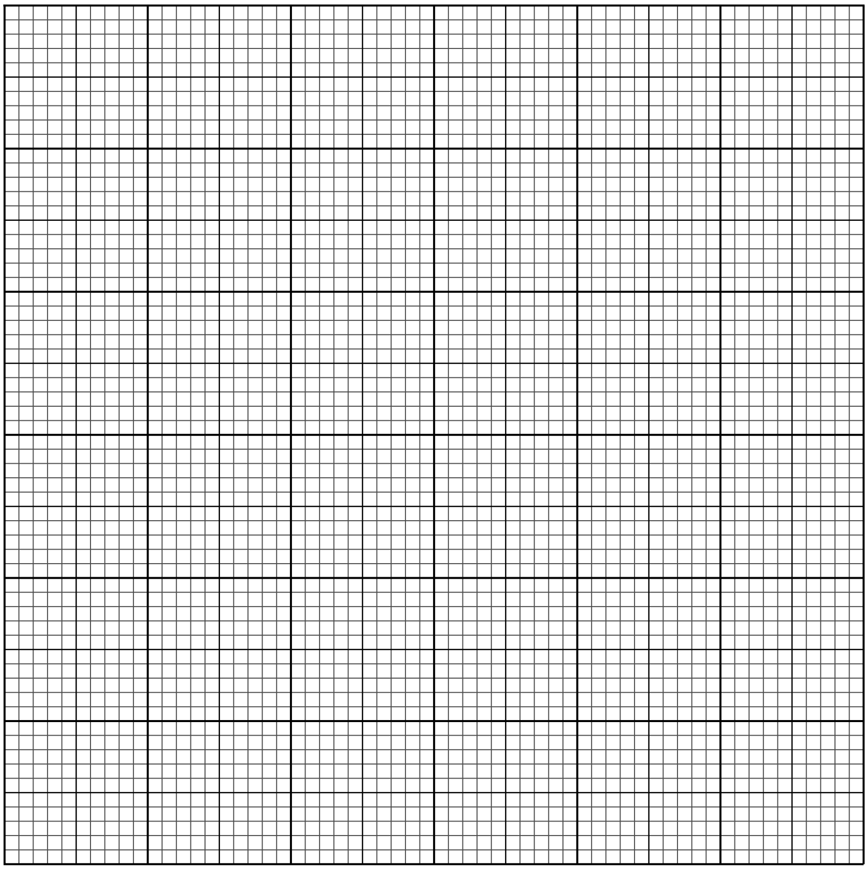
(c) Given that an estimate of the mean time is 61.05 minutes, state what feature of the distribution accounts for the median and the mean being different. [1]

29. [9709/s22/51/q3]

The times taken to travel to college by 2500 students are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 90$
Frequency	440	720	920	300	120

(a) Draw a histogram to represent this information. [4]



From the data, the estimate of the mean value of t is 31.44.

(b) Calculate an estimate of the standard deviation of the times taken to travel to college. [3]

(c) In which class interval does the upper quartile lie? [1]

It was later discovered that the times taken to travel to college by two students were incorrectly recorded. One student's time was recorded as 15 instead of 5 and the other's time was recorded as 65 instead of 75.

(d) Without doing any further calculations, state with a reason whether the estimate of the standard deviation in part **(b)** would be increased, decreased or stay the same. [1]

30. [9709/s22/52/q1]

For n values of the variable x, it is given that

$$\Sigma(x - 200) = 446 \quad \text{and} \quad \Sigma x = 6846.$$

Find the value of n. [3]

31. [9709/s22/52/q3]

The back-to-back stem-and-leaf diagram shows the diameters, in cm, of 19 cylindrical pipes produced by each of two companies, A and B.

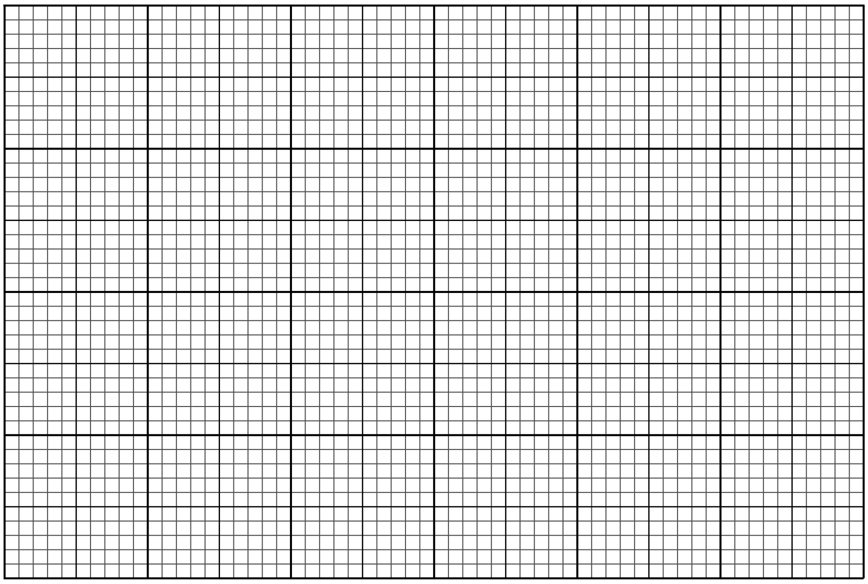
Company A							Company B				
					4	33	1	2	8		
	9	8	3	2	0	34	1	6	8	9	9
8	7	5	4	1	1	35	1	2	2	3	
		9	6	5	2	36	5	6			
			4	3	1	37	0	3	4		
						38	2	8			

Key: 1 | 35 | 3 means the pipe diameter from company A is 0.351 cm and from company B is 0.353 cm.

(a) Find the median and interquartile range of the pipes produced by company A. [3]

It is given that for the pipes produced by company B the lower quartile, median and upper quartile are 0.346 cm, 0.352 cm and 0.370 cm respectively.

(b) Draw box-and-whisker plots for companies A and B on the grid below. [3]



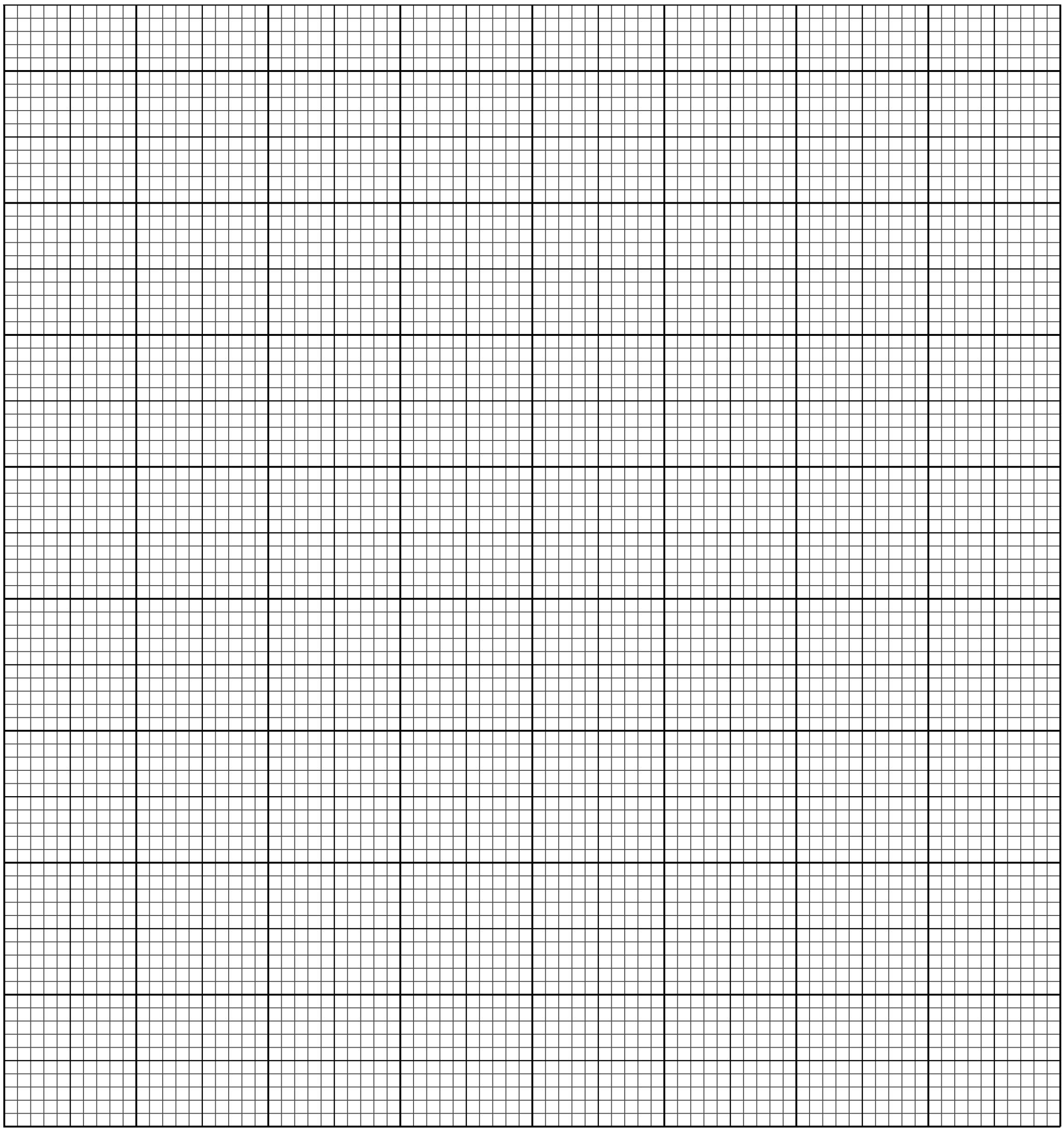
(c) Make one comparison between the diameters of the pipes produced by companies A and B. [1]

32. [9709/s22/53/q1]

The time taken, t minutes, to complete a puzzle was recorded for each of 150 students. These times are summarised in the table.

Time taken (t minutes)	$t \leqslant 25$	$t \leqslant 50$	$t \leqslant 75$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$
Cumulative frequency	16	44	86	104	132	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 20th percentile of the data. [1]

33. [9709/s22/53/q2]

Twenty children were asked to estimate the height of a particular tree. Their estimates, in metres, were as follows.

4.1	4.2	4.4	4.5	4.6	4.8	5.0	5.2	5.3	5.4
5.5	5.8	6.0	6.2	6.3	6.4	6.6	6.8	6.9	19.4

(a) Find the mean of the estimated heights. [1]

(b) Find the median of the estimated heights. [1]

(c) Give a reason why the median is likely to be more suitable than the mean as a measure of the central tendency for this information. [1]

34. [9709/w22/51/q3]

The Lions and the Tigers are two basketball clubs. The heights, in cm, of the 11 players in each of their first team squads are given in the table.

Lions	178	186	181	187	179	190	189	190	180	169	196
Tigers	194	179	187	190	183	201	184	180	195	191	197

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with the Lions on the left. [4]

(b) Find the median and the interquartile range of the heights of the Lions first team squad. [3]

It is given that for the Tigers, the lower quartile is 183 cm, the median is 190 cm and the upper quartile is 195 cm.

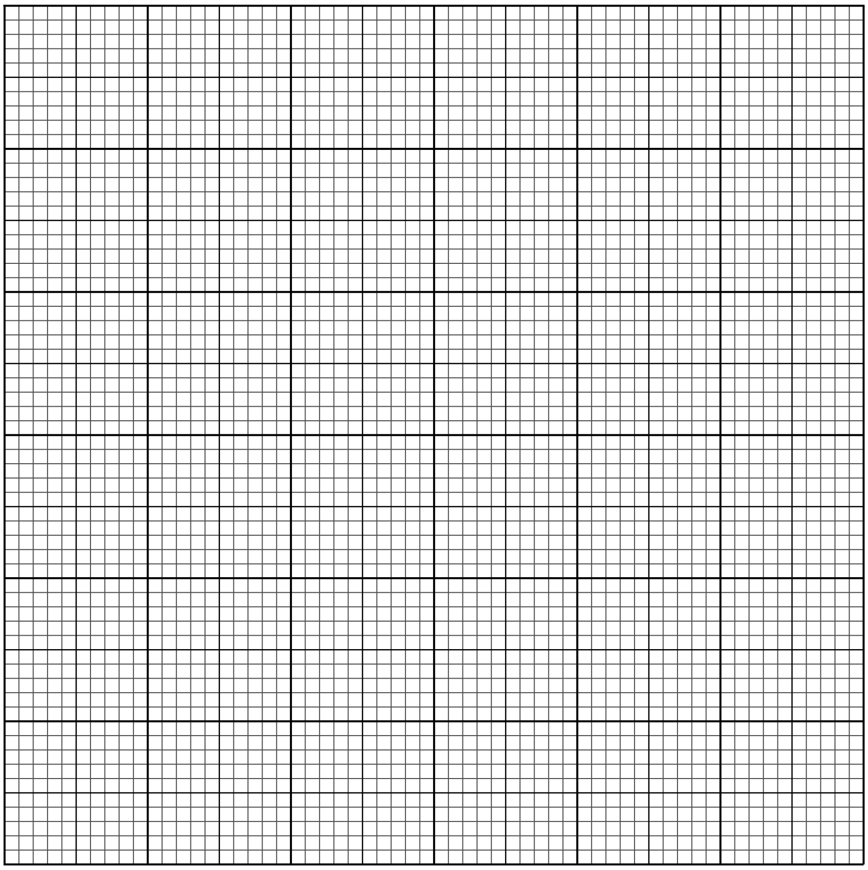
(c) Make two comparisons between the heights of the players in the Lions first team squad and the heights of the players in the Tigers first team squad. [2]

35. [9709/w22/52/q4]

The times taken, in minutes, to complete a word processing task by 250 employees at a particular company are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 60$	$60 \leqslant t < 100$
Frequency	32	46	96	52	24

(a) Draw a histogram to represent this information. [4]



From the data, the estimate of the mean time taken by these 250 employees is 43.2 minutes.

(b) Calculate an estimate for the standard deviation of these times. [3]

36. [9709/w22/53/q1]

50 values of the variable x are summarised by

$$\Sigma(x-20) = 35 \qquad \text{and} \qquad \Sigma x^2 = 25\,036.$$

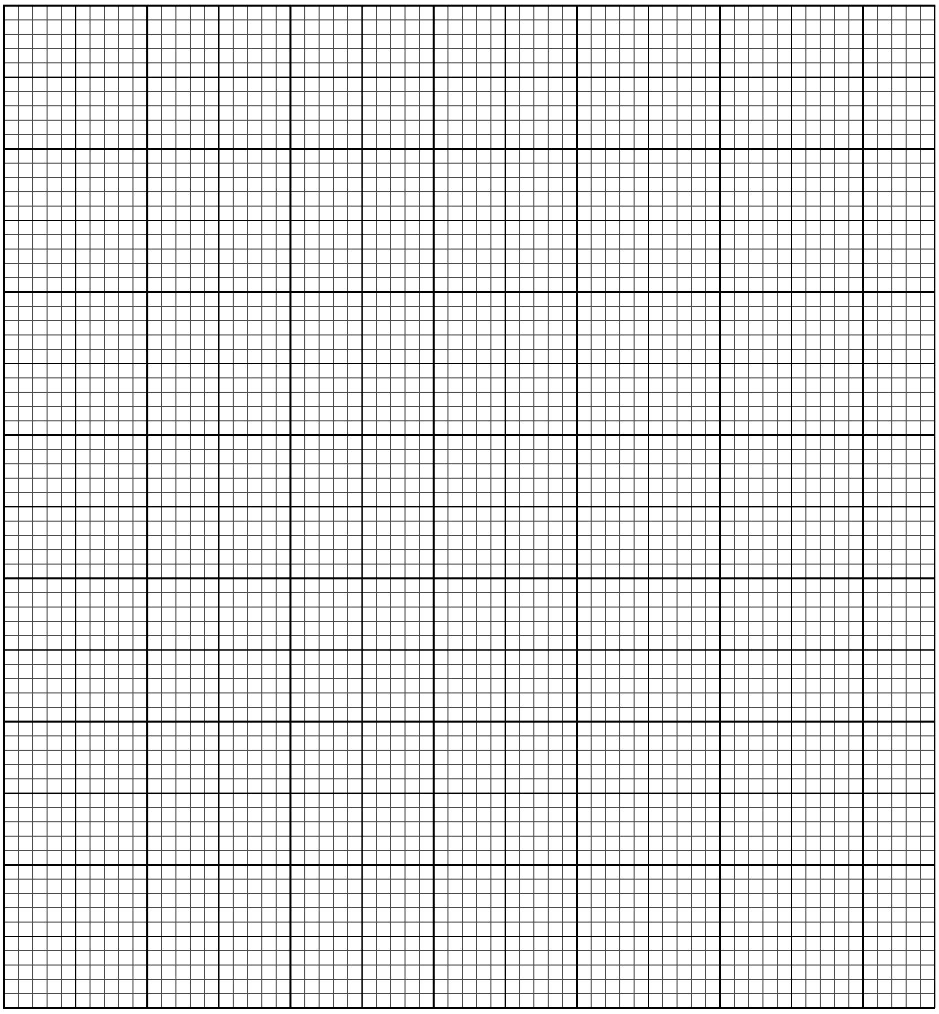
Find the variance of these 50 values. [3]

37. [9709/w22/53/q3]

The times, t minutes, taken to complete a walking challenge by 250 members of a club are summarised in the table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 60$
Cumulative frequency	32	66	112	178	228	250

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 60th percentile of the data. [1]

It is given that an estimate for the mean time taken to complete the challenge by these 250 members is 34.4 minutes.

(c) Calculate an estimate for the standard deviation of the times taken to complete the challenge by these 250 members. [4]

38. [9709/m21/52/q5]

A driver records the distance travelled in each of 150 journeys. These distances, correct to the nearest km, are summarised in the following table.

Distance (km)	0 – 4	5 – 10	11 – 20	21 – 30	31 – 40	41 – 60
Frequency	12	16	32	66	20	4

(a) Draw a cumulative frequency graph to illustrate the data. [4]

(b) For 30% of these journeys the distance travelled is d km or more.

Use your graph to estimate the value of d. [2]

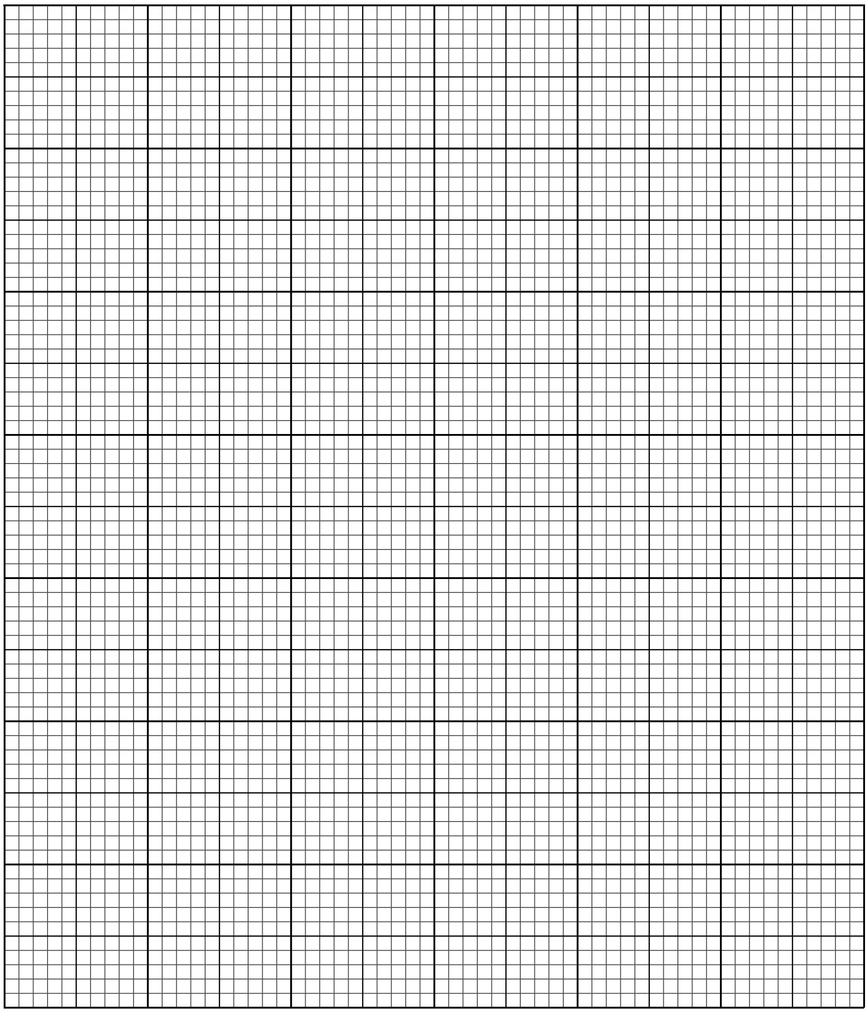
(c) Calculate an estimate of the mean distance travelled for the 150 journeys. [3]

39. [9709/s21/51/q5]

The times taken by 200 players to solve a computer puzzle are summarised in the following table.

Time (t seconds)	$0 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 100$
Number of players	16	54	78	32	20

(a) Draw a histogram to represent this information. [4]



(b) Calculate an estimate of the mean time taken by these 200 players. [2]

(c) Find the greatest possible value of the interquartile range of these times. [2]

40. [9709/s21/52/q7]

The heights, in cm, of the 11 basketball players in each of two clubs, the Amazons and the Giants, are shown below.

Amazons	205	198	181	182	190	215	201	178	202	196	184
Giants	175	182	184	187	189	192	193	195	195	195	204

(a) State an advantage of using a stem-and-leaf diagram compared to a box-and-whisker plot to illustrate this information. [1]

(b) Represent the data by drawing a back-to-back stem-and-leaf diagram with Amazons on the left-hand side of the diagram. [4]

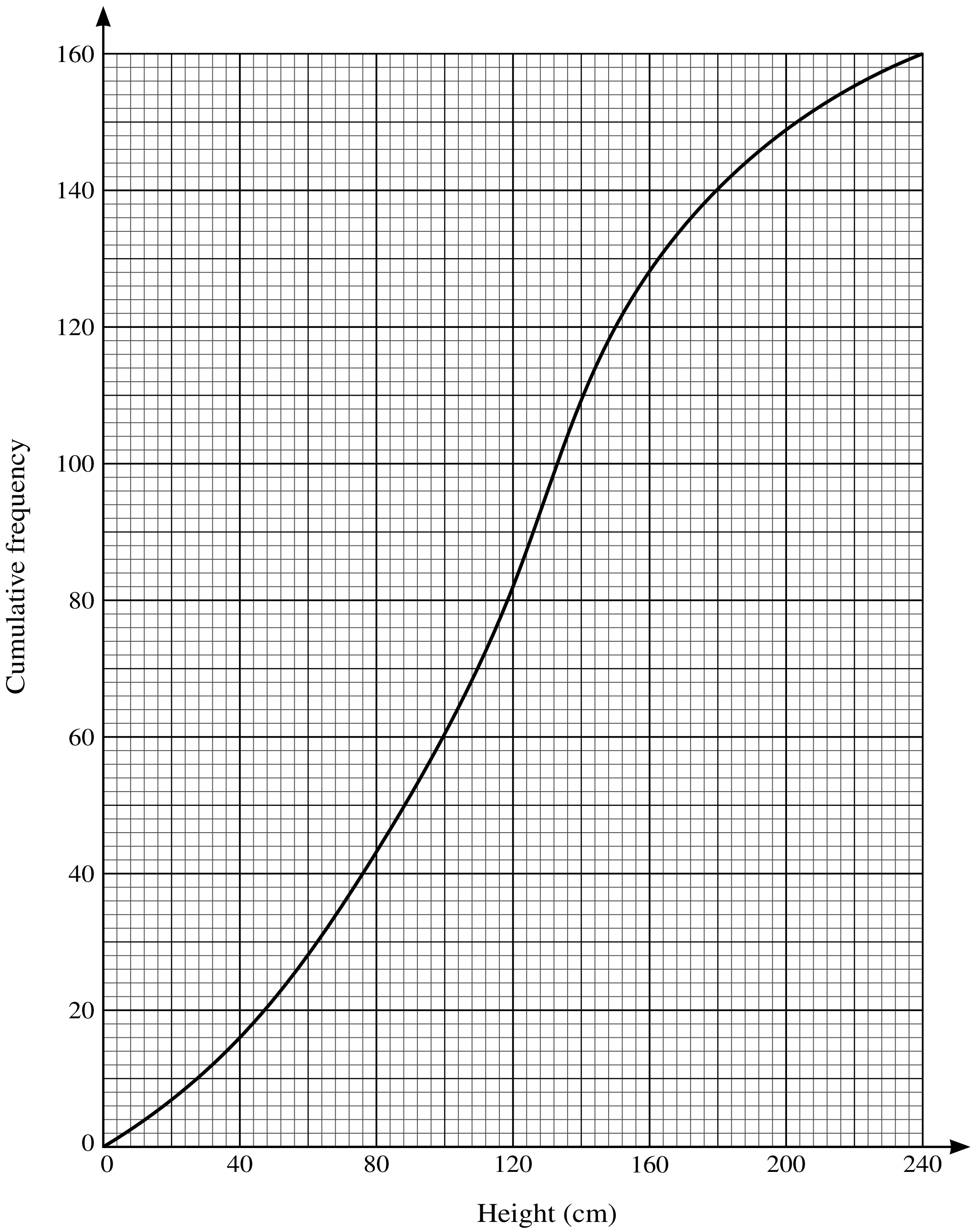
(c) Find the interquartile range of the heights of the players in the Amazons. [2]

Four new players join the Amazons. The mean height of the 15 players in the Amazons is now 191.2 cm. The heights of three of the new players are 180 cm, 185 cm and 190 cm.

(d) Find the height of the fourth new player. [3]

41. [9709/s21/53/q1]

The heights in cm of 160 sunflower plants were measured. The results are summarised on the following cumulative frequency curve.



(a) Use the graph to estimate the number of plants with heights less than 100 cm. [1]

(b) Use the graph to estimate the 65th percentile of the distribution. [2]

(c) Use the graph to estimate the interquartile range of the heights of these plants. [2]

42. [9709/s21/53/q3]

A sports club has a volleyball team and a hockey team. The heights of the 6 members of the volleyball team are summarised by $\Sigma x = 1050$ and $\Sigma x^2 = 193\,700$, where x is the height of a member in cm. The heights of the 11 members of the hockey team are summarised by $\Sigma y = 1991$ and $\Sigma y^2 = 366\,400$, where y is the height of a member in cm.

(a) Find the mean height of all 17 members of the club. [2]

(b) Find the standard deviation of the heights of all 17 members of the club. [3]

43. [9709/w21/51/q2]

A summary of 40 values of x gives the following information:

$$\Sigma(x-k) = 520, \qquad \Sigma(x-k)^2 = 9640,$$

where k is a constant.

(a) Given that the mean of these 40 values of x is 34, find the value of k. [2]

(b) Find the variance of these 40 values of x. [2]

44. [9709/w21/51/q6]

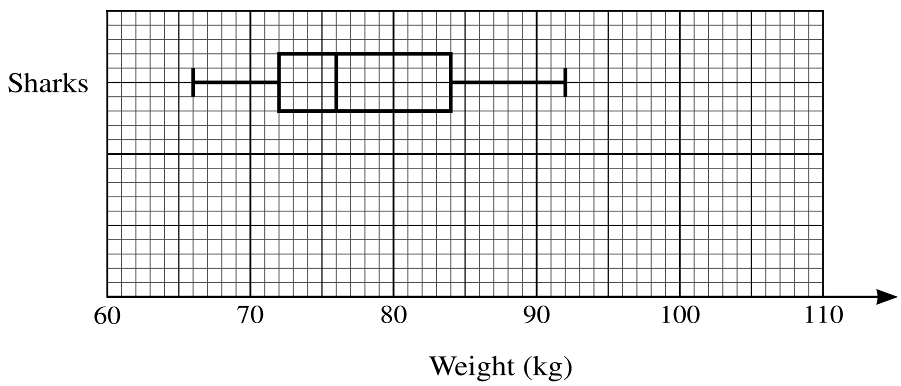
The weights, in kg, of 15 rugby players in the Rebels club and 15 soccer players in the Sharks club are shown below.

Rebels	75	78	79	80	82	82	83	84	85	86	89	93	95	99	102
Sharks	66	68	71	72	74	75	75	76	78	83	83	84	85	86	92

(a) Represent the data by drawing a back-to-back stem-and-leaf diagram with Rebels on the left-hand side of the diagram. [4]

(b) Find the median and the interquartile range for the Rebels. [3]

A box-and-whisker plot for the Sharks is shown below.



(c) On the same diagram, draw a box-and-whisker plot for the Rebels. [2]

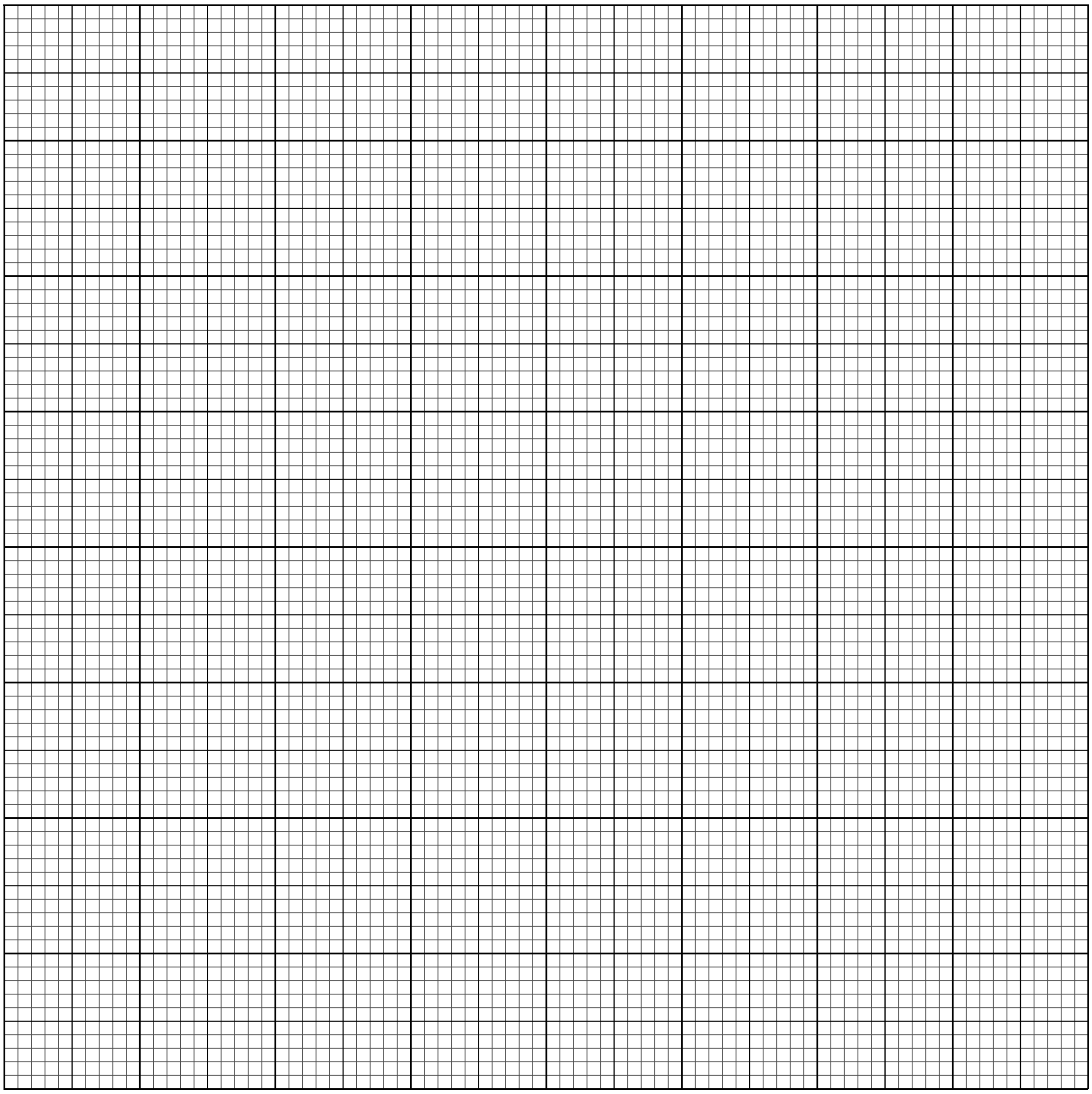
(d) Make one comparison between the weights of the players in the Rebels club and the weights of the players in the Sharks club. [1]

45. [9709/w21/52/q7]

The distances, x m, travelled to school by 140 children were recorded. The results are summarised in the table below.

Distance, x m	$x \leqslant 200$	$x \leqslant 300$	$x \leqslant 500$	$x \leqslant 900$	$x \leqslant 1200$	$x \leqslant 1600$
Cumulative frequency	16	46	88	122	134	140

(a) On the grid, draw a cumulative frequency graph to represent these results. [2]



(b) Use your graph to estimate the interquartile range of the distances. [2]

(c) Calculate estimates of the mean and standard deviation of the distances. [6]

46. [9709/w21/53/q2]

Lakeview and Riverside are two schools. The pupils at both schools took part in a competition to see how far they could throw a ball. The distances thrown, to the nearest metre, by 11 pupils from each school are shown in the following table.

Lakeview	10	14	19	22	26	27	28	30	32	33	41
Riverside	23	36	21	18	37	25	18	20	24	30	25

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Lakeview on the left-hand side. [4]

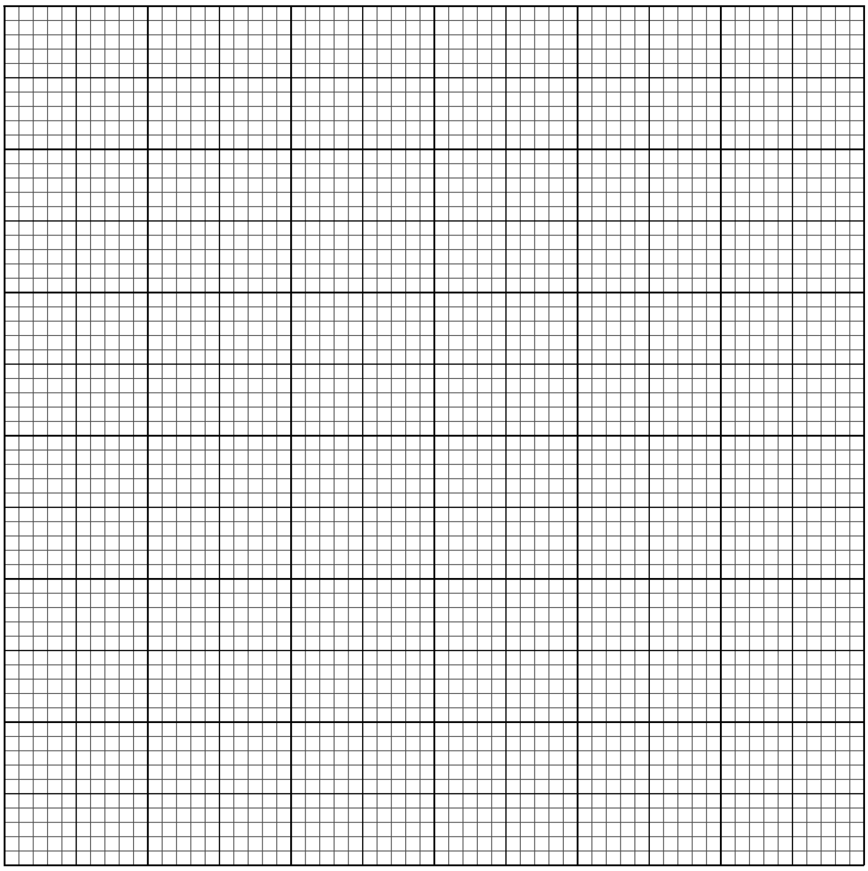
(b) Find the interquartile range of the distances thrown by the 11 pupils at Lakeview school. [2]

47. [9709/w21/53/q3]

The times taken, in minutes, by 360 employees at a large company to travel from home to work are summarised in the following table.

Time, t minutes	$0 \leqslant t < 5$	$5 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 50$
Frequency	23	102	135	76	24

(a) Draw a histogram to represent this information. [4]



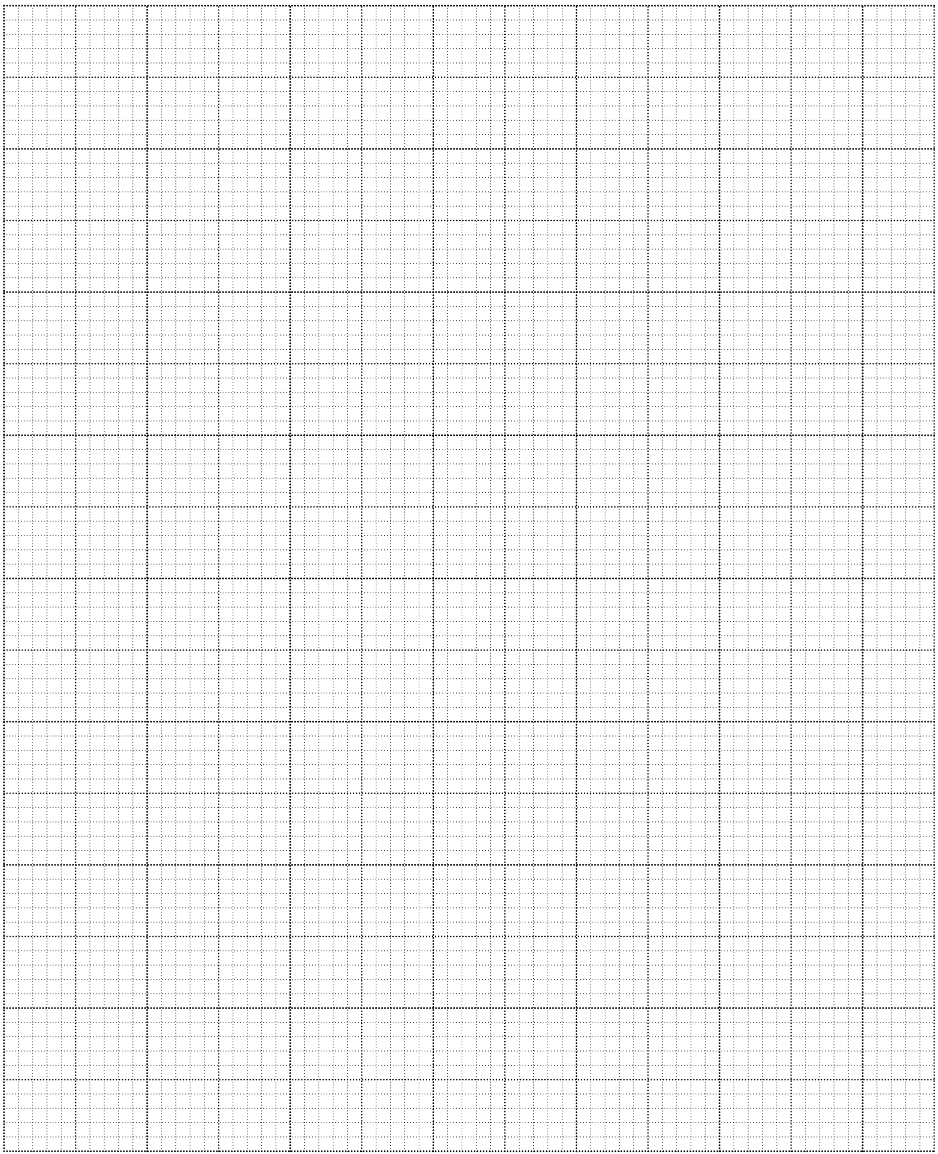
(b) Calculate an estimate of the mean time taken by an employee to travel to work. [2]

48. [9709/m20/52/q7]

Helen measures the lengths of 150 fish of a certain species in a large pond. These lengths, correct to the nearest centimetre, are summarised in the following table.

Length (cm)	0 – 9	10 – 14	15 – 19	20 – 30
Frequency	15	48	66	21

(a) Draw a cumulative frequency graph to illustrate the data. [4]



(b) 40% of these fish have a length of d cm or more. Use your graph to estimate the value of d. [2]

The mean length of these 150 fish is 15.295 cm.

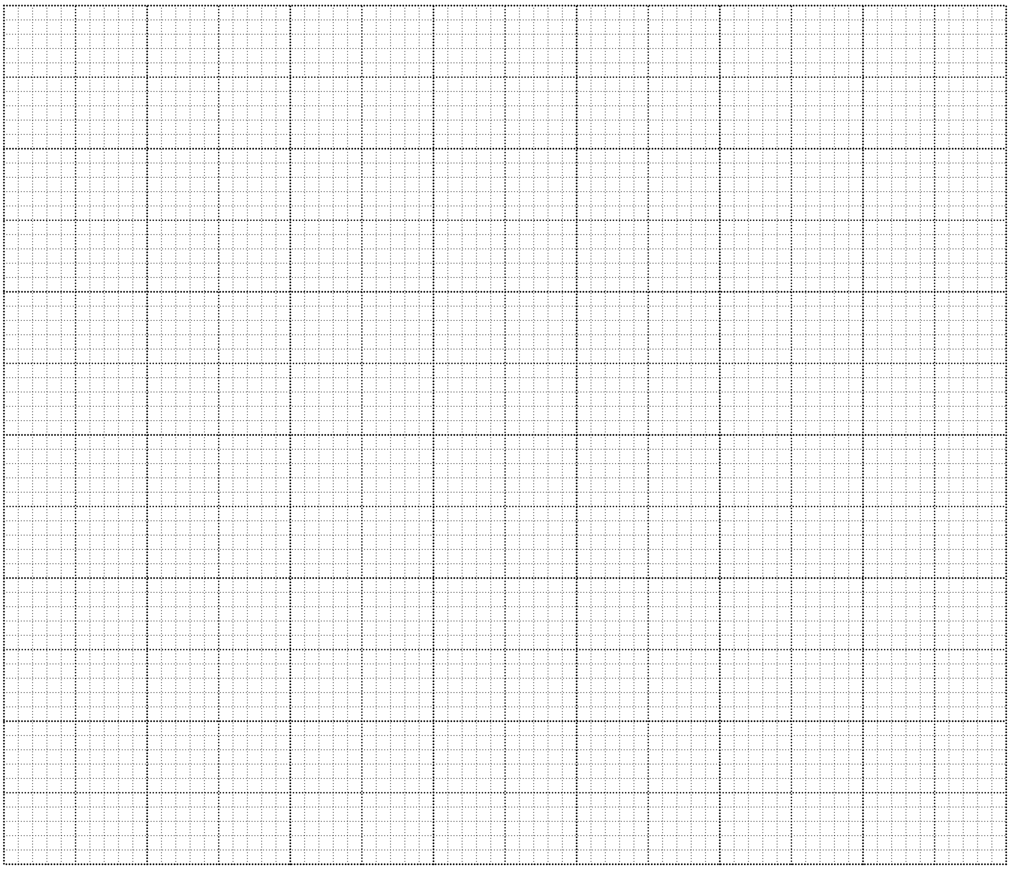
(c) Calculate an estimate for the variance of the lengths of the fish. [3]

49. [9709/s20/51/q7]

The numbers of chocolate bars sold per day in a cinema over a period of 100 days are summarised in the following table.

Number of chocolate bars sold	1 – 10	11 – 15	16 – 30	31 – 50	51 – 60
Number of days	18	24	30	20	8

(a) Draw a histogram to represent this information. [5]



(b) What is the greatest possible value of the interquartile range for the data? [2]

(c) Calculate estimates of the mean and standard deviation of the number of chocolate bars sold. [4]

50. [9709/s20/52/q1]

For n values of the variable x, it is given that

$$\Sigma(x - 50) = 144 \quad \text{and} \quad \Sigma x = 944.$$

Find the value of n. [3]

51. [9709/s20/52/q3]

Two machines, A and B, produce metal rods of a certain type. The lengths, in metres, of 19 rods produced by machine A and 19 rods produced by machine B are shown in the following back-to-back stem-and-leaf diagram.

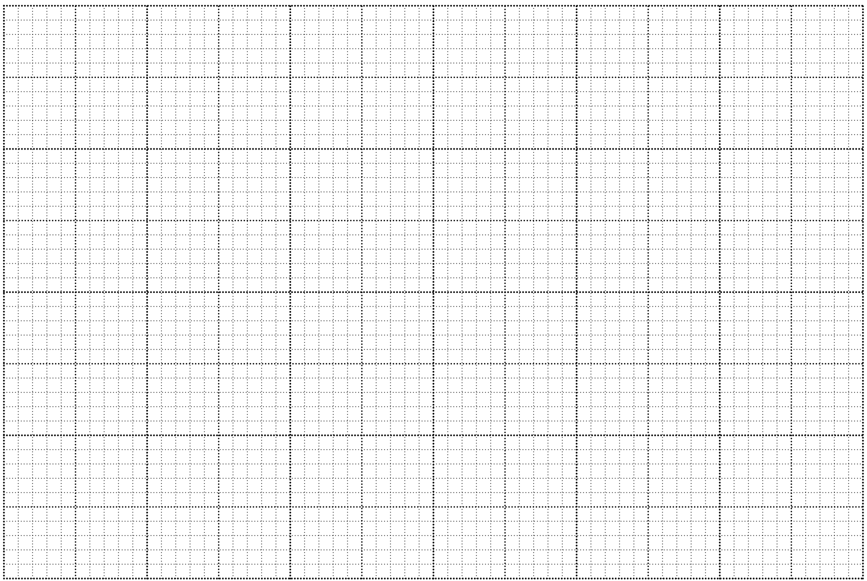
A		B
	21	1 2 4
7 6 3 0	22	2 4 5 5 6
8 7 4 3 1 1	23	0 2 6 8 9 9
5 5 5 3 2	24	3 3 4 6
4 3 1 0	25	6

Key: 7 | 22 | 4 means 0.227 m for machine A and 0.224 m for machine B.

(a) Find the median and the interquartile range for machine A. [3]

It is given that for machine B the median is 0.232 m, the lower quartile is 0.224 m and the upper quartile is 0.243 m.

(b) Draw box-and-whisker plots for A and B. [3]



(c) Hence make two comparisons between the lengths of the rods produced by machine A and those produced by machine B. [2]

52. [9709/s20/53/q6]

The annual salaries, in thousands of dollars, for 11 employees at each of two companies A and B are shown below.

Company A	30	32	35	41	41	42	47	49	52	53	64
Company B	26	47	30	52	41	38	35	42	49	31	42

(a) Represent the data by drawing a back-to-back stem-and-leaf diagram with company A on the left-hand side of the diagram. [4]

(b) Find the median and the interquartile range of the salaries of the employees in company A. [3]

A new employee joins company B. The mean salary of the 12 employees is now \$38 500.

(c) Find the salary of the new employee. [3]

53. [9709/w20/51/q6]

The times, t minutes, taken by 150 students to complete a particular challenge are summarised in the following cumulative frequency table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 40$	$t \leqslant 60$	$t \leqslant 100$
Cumulative frequency	12	48	106	134	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]

(b) 24% of the students take k minutes or longer to complete the challenge. Use your graph to estimate the value of k. [2]

(c) Calculate estimates of the mean and the standard deviation of the time taken to complete the challenge. [6]

54. [9709/w20/52/q5]

The following table gives the weekly snowfall, in centimetres, for 11 weeks in 2018 at two ski resorts, Dados and Linva.

Dados	6	8	12	15	10	36	42	28	10	22	16
Linva	2	11	15	16	0	32	36	40	10	12	9

(a) Represent the information in a back-to-back stem-and-leaf diagram. [4]

(b) Find the median and the interquartile range for the weekly snowfall in Dados. [3]

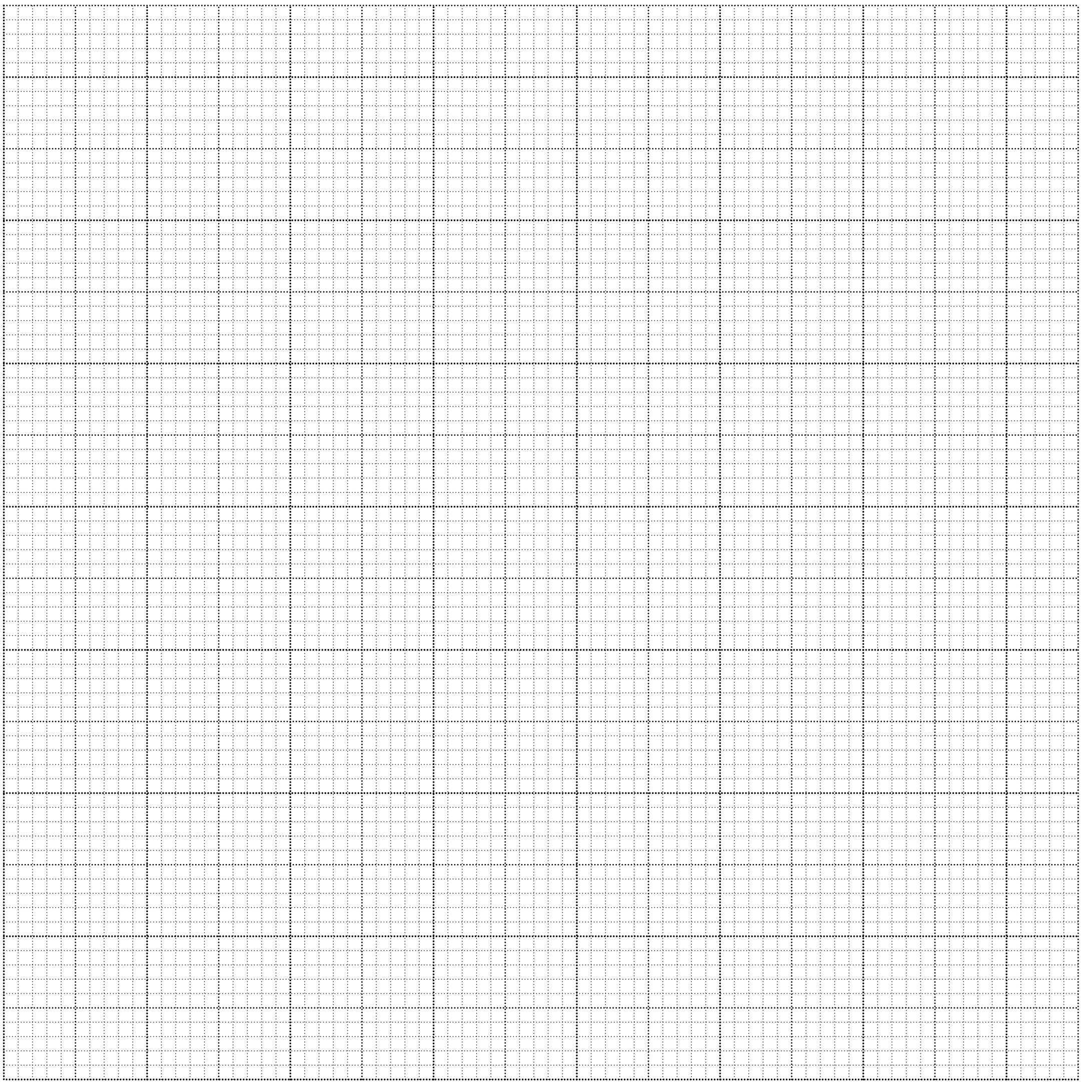
(c) The median, lower quartile and upper quartile of the weekly snowfall for Linva are 12, 9 and 32 cm respectively. Use this information and your answers to part **(b)** to compare the central tendency and the spread of the weekly snowfall in Dados and Linva. [2]

55. [9709/w20/53/q7]

A particular piece of music was played by 91 pianists and for each pianist, the number of incorrect notes was recorded. The results are summarised in the table.

Number of incorrect notes	1 – 5	6 – 10	11 – 20	21 – 40	41 – 70
Frequency	10	5	26	32	18

(a) Draw a histogram to represent this information. [5]



(b) State which class interval contains the lower quartile and which class interval contains the upper quartile.

Hence find the greatest possible value of the interquartile range. [2]

(c) Calculate an estimate for the mean number of incorrect notes. [3]

56. [9709/m19/62/q2]

For 40 values of the variable x, it is given that $\Sigma(x-c)^2 = 3099.2$, where c is a constant. The standard deviation of these values of x is 3.2.

(i) Find the value of $\Sigma(x-c)$. [3]

(ii) Given that $c = 50$, find the mean of these values of x. [1]

57. [9709/m19/62/q5]

The weights, in kg, of the 11 members of the Dolphins swimming team and the 11 members of the Sharks swimming team are shown below.

Dolphins	62	75	69	82	63	80	65	65	73	82	72
Sharks	68	84	59	70	71	64	77	80	66	74	72

(i) Draw a back-to-back stem-and-leaf diagram to represent this information, with Dolphins on the left-hand side of the diagram and Sharks on the right-hand side. [4]

(ii) Find the median and interquartile range for the Dolphins. [3]

58. [9709/s19/61/q1]

The times, t seconds, taken to swim 100 m were recorded for a group of 9 swimmers and were found to be as follows.

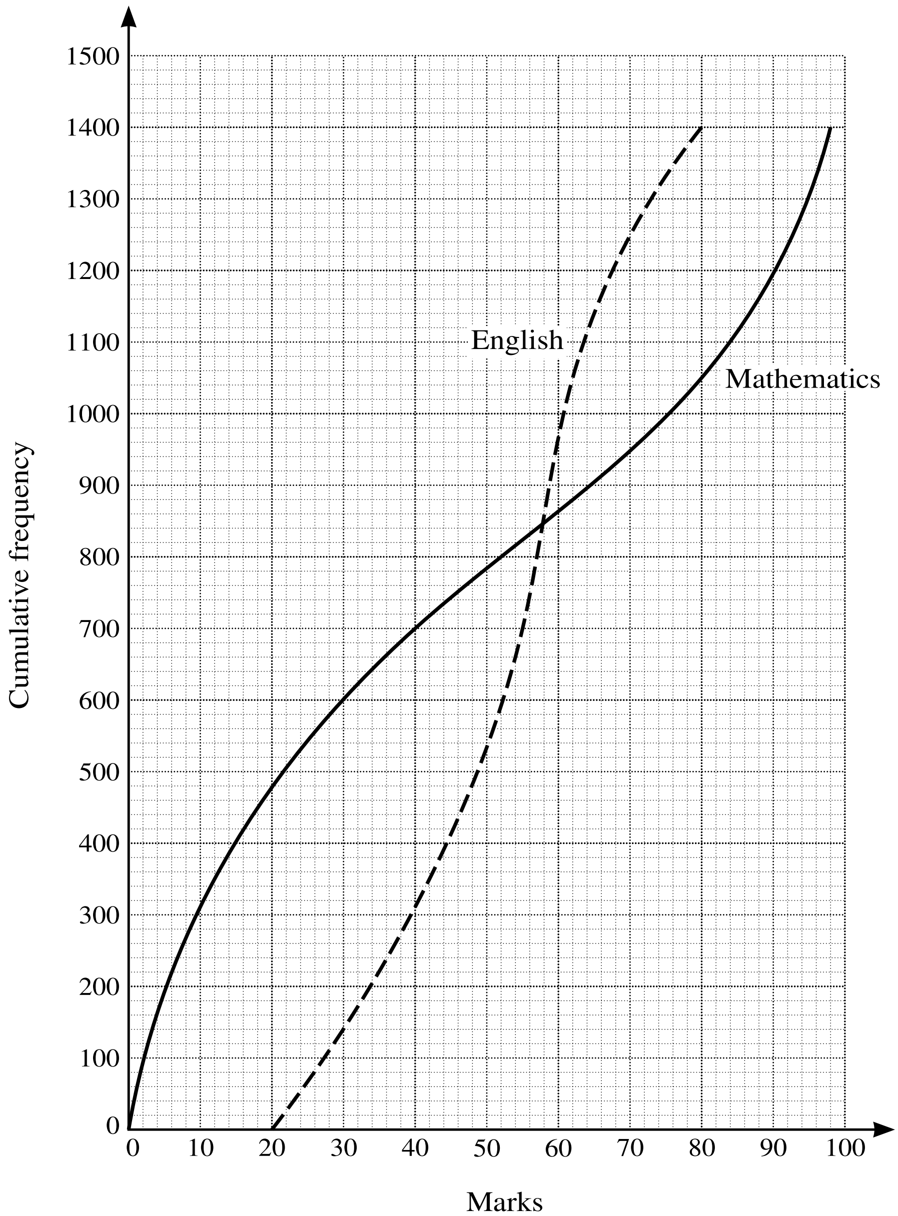
95	126	117	135	120	125	114	119	136

(i) Find the values of $\Sigma(t-120)$ and $\Sigma(t-120)^2$. [2]

(ii) Using your values found in part **(i)**, calculate the variance of t. [2]

59. [9709/s19/61/q4]

The Mathematics and English A-level marks of 1400 pupils all taking the same examinations are shown in the cumulative frequency graphs below. Both examinations are marked out of 100.



Use suitable data from these graphs to compare the central tendency and spread of the marks in Mathematics and English. [6]

60. [9709/s19/62/q6]

(i) Give one advantage and one disadvantage of using a box-and-whisker plot to represent a set of data. [2]

(ii) The times in minutes taken to run a marathon were recorded for a group of 13 marathon runners and were found to be as follows.

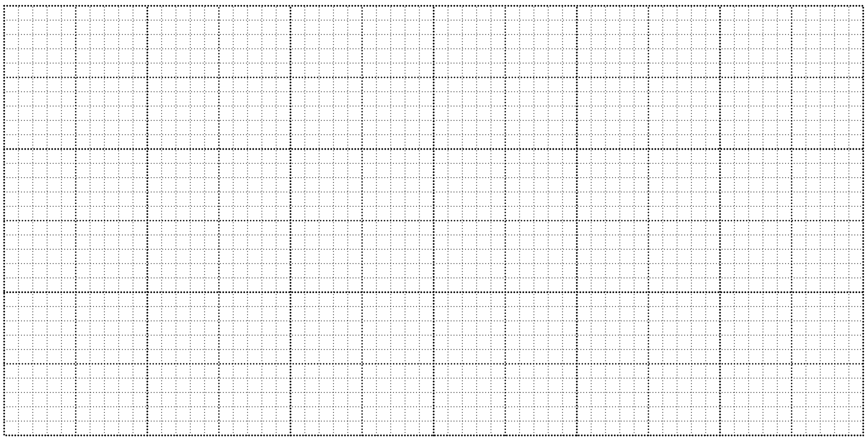
180	275	235	242	311	194	246	229	238	768	332	227	228

State which of the mean, mode or median is most suitable as a measure of central tendency for these times. Explain why the other measures are less suitable. [3]

(iii) Another group of 33 people ran the same marathon and their times in minutes were as follows.

190	203	215	246	249	253	255	254	258	260	261
263	267	269	274	276	280	288	283	287	294	300
307	318	327	331	336	345	351	353	360	368	375

(a) On the grid below, draw a box-and-whisker plot to illustrate the times for these 33 people. [4]



(b) Find the interquartile range of these times. [1]

61. [9709/s19/63/q7]

The times in minutes taken by 13 pupils at each of two schools in a cross-country race are recorded in the table below.

Thaters School	38	43	48	52	54	56	57	58	58	61	62	66	75
Whitefay Park School	45	47	53	56	56	61	64	66	69	73	75	78	83

(i) Draw a back-to-back stem-and-leaf diagram to illustrate these times with Thaters School on the left. [4]

(ii) Find the interquartile range of the times for pupils at Thaters School. [2]

The times taken by pupils at Whitefay Park School are denoted by x minutes.

(iii) Find the value of $\Sigma(x-60)^2$. [2]

(iv) It is given that $\Sigma(x-60) = 46$. Use this result, together with your answer to part **(iii)**, to find the variance of x. [2]

62. [9709/w19/61/q3]

The mean and standard deviation of 20 values of x are 60 and 4 respectively.

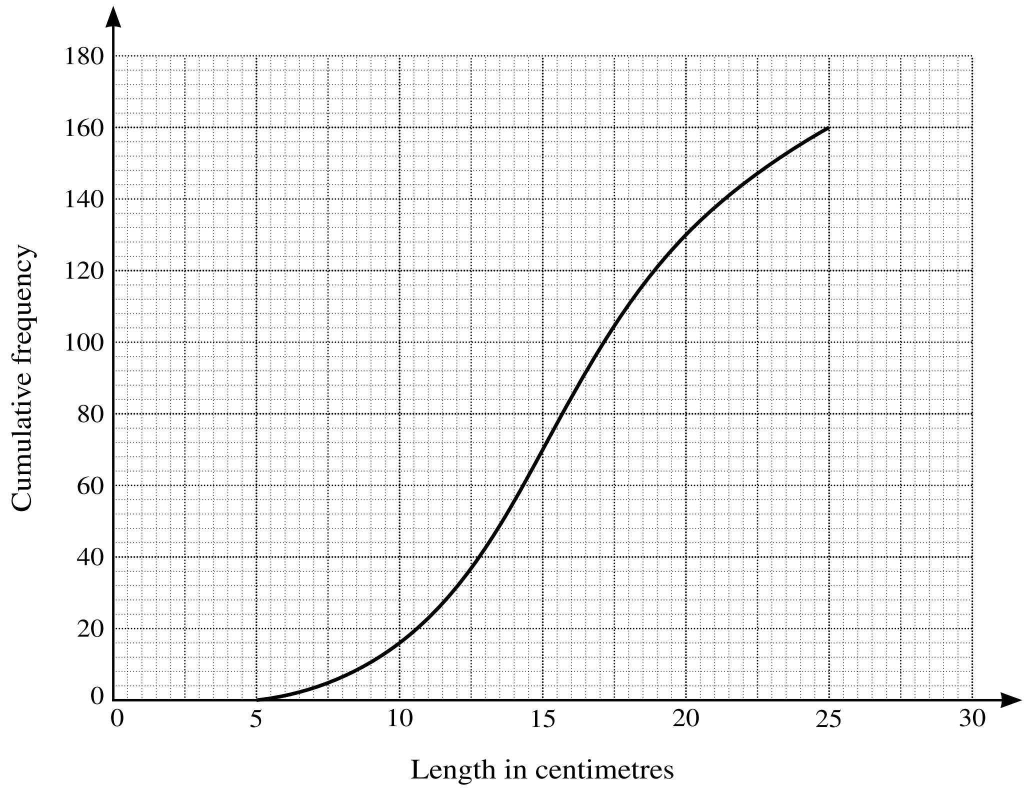
(i) Find the values of Σx and Σx^2. [3]

Another 10 values of x are such that their sum is 550 and the sum of their squares is 40 500.

(ii) Find the mean and standard deviation of all these 30 values of x. [4]

63. [9709/w19/61/q5]

Ransha measured the lengths, in centimetres, of 160 palm leaves. His results are illustrated in the cumulative frequency graph below.

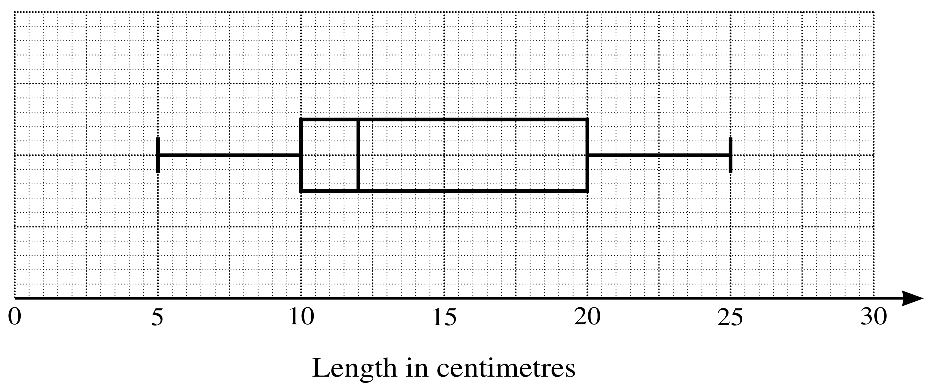


(i) Estimate how many leaves have a length between 14 and 24 centimetres. [1]

(ii) 10% of the leaves have a length of L centimetres or more. Estimate the value of L. [2]

(iii) Estimate the median and the interquartile range of the lengths. [3]

Sharim measured the lengths, in centimetres, of 160 palm leaves of a different type. He drew a box-and-whisker plot for the data, as shown on the grid below.



(iv) Compare the central tendency and the spread of the two sets of data. [2]

64. [9709/w19/62/q1]

Twelve tourists were asked to estimate the height, in metres, of a new building. Their estimates were as follows.

50 45 62 30 40 55 110 38 52 60 55 40

(i) Find the median and the interquartile range for the data. [3]

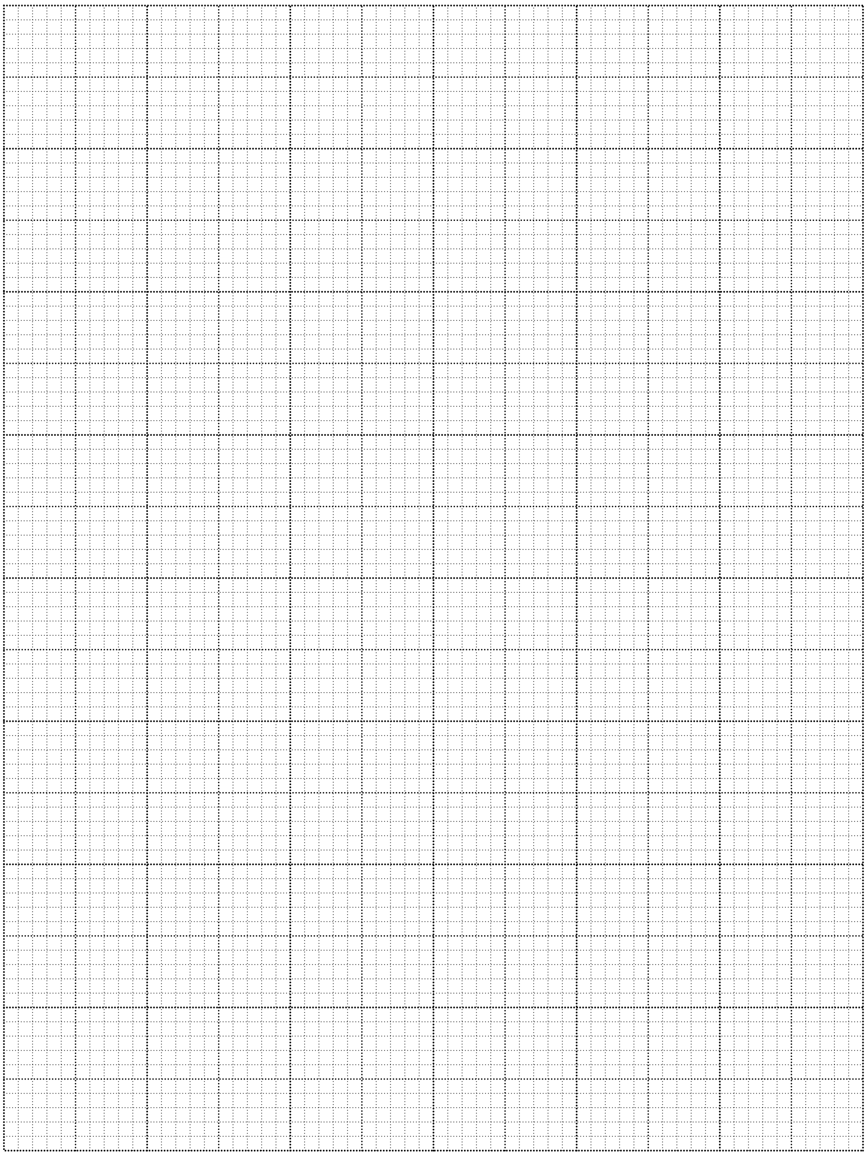
(ii) Give a disadvantage of using the mean as a measure of the central tendency in this case. [1]

65. [9709/w19/62/q3]

The speeds, in km h^{-1}, of 90 cars as they passed a certain marker on a road were recorded, correct to the nearest km h^{-1}. The results are summarised in the following table.

Speed (km h^{-1})	10 – 29	30 – 39	40 – 49	50 – 59	60 – 89
Frequency	10	24	30	14	12

(i) On the grid, draw a histogram to illustrate the data in the table. [4]



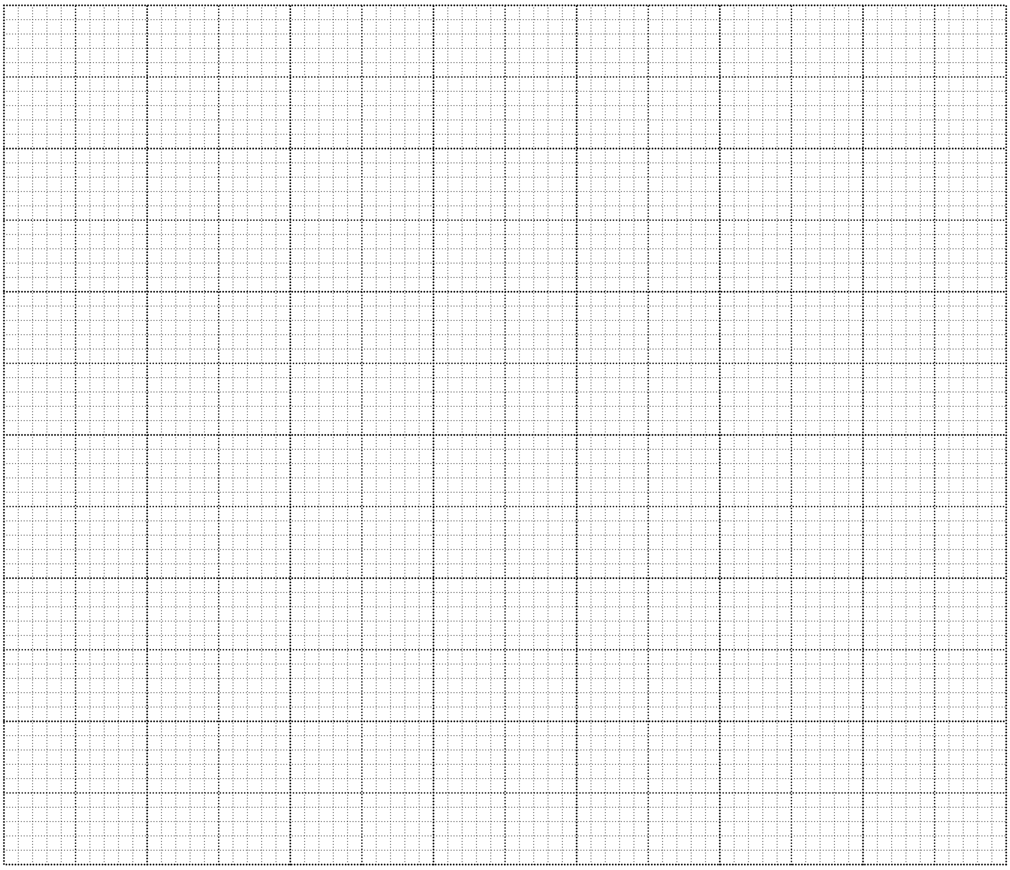
(ii) Calculate an estimate for the mean speed of these 90 cars as they pass the marker. [2]

66. [9709/w19/63/q5]

Last Saturday, 200 drivers entering a car park were asked the time, in minutes, that it had taken them to travel from home to the car park. The results are summarised in the following cumulative frequency table.

Time (t minutes)	$t \leqslant 10$	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 50$	$t \leqslant 70$	$t \leqslant 90$
Cumulative frequency	16	50	106	146	176	200

(i) On the grid, draw a cumulative frequency graph to illustrate the data. [2]



(ii) Use your graph to estimate the median of the data. [1]

(iii) For 80 of the drivers, the time taken was at least T minutes. Use your graph to estimate the value of T. [2]

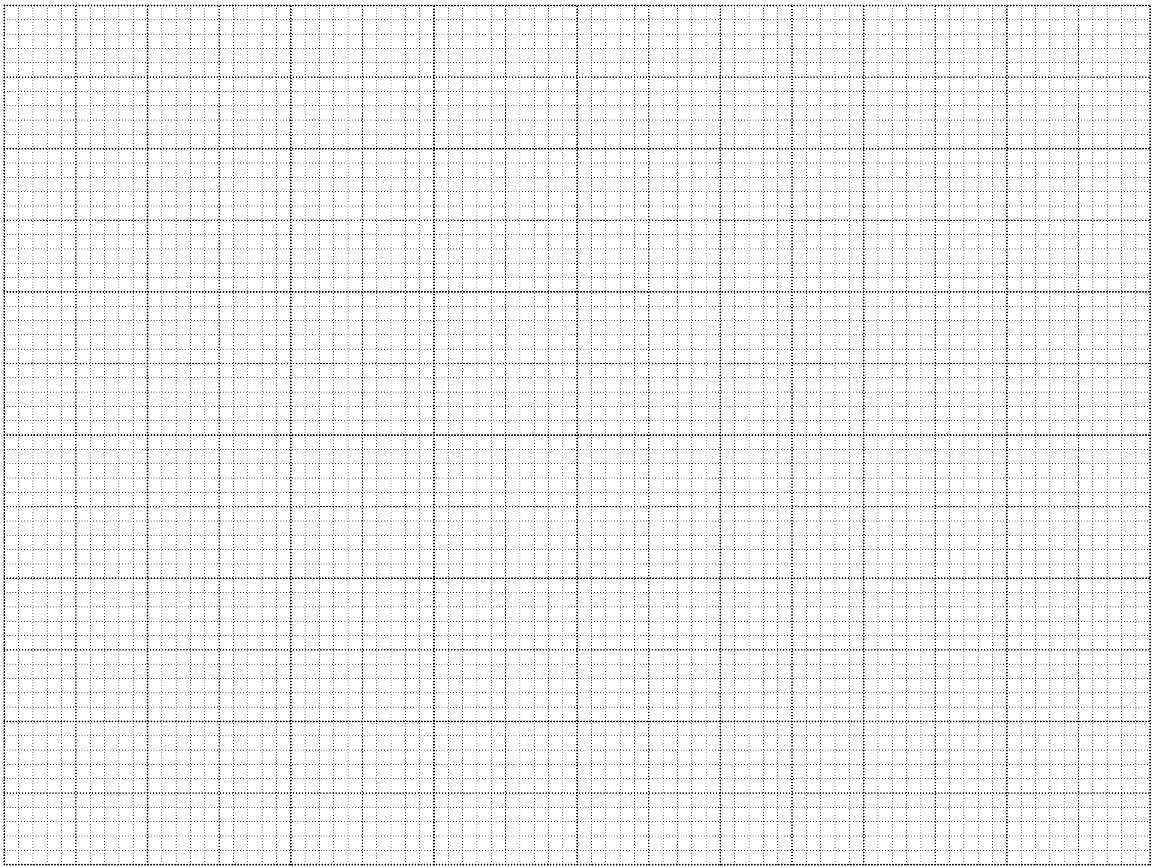
(iv) Calculate an estimate of the mean time taken by all 200 drivers to travel to the car park. [4]

67. [9709/m18/62/q1]

There are 900 students in a certain year-group. An identical puzzle is given to each student and the time taken, t minutes, to complete the puzzle is recorded. These times are summarised in the following frequency table.

Time taken, t minutes	$t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 6$	$6 < t \leqslant 8$	$8 < t \leqslant 10$	$10 < t \leqslant 14$
Frequency	120	180	200	160	110	80	50

On the grid, draw a cumulative frequency graph to represent the data. Use your graph to estimate the median time taken by these students to complete the puzzle. [4]



68. [9709/m18/62/q5]

A summary of n values of x gave the following information:

$$\Sigma(x-20)=136, \qquad \Sigma(x-20)^2=2888.$$

The mean of the n values of x is 24.25.

(i) Find the value of n. [2]

(ii) Find Σx^2. [4]

69. [9709/s18/61/q1]

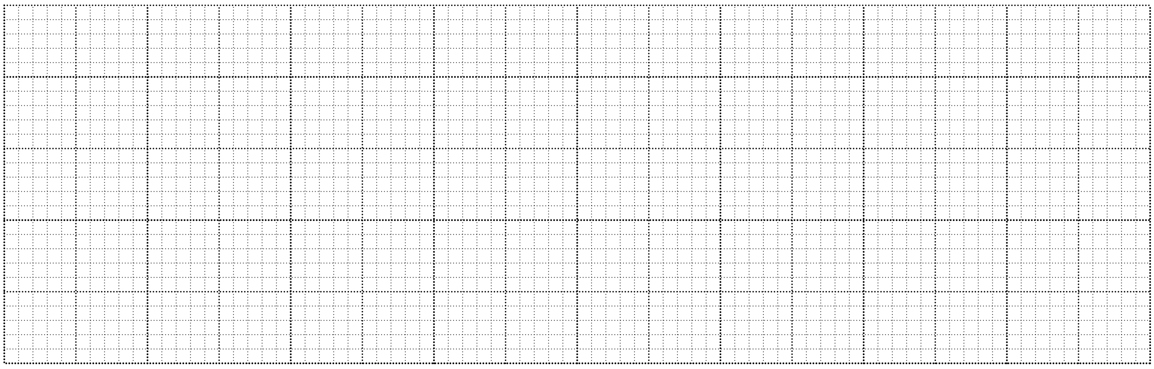
In a statistics lesson 12 people were asked to think of a number, x, between 1 and 20 inclusive. From the results Tom found that $\Sigma x = 186$ and that the standard deviation of x is 4.5. Assuming that Tom's calculations are correct, find the values of $\Sigma(x - 10)$ and $\Sigma(x - 10)^2$. [3]

70. [9709/s18/61/q2]

In a survey 55 students were asked to record, to the nearest kilometre, the total number of kilometres they travelled to school in a particular week. The results are shown below.

5	5	9	10	13	13	13	15	15	15	15
16	18	18	18	19	19	20	20	20	20	21
21	21	21	23	25	25	27	27	29	30	33
35	38	39	40	42	45	48	50	50	51	51
52	55	57	57	60	61	64	65	66	69	70

(i) On the grid, draw a box-and-whisker plot to illustrate the data. [5]



An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(ii) Show that there are no outliers. [2]

71. [9709/s18/62/q1]

Each of a group of 10 boys estimates the length of a piece of string. The estimates, in centimetres, are as follows.

37	40	45	38	36	38	42	38	40	39

(i) Find the mode. [1]

(ii) Find the median and the interquartile range. [3]

72. [9709/s18/62/q5]

The lengths, t minutes, of 242 phone calls made by a family over a period of 1 week are summarised in the frequency table below.

Length of phone call (t minutes)	$0 < t \leqslant 1$	$1 < t \leqslant 2$	$2 < t \leqslant 5$	$5 < t \leqslant 10$	$10 < t \leqslant 30$
Frequency	14	46	102	a	40

(i) Find the value of a. [1]

(ii) Calculate an estimate of the mean length of these phone calls. [2]

(iii) On the grid, draw a histogram to illustrate the data in the table. [4]

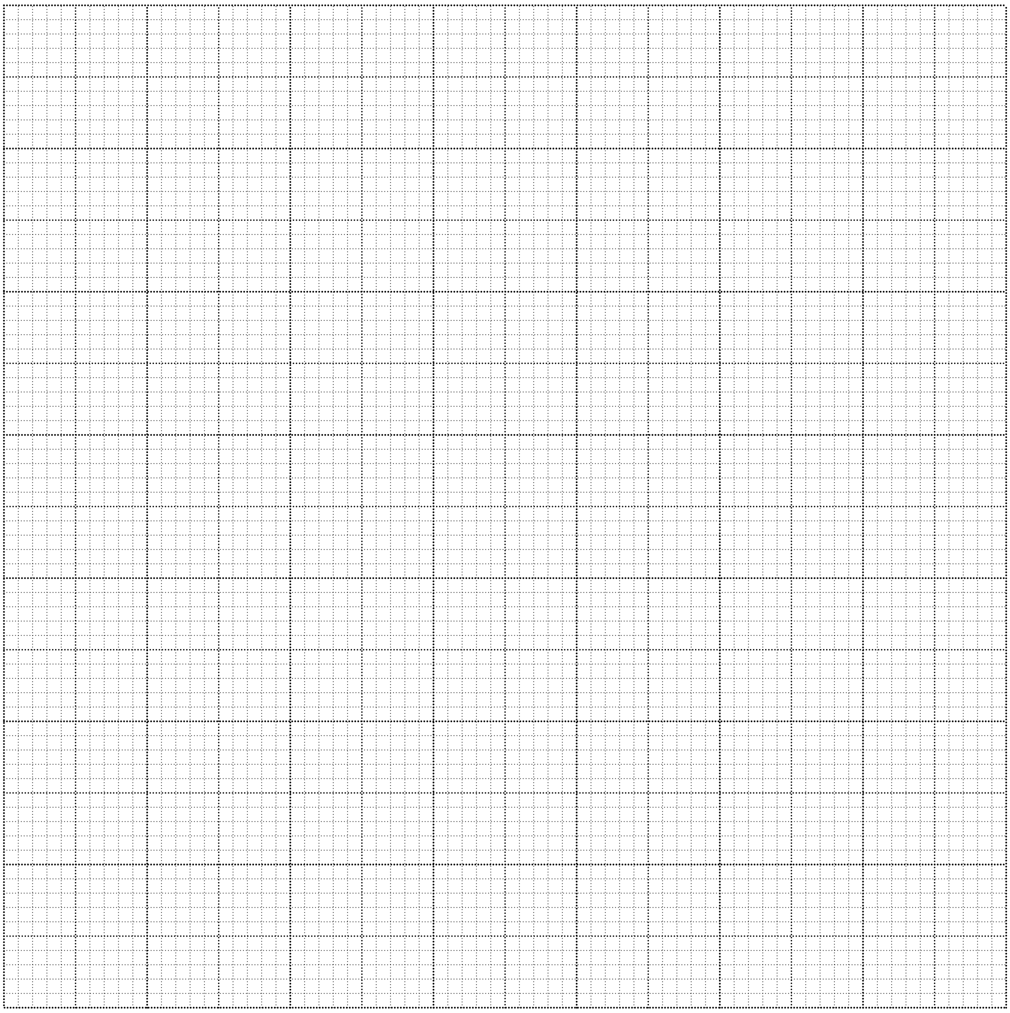
73. [9709/s18/63/q1]

The masses in kilograms of 50 children having a medical check-up were recorded correct to the nearest kilogram. The results are shown in the table.

Mass (kg)	10 – 14	15 – 19	20 – 24	25 – 34	35 – 59
Frequency	6	12	14	10	8

(i) Find which class interval contains the lower quartile. [1]

(ii) On the grid, draw a histogram to illustrate the data in the table. [4]



74. [9709/s18/63/q4]

Farfield Travel and Lacket Travel are two travel companies which arrange tours abroad. The numbers of holidays arranged in a certain week are recorded in the table below, together with the means and standard deviations of the prices.

	Number of holidays	Mean price (\$)	Standard deviation (\$)
Farfield Travel	30	1500	230
Lacket Travel	21	2400	160

(i) Calculate the mean price of all 51 holidays. [2]

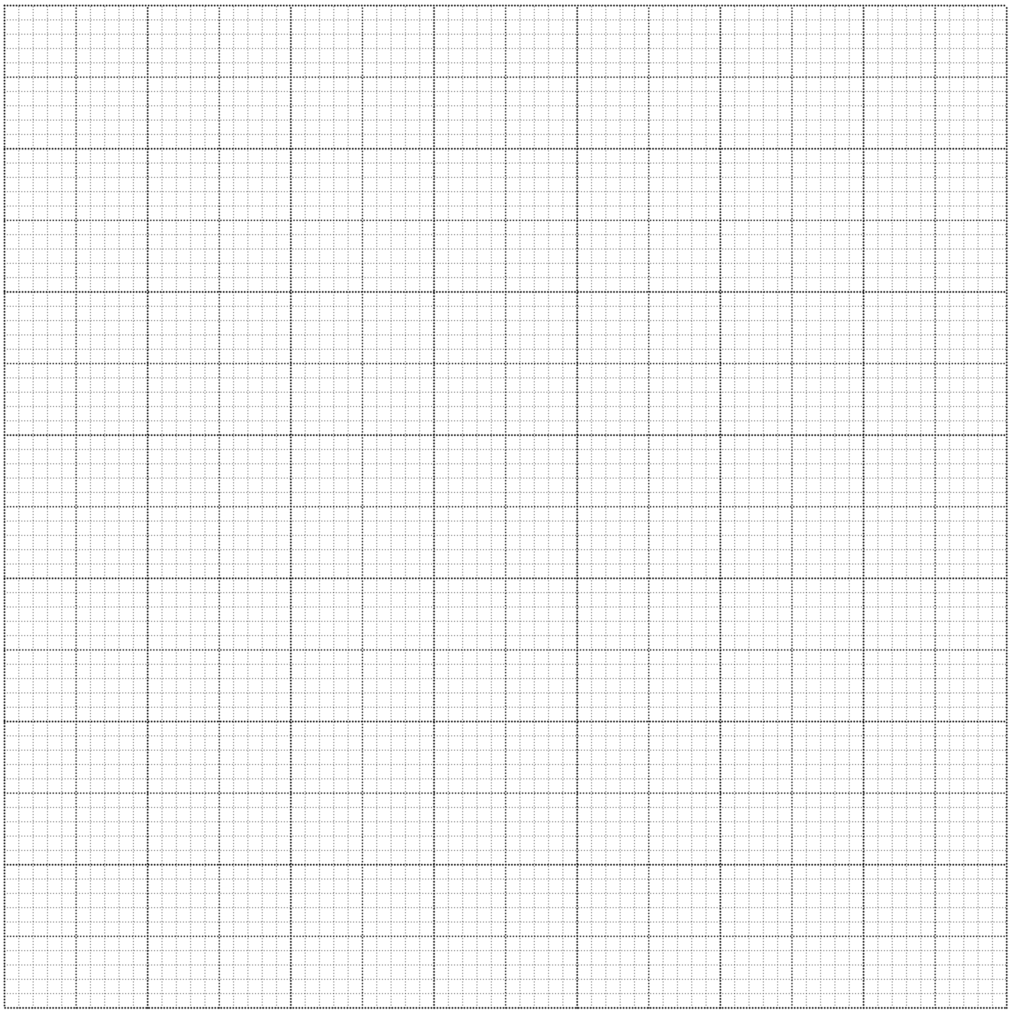
(ii) The prices of individual holidays with Farfield Travel are denoted by $\$x_F$ and the prices of individual holidays with Lacket Travel are denoted by $\$x_L$. By first finding Σx_F^2 and Σx_L^2, find the standard deviation of the prices of all 51 holidays. [5]

75. [9709/w18/61/q6]

The daily rainfall, x mm, in a certain village is recorded on 250 consecutive days. The results are summarised in the following cumulative frequency table.

Rainfall, x mm	$x \leqslant 20$	$x \leqslant 30$	$x \leqslant 40$	$x \leqslant 50$	$x \leqslant 70$	$x \leqslant 100$
Cumulative frequency	52	94	142	172	222	250

(i) On the grid, draw a cumulative frequency graph to illustrate the data. [2]



(ii) On 100 of the days, the rainfall was k mm or more. Use your graph to estimate the value of k. [2]

(iii) Calculate estimates of the mean and standard deviation of the daily rainfall in this village. [6]

76. [9709/w18/62/q2]

The following back-to-back stem-and-leaf diagram shows the reaction times in seconds in an experiment involving two groups of people, A and B.

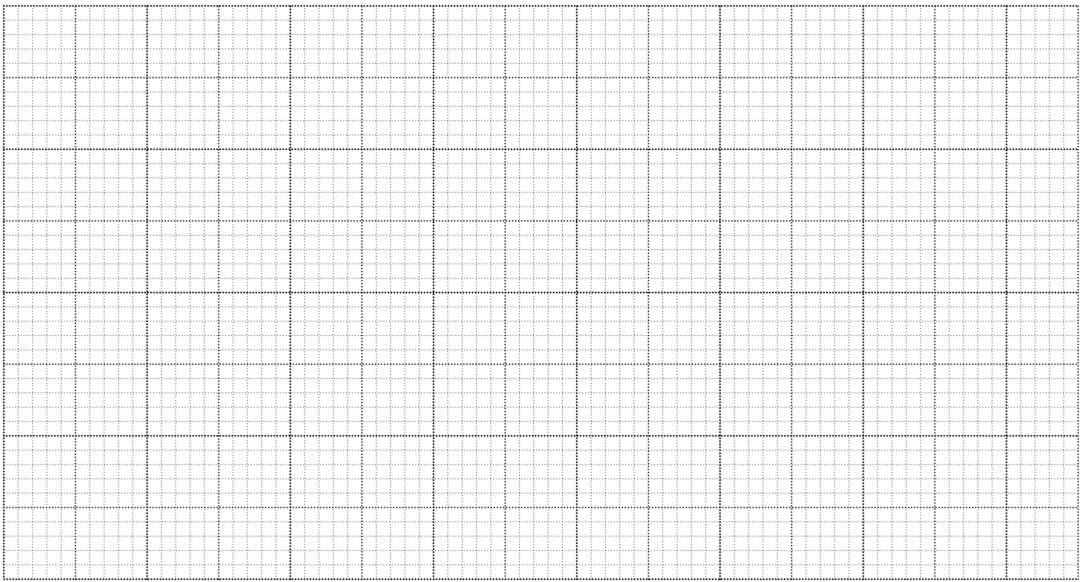
	A		B	
(4)	4 2 0 0	20	5 6 7	(3)
(5)	9 8 5 0 0	21	1 2 2 3 7 7	(6)
(8)	9 8 7 5 3 2 2 2	22	1 3 5 6 6 8 9	(7)
(6)	8 7 6 5 2 1	23	4 5 7 8 8 9 9 9	(8)
(3)	8 6 3	24	2 4 5 6 7 8 8	(7)
(1)	0	25	0 2 7 8	(4)

Key: 5 | 22 | 6 means a reaction time of 0.225 seconds for A and 0.226 seconds for B

(i) Find the median and the interquartile range for group A. [3]

The median value for group B is 0.235 seconds, the lower quartile is 0.217 seconds and the upper quartile is 0.245 seconds.

(ii) Draw box-and-whisker plots for groups A and B on the grid. [3]



77. [9709/w18/62/q5]

The Quivers Archery club has 12 Junior members and 20 Senior members. For the Junior members, the mean age is 15.5 years and the standard deviation of the ages is 1.2 years. The ages of the Senior members are summarised by $\Sigma y = 910$ and $\Sigma y^2 = 42\,850$, where y is the age of a Senior member in years.

(i) Find the mean age of all 32 members of the club. [2]

(ii) Find the standard deviation of the ages of all 32 members of the club. [4]

78. [9709/w18/63/q7]

The heights, in cm, of the 11 members of the Anvils athletics team and the 11 members of the Brecons swimming team are shown below.

Anvils	173	158	180	196	175	165	170	169	181	184	172
Brecons	166	170	171	172	172	178	181	182	183	183	192

(i) Draw a back-to-back stem-and-leaf diagram to represent this information, with Anvils on the left-hand side of the diagram and Brecons on the right-hand side. [4]

(ii) Find the median and the interquartile range for the heights of the Anvils. [3]

The heights of the 11 members of the Anvils are denoted by x cm. It is given that $\Sigma x = 1923$ and $\Sigma x^2 = 337\,221$. The Anvils are joined by 3 new members whose heights are 166 cm, 172 cm and 182 cm.

(iii) Find the standard deviation of the heights of all 14 members of the Anvils. [4]

79. [9709/m17/62/q1]

Twelve values of x are shown below.

1761.6	1758.5	1762.3	1761.4	1759.4	1759.1
1762.5	1761.9	1762.4	1761.9	1762.8	1761.0

Find the mean and standard deviation of $(x - 1760)$. Hence find the mean and standard deviation of x. [4]

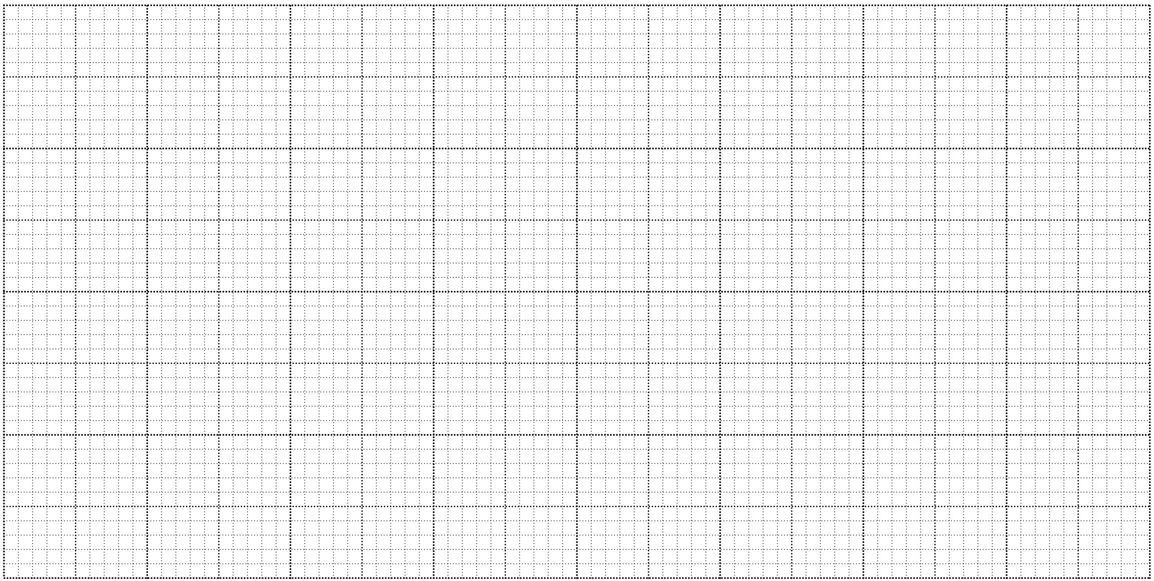
80. [9709/m17/62/q4]

The weights in kilograms of packets of cereal were noted correct to 4 significant figures. The following stem-and-leaf diagram shows the data.

747	3	(1)
748	1 2 5 7 7 9	(6)
749	0 2 2 2 3 5 5 5 6 7 8 9	(12)
750	1 1 2 2 2 3 4 4 5 6 7 7 8 8 9	(15)
751	0 0 2 3 3 4 4 4 5 5 7 7 9	(13)
752	0 0 0 1 1 2 2 3 4 4 4	(11)
753	2	(1)

Key: 748 | 5 represents 0.7485 kg.

(i) On the grid, draw a box-and-whisker plot to represent the data. [5]



(ii) Name a distribution that might be a suitable model for the weights of this type of cereal packet. Justify your answer. [2]

81. [9709/s17/61/q1]

Kadijat noted the weights, x grams, of 30 chocolate buns. Her results are summarised by

$$\Sigma(x-k) = 315, \qquad \Sigma(x-k)^2 = 4022,$$

where k is a constant. The mean weight of the buns is 50.5 grams.

(i) Find the value of k. [2]

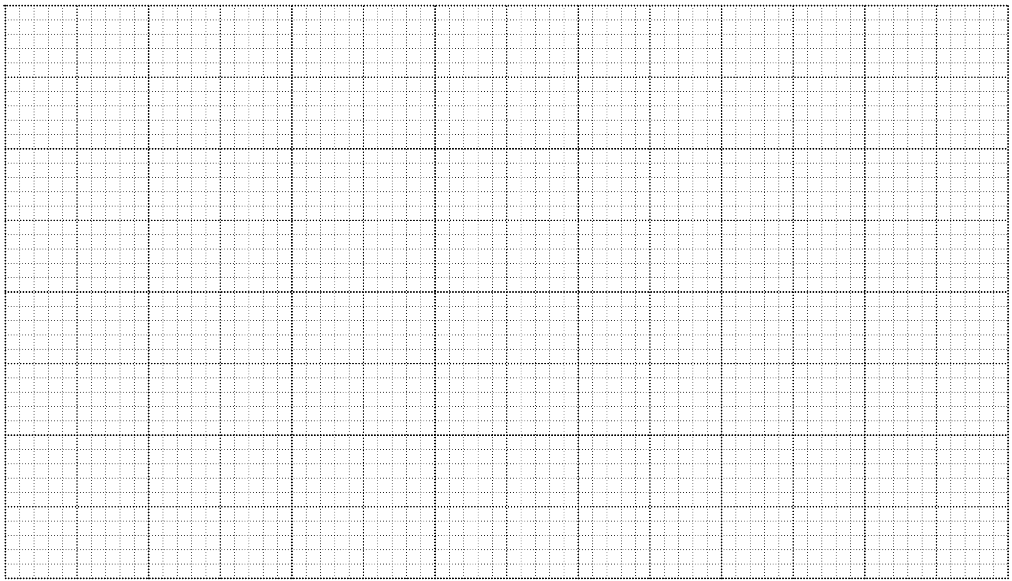
(ii) Find the standard deviation of x. [2]

82. [9709/s17/61/q4]

The times taken, t seconds, by 1140 people to solve a puzzle are summarised in the table.

Time (t seconds)	$0 \leqslant t < 20$	$20 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 100$	$100 \leqslant t < 140$
Number of people	320	280	220	220	100

(i) On the grid, draw a histogram to illustrate this information. [4]



(ii) Calculate an estimate of the mean of t. [2]

83. [9709/s17/62/q1]

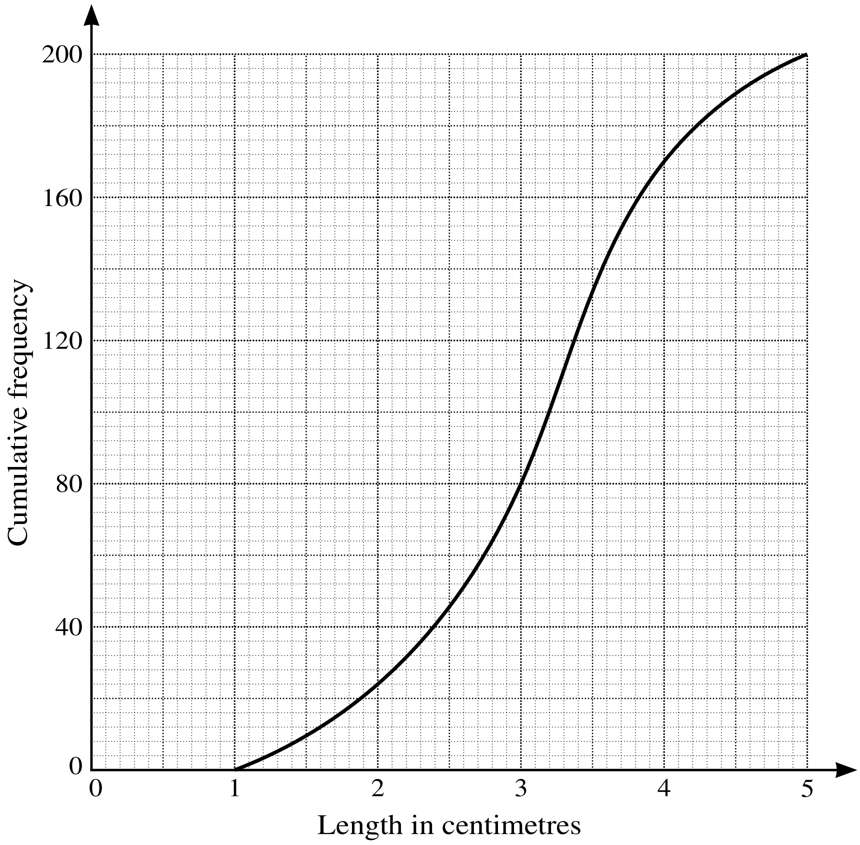
Rani and Diksha go shopping for clothes.

(i) Rani buys 4 identical vests, 3 identical sweaters and 1 coat. Each vest costs \$5.50 and the coat costs \$90. The mean cost of Rani's 8 items is \$29. Find the cost of a sweater. [3]

(ii) Diksha buys 1 hat and 4 identical shirts. The mean cost of Diksha's 5 items is \$26 and the standard deviation is \$0. Explain how you can tell that Diksha spends \$104 on shirts. [2]

84. [9709/s17/62/q2]

Anabel measured the lengths, in centimetres, of 200 caterpillars. Her results are illustrated in the cumulative frequency graph below.



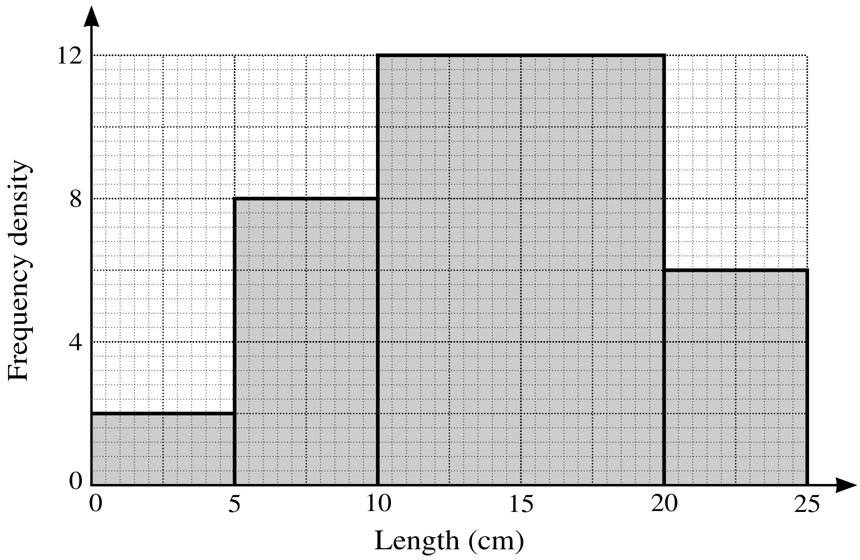
(i) Estimate the median and the interquartile range of the lengths. [3]

(ii) Estimate how many caterpillars had a length of between 2 and 3.5 cm. [1]

(iii) 6% of caterpillars were of length l centimetres or more. Estimate l. [2]

85. [9709/s17/63/q7]

The following histogram represents the lengths of worms in a garden.



(i) Calculate the frequencies represented by each of the four histogram columns. [2]

(ii) On the grid on the next page, draw a cumulative frequency graph to represent the lengths of worms in the garden. [4]

(iii) Use your graph to estimate the median and interquartile range of the lengths of worms in the garden. [3]

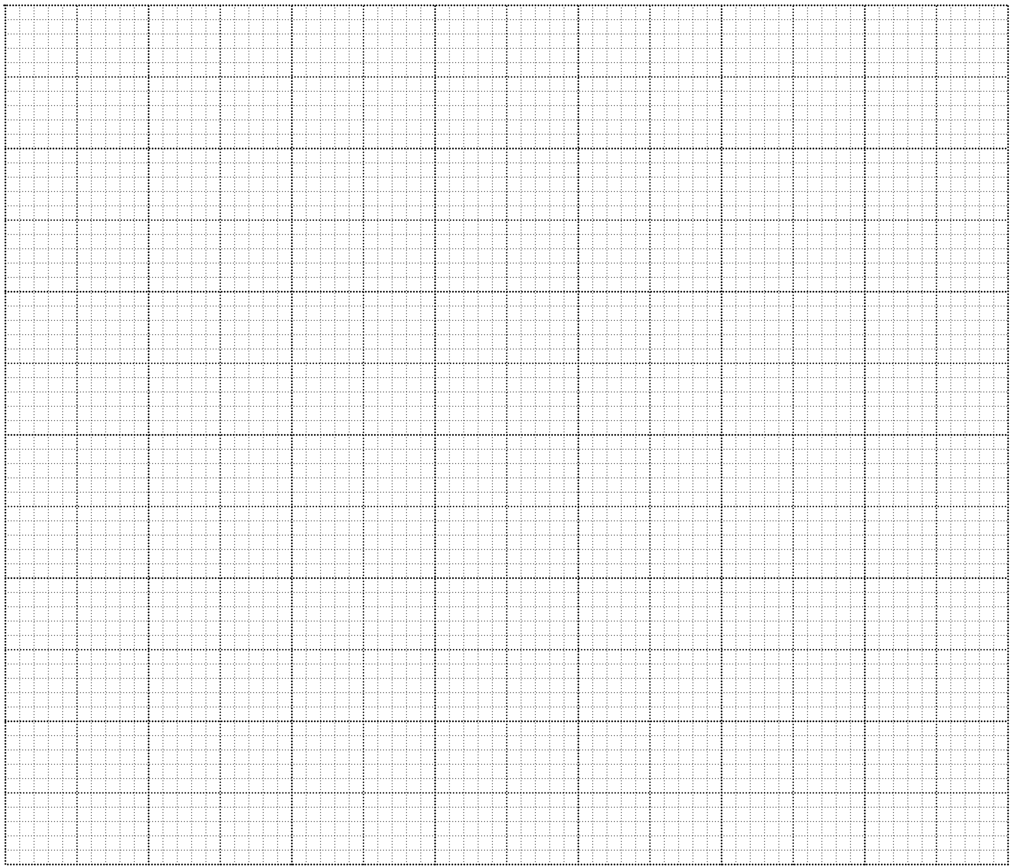
(iv) Calculate an estimate of the mean length of worms in the garden. [2]

86. [9709/w17/61/q2]

The time taken by a car to accelerate from 0 to 30 metres per second was measured correct to the nearest second. The results from 48 cars are summarised in the following table.

Time (seconds)	3 – 5	6 – 8	9 – 11	12 – 16	17 – 25
Frequency	10	15	17	4	2

(i) On the grid, draw a cumulative frequency graph to represent this information. [3]



(ii) 35 of these cars accelerated from 0 to 30 metres per second in a time more than t seconds. Estimate the value of t. [2]

87. [9709/w17/61/q4]

The ages of a group of 12 people at an Art class have mean 48.7 years and standard deviation 7.65 years. The ages of a group of 7 people at another Art class have mean 38.1 years and standard deviation 4.2 years.

(i) Find the mean age of all 19 people. [2]

(ii) The individual ages in years of people in the first Art class are denoted by x and those in the second Art class by y. By first finding Σx^2 and Σy^2, find the standard deviation of the ages of all 19 people. [4]

88. [9709/w17/62/q1]

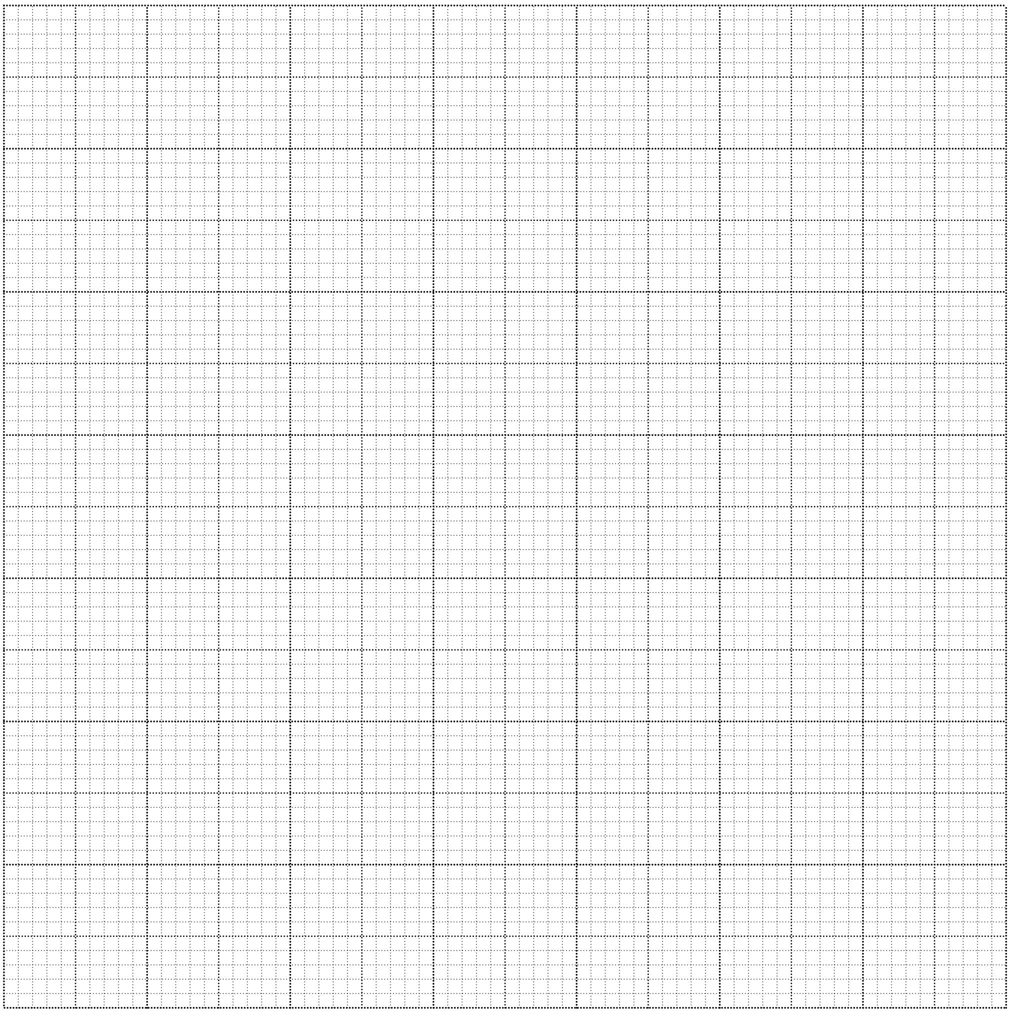
Andy counts the number of emails, x, he receives each day and notes that, over a period of n days, $\Sigma(x - 10) = 27$ and the mean number of emails is 11.5. Find the value of n. [3]

89. [9709/w17/62/q2]

The circumferences, c cm, of some trees in a wood were measured. The results are summarised in the table.

Circumference (c cm)	$40 < c \leqslant 50$	$50 < c \leqslant 80$	$80 < c \leqslant 100$	$100 < c \leqslant 120$
Frequency	14	48	70	8

(i) On the grid, draw a cumulative frequency graph to represent the information. [3]



(ii) Estimate the percentage of trees which have a circumference larger than 75 cm. [2]

90. [9709/w17/63/q2]

Tien measured the arm lengths, x cm, of 20 people in his class. He found that $\Sigma x = 1218$ and the standard deviation of x was 4.2. Calculate $\Sigma(x - 45)$ and $\Sigma(x - 45)^2$. [3]

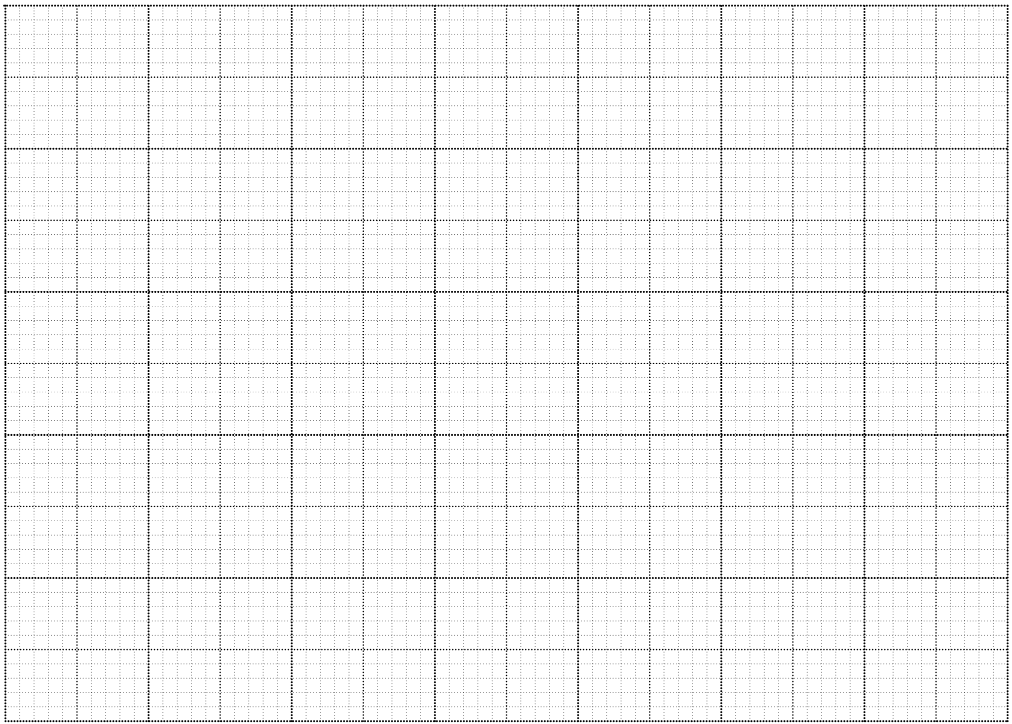
91. [9709/w17/63/q5]

The number of Olympic medals won in the 2012 Olympic Games by the top 27 countries is shown below.

104	88	82	65	44	38	35	34	28
28	18	18	17	17	14	13	13	12
12	10	10	10	9	6	5	2	2

(i) Draw a stem-and-leaf diagram to illustrate the data. [4]

(ii) Find the median and quartiles and draw a box-and-whisker plot on the grid. [5]



92. [9709/m16/62/q1]

For 10 values of x the mean is 86.2 and $\Sigma(x - a) = 362$. Find the value of

(i) Σx, [1]

(ii) the constant a. [2]

93. [9709/m16/62/q4]

A survey was made of the journey times of 63 people who cycle to work in a certain town. The results are summarised in the following cumulative frequency table.

Journey time (minutes)	$\leqslant 10$	$\leqslant 25$	$\leqslant 45$	$\leqslant 60$	$\leqslant 80$
Cumulative frequency	0	18	50	59	63

(i) State how many journey times were between 25 and 45 minutes. [1]

(ii) Draw a histogram on graph paper to represent the data. [4]

(iii) Calculate an estimate of the mean journey time. [2]

94. [9709/s16/61/q7]

The amounts spent by 160 shoppers at a supermarket are summarised in the following table.

Amount spent ($\$x$)	$0 < x \leqslant 30$	$30 < x \leqslant 50$	$50 < x \leqslant 70$	$70 < x \leqslant 90$	$90 < x \leqslant 140$
Number of shoppers	16	40	48	26	30

(i) Draw a cumulative frequency graph of this distribution. [4]

(ii) Estimate the median and the interquartile range of the amount spent. [3]

(iii) Estimate the number of shoppers who spent more than $115. [2]

(iv) Calculate an estimate of the mean amount spent. [2]

95. [9709/s16/62/q5]

The following are the maximum daily wind speeds in kilometres per hour for the first two weeks in April for two towns, Bronlea and Rogate.

Bronlea	21	45	6	33	27	3	32	14	28	24	13	17	25	22
Rogate	7	5	4	15	23	7	11	13	26	18	23	16	10	34

(i) Draw a back-to-back stem-and-leaf diagram to represent this information. [5]

(ii) Write down the median of the maximum wind speeds for Bronlea and find the interquartile range for Rogate. [3]

(iii) Use your diagram to make one comparison between the maximum wind speeds in the two towns. [1]

96. [9709/s16/63/q2]

A group of children played a computer game which measured their time in seconds to perform a certain task. A summary of the times taken by girls and boys in the group is shown below.

	Minimum	Lower quartile	Median	Upper quartile	Maximum
Girls	5	5.5	7	9	13
Boys	4	6	8.5	11	16

(i) On graph paper, draw two box-and-whisker plots in a single diagram to illustrate the times taken by girls and boys to perform this task. [3]

(ii) State two comparisons of the times taken by girls and boys. [2]

97. [9709/s16/63/q4]

The monthly rental prices, $\$x$, for 9 apartments in a certain city are listed and are summarised as follows.

$$\Sigma(x-c) = 1845 \qquad \Sigma(x-c)^2 = 477\,450$$

The mean monthly rental price is \$2205.

(i) Find the value of the constant c. [2]

(ii) Find the variance of these values of x. [2]

(iii) Another apartment is added to the list. The mean monthly rental price is now \$2120.50. Find the rental price of this additional apartment. [2]

98. [9709/w16/61/q7]

The masses, in grams, of components made in factory *A* and components made in factory *B* are shown below.

Factory *A*	0.049	0.050	0.053	0.054	0.057	0.058	0.058
	0.059	0.061	0.061	0.061	0.063	0.065	
Factory *B*	0.031	0.056	0.049	0.044	0.038	0.048	0.051
	0.064	0.035	0.042	0.047	0.054	0.058	

(i) Draw a back-to-back stem-and-leaf diagram to represent the masses of components made in the two factories. [5]

(ii) Find the median and the interquartile range for the masses of components made in factory *B*. [3]

(iii) Make two comparisons between the masses of components made in factory *A* and the masses of those made in factory *B*. [2]

99. [9709/w16/62/q5]

The number of people a football stadium can hold is called the 'capacity'. The capacities of 130 football stadiums in the UK, to the nearest thousand, are summarised in the table.

Capacity	3000–7000	8000–12 000	13 000–22 000	23 000–42 000	43 000–82 000
Number of stadiums	40	30	18	34	8

(i) On graph paper, draw a histogram to represent this information. Use a scale of 2 cm for a capacity of 10 000 on the horizontal axis. [5]

(ii) Calculate an estimate of the mean capacity of these 130 stadiums. [2]

(iii) Find which class in the table contains the median and which contains the lower quartile. [2]

100. [9709/w16/63/q5]

The tables summarise the heights, h cm, of 60 girls and 60 boys.

Height of girls (cm)	$140 < h \leqslant 150$	$150 < h \leqslant 160$	$160 < h \leqslant 170$	$170 < h \leqslant 180$	$180 < h \leqslant 190$
Frequency	12	21	17	10	0

Height of boys (cm)	$140 < h \leqslant 150$	$150 < h \leqslant 160$	$160 < h \leqslant 170$	$170 < h \leqslant 180$	$180 < h \leqslant 190$
Frequency	0	20	23	12	5

(i) On graph paper, using the same set of axes, draw two cumulative frequency graphs to illustrate the data. [4]

(ii) On a school trip the students have to enter a cave which is 165 cm high. Use your graph to estimate the percentage of the girls who will be unable to stand upright. [3]

(iii) The students are asked to compare the heights of the girls and the boys. State one advantage of using a pair of box-and-whisker plots instead of the cumulative frequency graphs to do this. [1]

101. [9709/s15/61/q2]

The table summarises the lengths in centimetres of 104 dragonflies.

Length (cm)	2.0 – 3.5	3.5 – 4.5	4.5 – 5.5	5.5 – 7.0	7.0 – 9.0
Frequency	8	25	28	31	12

(i) State which class contains the upper quartile. [1]

(ii) Draw a histogram, on graph paper, to represent the data. [4]

102. [9709/s15/61/q5]

The table shows the mean and standard deviation of the weights of some turkeys and geese.

	Number of birds	Mean (kg)	Standard deviation (kg)
Turkeys	9	7.1	1.45
Geese	18	5.2	0.96

(i) Find the mean weight of the 27 birds. [2]

(ii) The weights of individual turkeys are denoted by x_t kg and the weights of individual geese by x_g kg. By first finding Σx_t^2 and Σx_g^2, find the standard deviation of the weights of all 27 birds. [5]

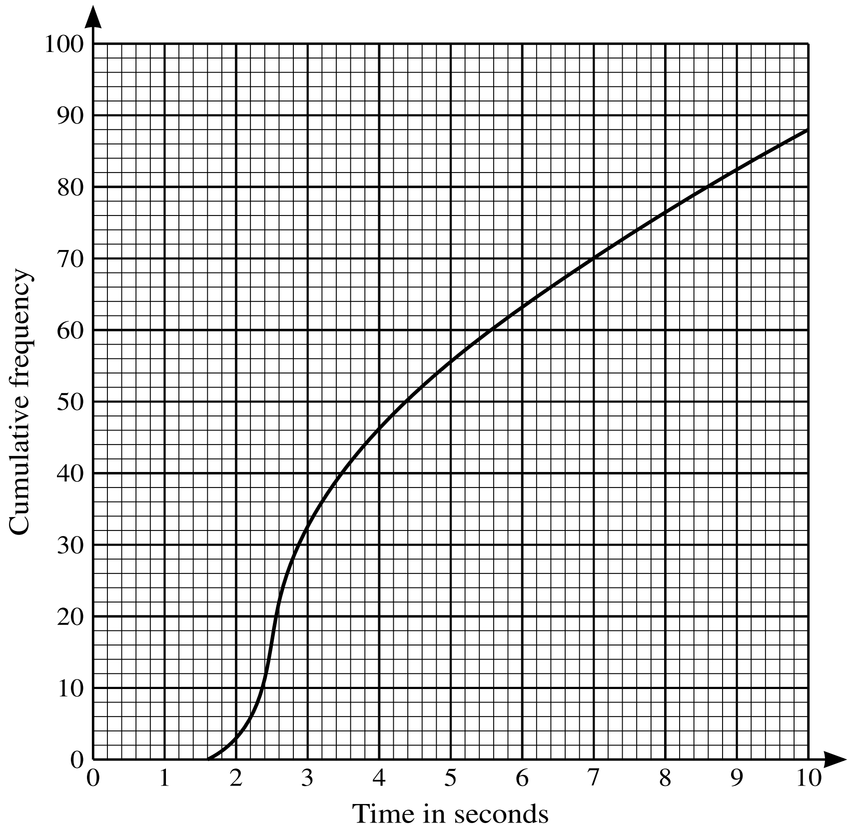
103. [9709/s15/62/q2]

120 people were asked to read an article in a newspaper. The times taken, to the nearest second, by the people to read the article are summarised in the following table.

Time (seconds)	1 – 25	26 – 35	36 – 45	46 – 55	56 – 90
Number of people	4	24	38	34	20

Calculate estimates of the mean and standard deviation of the reading times. [5]

104. [9709/s15/62/q3]



In an open-plan office there are 88 computers. The times taken by these 88 computers to access a particular web page are represented in the cumulative frequency diagram.

(i) On graph paper draw a box-and-whisker plot to summarise this information. [4]

An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(ii) Show that there are no outliers. [2]

105. [9709/s15/63/q6]

Seventy samples of fertiliser were collected and the nitrogen content was measured for each sample. The cumulative frequency distribution is shown in the table below.

Nitrogen content	$\leqslant 3.5$	$\leqslant 3.8$	$\leqslant 4.0$	$\leqslant 4.2$	$\leqslant 4.5$	$\leqslant 4.8$
Cumulative frequency	0	6	18	41	62	70

(i) On graph paper draw a cumulative frequency graph to represent the data. [3]

(ii) Estimate the percentage of samples with a nitrogen content greater than 4.4. [2]

(iii) Estimate the median. [1]

(iv) Construct the frequency table for these results and draw a histogram on graph paper. [5]

106. [9709/w15/61/q3]

Robert has a part-time job delivering newspapers. On a number of days he noted the time, correct to the nearest minute, that it took him to do his job. Robert used his results to draw up the following table; two of the values in the table are denoted by a and b.

Time (t minutes)	60 – 62	63 – 64	65 – 67	68 – 71
Frequency (number of days)	3	9	6	b
Frequency density	1	a	2	1.5

(i) Find the values of a and b. [3]

(ii) On graph paper, draw a histogram to represent Robert's times. [3]

107. [9709/w15/61/q4]

(a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x-80) = -147 \qquad \Sigma(x-80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

(b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute. [3]

108. [9709/w15/62/q1]

For n values of the variable x, it is given that $\Sigma(x - 100) = 216$ and $\Sigma x = 2416$. Find the value of n. [3]

109. [9709/w15/62/q5]

The weights, in kilograms, of the 15 rugby players in each of two teams, A and B, are shown below.

Team A	97	98	104	84	100	109	115	99	122	82	116	96	84	107	91
Team B	75	79	94	101	96	77	111	108	83	84	86	115	82	113	95

(i) Represent the data by drawing a back-to-back stem-and-leaf diagram with team A on the left-hand side of the diagram and team B on the right-hand side. [4]

(ii) Find the interquartile range of the weights of the players in team A. [2]

(iii) A new player joins team B as a substitute. The mean weight of the 16 players in team B is now 93.9 kg. Find the weight of the new player. [3]

110. [9709/w15/63/q1]

The time taken, t hours, to deliver letters on a particular route each day is measured on 250 working days. The mean time taken is 2.8 hours. Given that $\Sigma(t - 2.5)^2 = 96.1$, find the standard deviation of the times taken. [3]

111. [9709/w15/63/q6]

The heights to the nearest metre of 134 office buildings in a certain city are summarised in the table below.

Height (m)	21 – 40	41 – 45	46 – 50	51 – 60	61 – 80
Frequency	18	15	21	52	28

(i) Draw a histogram on graph paper to illustrate the data. [4]

(ii) Calculate estimates of the mean and standard deviation of these heights. [5]

Chapter 2

Permutations and combinations

1. [9709/m25/52/q6]

Alissa has 10 different books from the series Squares and Circles. The books look similar except for their colour. There are 3 blue books, 2 red books, 2 yellow books, 1 orange book, 1 purple book and 1 green book.

Alissa places the books in a row on her shelf. She is only interested in the arrangement of the colours.

(a) How many different colour arrangements are there of the 10 books? [1]

(b) How many different colour arrangements are there of the 10 books in which the 3 blue books are together, but the 2 yellow books are **not** next to each other? [2]

(c) How many different colour arrangements are there of the 10 books with exactly 4 books between the 2 yellow books? [3]

Alissa selects 4 books from her 10 different books from the series Squares and Circles.

(d) Find the number of different selections if the 4 books include at least 1 red book, at most 1 blue book and exactly 1 yellow book. [4]

2. [9709/s25/51/q5]

In a group of 20 musicians, there are 9 guitarists, 6 pianists and 5 drummers.

6 musicians are selected from these 20 to perform at a concert.

(a) Find the number of different ways in which the 6 musicians can be selected if there must be at least 3 guitarists, at most 2 pianists and exactly 1 drummer. [4]

Three bands will be selected from the original group of 20 musicians. Each band will consist of 3 guitarists, 1 pianist and 1 drummer. No musician can be in more than one band. The first band selected will play at a concert in France, the second band selected will play in Italy and the third band selected will play in Spain.

(b) Find the number of different ways in which these three bands can be selected. [3]

3. [9709/s25/52/q6]

(a) Find the number of different ways in which the 10 letters in the word AMALGAMATE can be arranged so that there is an M at the beginning, an M at the end and no As are together. [3]

(b) Find the number of different ways in which the 10 letters in the word AMALGAMATE can be arranged with exactly 3 letters between the two Ms. [3]

Five letters are selected from the 10 letters in the word AMALGAMATE.

(c) Find the number of different selections in which the five letters include at least one M and at least two As. [3]

4. [9709/s25/53/q7]

A set of friends consists of 7 men and 4 women. Three of the men are brothers: Ali, Ben and Charlie.

(a) Find the number of different arrangements of the 7 men in a line in which Ali and Ben do **not** stand next to each other. [3]

(b) Find the number of different arrangements of the 7 men and 4 women in a line in which all the men stand together and all the women stand together. [3]

(c) In how many ways can the 7 men and 4 women be divided into a group of 6, a group of 3 and a group of 2 if there are no restrictions? [2]

(d) The 7 men and 4 women are divided at random into a group of 6, a group of 3 and a group of 2.

Find the probability that Ali, Ben and Charlie are all in the same group. [4]

5. [9709/s25/55/q6]

A darts club has 12 members made up of 7 men and 5 women.

Every Monday, a team of 4 is chosen at random to represent the club in a competition.

(a) Find the probability that, on a particular Monday, the team consists of 1 man and 3 women. [3]

Every Tuesday, the darts club chooses 3 teams of 4. Each team enters a competition in a different town.

(b) In how many different ways can the teams be chosen if there are no restrictions? [2]

(c) In how many different ways can the teams be chosen if each team must contain at least 1 man and at least 1 woman? [3]

The 7 men stand in a line for a photograph. Two of them are brothers, George and Harry.

(d) How many different arrangements are there of the 7 men in which there are exactly 2 men between George and Harry? [2]

6. [9709/w25/51/q7]

(a) How many different arrangements are there of the 10 letters in the word SEYCHELLES? [1]

(b) How many different arrangements are there of the 10 letters in the word SEYCHELLES in which there are exactly two letters between the Ss and one of these two letters is C? [3]

(c) How many different arrangements are there of the 10 letters in the word SEYCHELLES in which there is an S at the beginning, an S at the end and the three Es are **not** all next to each other? [3]

5 letters are selected at random from the 10 letters in the word SEYCHELLES.

(d) Find the probability that these 5 letters include the three Es. [3]

7. [9709/w25/52/q7]

(a) Find the number of different arrangements of the 10 letters in the word ZOOLOGICAL in which the three Os are together and the two Ls are **not** next to each other. [4]

(b) Find the number of different arrangements of the 10 letters in the word ZOOLOGICAL in which there are exactly 5 letters between the two Ls. [3]

Two letters are chosen at random from the 10 letters in the word ZOOLOGICAL.

(c) Find the probability that these two letters are different. [3]

8. [9709/w25/53/q2]

The Splash Club has 26 members, of whom 16 are swimmers and 10 are divers. No member is both a swimmer and a diver. The club committee consists of 6 of these 26 members.

In how many ways can the club committee be selected if it must include at least 2 swimmers and at least 2 divers? [4]

9. [9709/w25/53/q3]

(a) Find the number of different arrangements of the 9 letters in the word DAFFODILS in which there is a D at each end and the two Fs are **not** next to each other. [3]

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DAFFODILS has exactly 4 letters between the two Ds. [3]

10. [9709/w25/55/q5]

In a group of 25 athletes, there are 8 sprinters, 5 hurdlers and 12 throwers.

(a) Find the number of different ways in which a team of 6 athletes can be selected if it consists of at least 3 sprinters, at most 2 hurdlers and at most 1 thrower. [4]

A group of 8 athletes chosen from the group of 25 athletes consists of 1 sprinter, 3 hurdlers and 4 throwers. These 8 athletes stand in a row.

(b) How many different arrangements of the 8 athletes are there if the 3 hurdlers do not stand all together? [3]

(c) How many different arrangements of the 8 athletes are there in which there are at least two athletes between any two hurdlers? [3]

11. [9709/m24/52/q6]

A new village social club has 10 members of whom 6 are men and 4 are women. The club committee will consist of 5 members.

(a) In how many ways can the committee of 5 members be chosen if it must include at least 2 men and at least 1 woman? [4]

The 10 members of the club stand in a line for a photograph.

(b) How many different arrangements are there of the 10 members if all the men stand together and all the women stand together? [2]

For a second photograph, the members stand in two rows, with 6 on the back row and 4 on the front row. Olly and his sister Petra are two of the members of the club.

(c) How many different arrangements are there of the 10 members in which Olly and Petra stand next to each other on the front row? [4]

12. [9709/s24/51/q7]

The eight digits 1, 2, 2, 3, 4, 4, 4, 5 are arranged in a line.

(a) How many different arrangements are there of these 8 digits? [1]

(b) Find the number of different arrangements of the 8 digits in which there is a 2 at the beginning, a 2 at the end and the three 4s are not all together. [4]

Three digits are selected at random from the eight digits 1, 2, 2, 3, 4, 4, 4, 5.

(c) Find the probability that the three digits are all different. [5]

13. [9709/s24/52/q7]

(a) How many different arrangements are there of the 10 letters in the word REGENERATE? [1]

(b) How many different arrangements are there of the 10 letters in the word REGENERATE in which the 4 Es are together and the 2 Rs have exactly 3 letters in between them? [4]

(c) Find the probability that a randomly chosen arrangement of the 10 letters in the word REGENERATE is one in which the consonants (G, N, R, R, T) and vowels (A, E, E, E, E) alternate, so that no two consonants are next to each other and no two vowels are next to each other. [5]

14. [9709/s24/53/q6]

(a) How many different arrangements are there of the 9 letters in the word RECORDERS? [1]

(b) How many different arrangements are there of the 9 letters in the word RECORDERS in which there is an E at the beginning, an E at the end and the three Rs are not all together? [3]

The 9 letters of the word RECORDERS are divided at random into two groups: a group of 5 letters and a group of 4 letters.

(c) Find the probability that the three Rs are in the same group. [4]

15. [9709/w24/51/q7]

(a) How many different arrangements are there of the 9 letters in the word INTELLECT in which the two Ts are together? [2]

(b) How many different arrangements are there of the 9 letters in the word INTELLECT in which there is a T at each end and the two Es are not next to each other? [3]

Four letters are selected at random from the 9 letters in the word INTELLECT.

(c) Find the percentage of the possible selections which contain at least one E and exactly one T. [4]

16. [9709/w24/52/q2]

(a) Find the number of different arrangements of the 9 letters in the word ALGEBRAIC. [1]

(b) Find the number of different arrangements of the 9 letters in the word ALGEBRAIC in which there are no more than two letters between the two As. [3]

17. [9709/w24/52/q5]

In a class of 21 students, there are 10 violinists, 6 guitarists and 5 pianists. A group of 7 is to be chosen from these 21 students. The group will consist of 4 violinists, 2 guitarists and 1 pianist.

(a) In how many ways can the group of 7 be chosen? [2]

On another occasion a group of 5 will be chosen from the 21 students. The group must contain at least 2 violinists, at least 1 guitarist and at most 1 pianist.

(b) In how many ways can the group of 5 be chosen? [4]

18. [9709/w24/53/q6]

(a) Find the number of different arrangements of the 9 letters in the word HAPPINESS. [1]

(b) Find the number of different arrangements of the 9 letters in the word HAPPINESS in which the first and last letters are not the same as each other. [3]

(c) Find the number of different arrangements of the 9 letters in the word HAPPINESS in which the two Ps are together and there are exactly two letters between the two Ss. [4]

The 9 letters in the word HAPPINESS are divided at random into a group of 5 and a group of 4.

(d) Find the probability that both Ps are in one group and both Ss are in the other group. [3]

19. [9709/m23/52/q7]

(a) Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three Es are together and the two Ds are **not** next to each other. [4]

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two Ds. [5]

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. [3]

20. [9709/s23/51/q2]

(a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

21. [9709/s23/51/q3]

(a) Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

(c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

22. [9709/s23/52/q6]

In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

(c) How many different arrangements are there of the 9 people if none of the cyclists stand next to each other? [4]

23. [9709/s23/53/q7]

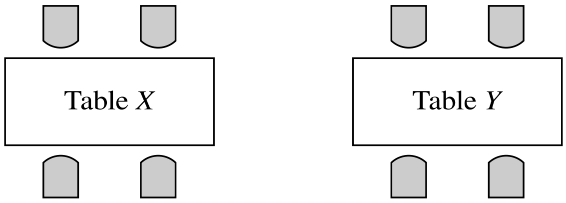
(a) Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

24. [9709/w23/51/q6]



In a restaurant, the tables are rectangular. Each table seats four people: two along each of the longer sides of the table (see diagram). Eight friends have booked two tables, X and Y. Rajid, Sue and Tan are three of these friends.

(a) The eight friends will be divided into two groups of 4, one group for table X and one group for table Y.

Find the number of ways in which this can be done if Rajid and Sue must sit at the same table as each other and Tan must sit at the other table. [3]

When the friends arrive at the restaurant, Rajid and Sue now decide to sit at table X on the same side as each other. Tan decides that he does not mind at which table he sits.

(b) Find the number of different seating arrangements for the 8 friends. [3]

As they leave the restaurant, the 8 friends stand in a line for a photograph.

(c) Find the number of different arrangements if Rajid and Sue stand next to each other, but neither is at an end of the line. [4]

25. [9709/w23/52/q7]

(a) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which no consonant is next to another consonant. (The letters D, M, N and R are consonants and the letters A, E and O are **not** consonants.) [3]

(b) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which there is an A at each end and the Ds are **not** together. [3]

Four letters are selected at random from the 9 letters in the word ANDROMEDA.

(c) Find the probability that this selection contains at least one D and exactly one A. [4]

26. [9709/w23/53/q6]

Jai and his wife Kaz are having a party. Jai has invited five friends and each friend will bring his wife.

(a) At the beginning of the party, the 12 people will stand in a line for a photograph.

(i) How many different arrangements are there of the 12 people if Jai stands next to Kaz and each friend stands next to his own wife? [3]

(ii) How many different arrangements are there of the 12 people if Jai and Kaz occupy the two middle positions in the line, with Jai's five friends on one side and the five wives of the friends on the other side? [2]

(b) For a competition during the party, the 12 people are divided at random into a group of 5, a group of 4 and a group of 3.

Find the probability that Jai and Kaz are in the same group as each other. [5]

27. [9709/m22/52/q5]

A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

28. [9709/s22/51/q1]

(a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

29. [9709/s22/51/q2]

There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

30. [9709/s22/52/q6]

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

31. [9709/s22/53/q7]

A group of 15 friends visit an adventure park. The group consists of four families.

- Mr and Mrs Kenny and their four children
- Mr and Mrs Lizo and their three children
- Mrs Martin and her child
- Mr and Mrs Nantes

The group travel to the park in three cars, one containing 6 people, one containing 5 people and one containing 4 people. The cars are driven by Mr Lizo, Mrs Martin and Mr Nantes respectively.

(a) In how many different ways can the remaining 12 members of the group be divided between the three cars? [3]

The group enter the park by walking through a gate one at a time.

(b) In how many different orders can the 15 friends go through the gate if Mr Lizo goes first and each family stays together? [3]

In the park, the group enter a competition which requires a team of 4 adults and 3 children.

(c) In how many ways can the team be chosen from the group of 15 so that the 3 children are all from different families? [2]

(d) In how many ways can the team be chosen so that at least one of Mr Kenny or Mr Lizo is included? [3]

32. [9709/w22/51/q6]

A Social Club has 15 members, of whom 8 are men and 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. [4]

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? [3]

In one photograph Abel, Betty, Cally, Doug, Eve, Freya and Gino are the 7 members in the back row.

(c) In how many different ways can these 7 members be arranged so that Abel and Betty are next to each other and Freya and Gino are not next to each other? [3]

33. [9709/w22/52/q7]

(a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

34. [9709/w22/53/q6]

(a) Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

Five letters are selected at random from the 9 letters in the word ACTIVATED.

(c) Find the probability that the selection does **not** contain more Ts than As. [5]

35. [9709/m21/52/q6]

(a) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR. [2]

(b) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR in which there is an R at the beginning and an R at the end, and the two As are not together. [4]

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contain both Rs and at least one A and at least one L. [4]

36. [9709/s21/51/q1]

A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.

How many different selections of 5 marbles contain at least 4 marbles of the same colour? [4]

37. [9709/s21/51/q3]

(a) How many different arrangements are there of the 8 letters in the word RELEASED? [1]

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

38. [9709/s21/52/q6]

(a) Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

39. [9709/s21/53/q6]

(a) How many different arrangements are there of the 11 letters in the word REQUIREMENT? [2]

(b) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which the two Rs are together and the three Es are together? [1]

(c) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs? [3]

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and at least one R? [4]

40. [9709/w21/51/q5]

Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? [1]

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? [3]

For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? [2]

(d) For a random division into a group of 5 and a group of 4, find the probability that Raman and Sanjay are in the same group as each other. [4]

41. [9709/w21/52/q2]

A group of 6 people is to be chosen from 4 men and 11 women.

(a) In how many different ways can a group of 6 be chosen if it must contain exactly 1 man? [2]

Two of the 11 women are sisters Jane and Kate.

(b) In how many different ways can a group of 6 be chosen if Jane and Kate cannot both be in the group? [3]

42. [9709/w21/52/q4]

(a) In how many different ways can the 9 letters of the word TELESCOPE be arranged? [2]

(b) In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the T and the C? [4]

43. [9709/w21/53/q1]

The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is only room for 20 people to attend.

In how many ways can the 20 people be chosen from the 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [2]

44. [9709/w21/53/q5]

A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from $\{A, B, C, D, E\}$ and the digits are chosen from $\{1, 2, 3, 4, 5, 6, 7\}$. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed? [2]

(b) Find the number of different codes that include the letter A or the digit 5 or both. [3]

A security code is formed at random.

(c) Find the probability that the code is DE followed by a number between 4500 and 5000. [3]

45. [9709/m20/52/q1]

The 40 members of a club include Ranuf and Saed. All 40 members will travel to a concert. 35 members will travel in a coach and the other 5 will travel in a car. Ranuf will be in the coach and Saed will be in the car.

In how many ways can the members who will travel in the coach be chosen? [3]

46. [9709/m20/52/q4]

Richard has 3 blue candles, 2 red candles and 6 green candles. The candles are identical apart from their colours. He arranges the 11 candles in a line.

(a) Find the number of different arrangements of the 11 candles if there is a red candle at each end. [2]

(b) Find the number of different arrangements of the 11 candles if all the blue candles are together and the red candles are not together. [4]

47. [9709/s20/51/q2]

(a) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

48. [9709/s20/51/q4]

In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.

How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists? [4]

49. [9709/s20/52/q6]

(a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

50. [9709/s20/53/q7]

(a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

51. [9709/w20/51/q7]

(a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

52. [9709/w20/52/q6]

Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

All 9 people stand in a line.

(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]

53. [9709/w20/53/q3]

A committee of 6 people is to be chosen from 9 women and 5 men.

(a) Find the number of ways in which the 6 people can be chosen if there must be more women than men on the committee. [3]

The 9 women and 5 men include a sister and brother.

(b) Find the number of ways in which the committee can be chosen if the sister and brother cannot both be on the committee. [3]

54. [9709/w20/53/q5]

The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

55. [9709/m19/62/q7]

Find the number of different arrangements that can be made of all 9 letters in the word CAMERAMAN in each of the following cases.

(i) There are no restrictions. [2]

(ii) The As occupy the 1st, 5th and 9th positions. [1]

(iii) There is exactly one letter between the Ms. [4]

Three letters are selected from the 9 letters of the word CAMERAMAN.

(iv) Find the number of different selections if the three letters include exactly one M and exactly one A. [1]

(v) Find the number of different selections if the three letters include at least one M. [3]

56. [9709/s19/61/q8]

Freddie has 6 toy cars and 3 toy buses, all different. He chooses 4 toys to take on holiday with him.

(i) In how many different ways can Freddie choose 4 toys? [1]

(ii) How many of these choices will include both his favourite car and his favourite bus? [2]

Freddie arranges these 9 toys in a line.

(iii) Find the number of possible arrangements if the buses are all next to each other. [3]

(iv) Find the number of possible arrangements if there is a car at each end of the line and no buses are next to each other. [3]

57. [9709/s19/62/q7]

(a) A group of 6 teenagers go boating. There are three boats available. One boat has room for 3 people, one has room for 2 people and one has room for 1 person. Find the number of different ways the group of 6 teenagers can be divided between the three boats. [3]

(b) Find the number of different 7-digit numbers which can be formed from the seven digits 2, 2, 3, 7, 7, 7, 8 in each of the following cases.

(i) The odd digits are together and the even digits are together. [3]

(ii) The 2s are not together. [4]

58. [9709/s19/63/q3]

Mr and Mrs Keene and their 5 children all go to watch a football match, together with their friends Mr and Mrs Uzuma and their 2 children. Find the number of ways in which all 11 people can line up at the entrance in each of the following cases.

(i) Mr Keene stands at one end of the line and Mr Uzuma stands at the other end. [2]

(ii) The 5 Keene children all stand together and the Uzuma children both stand together. [3]

59. [9709/s19/63/q4]

(i) Find the number of ways a committee of 6 people can be chosen from 8 men and 4 women if there must be at least twice as many men as there are women on the committee. [3]

(ii) Find the number of ways a committee of 6 people can be chosen from 8 men and 4 women if 2 particular men refuse to be on the committee together. [3]

60. [9709/w19/61/q6]

(i) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that all four Es are together. [1]

(ii) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that the Ss are not next to each other. [4]

Four letters are selected from the 12 letters of the word STEEPLECHASE.

(iii) Find the number of different selections if the four letters include exactly one S. [4]

61. [9709/w19/63/q2]

(i) How many different arrangements are there of the 9 letters in the word CORRIDORS? [2]

(ii) How many different arrangements are there of the 9 letters in the word CORRIDORS in which the first letter is D and the last letter is R or O? [3]

62. [9709/w19/63/q3]

A sports team of 7 people is to be chosen from 6 attackers, 5 defenders and 4 midfielders. The team must include at least 3 attackers, at least 2 defenders and at least 1 midfielder.

(i) In how many different ways can the team of 7 people be chosen? [4]

The team of 7 that is chosen travels to a match in two cars. A group of 4 travel in one car and a group of 3 travel in the other car.

(ii) In how many different ways can the team of 7 be divided into a group of 4 and a group of 3? [2]

63. [9709/m18/62/q2]

A selection of 3 letters from the 8 letters of the word COLLIDER is made.

(i) How many different selections of 3 letters can be made if there is exactly one L? [1]

(ii) How many different selections of 3 letters can be made if there are no restrictions? [3]

64. [9709/m18/62/q6]

The digits 1, 3, 5, 6, 6, 6, 8 can be arranged to form many different 7-digit numbers.

(i) How many of the 7-digit numbers have all the even digits together and all the odd digits together? [3]

(ii) How many of the 7-digit numbers are even? [3]

65. [9709/s18/61/q7]

Find the number of different ways in which all 9 letters of the word MINCEMEAT can be arranged in each of the following cases.

(i) There are no restrictions. [1]

(ii) No vowel (A, E, I are vowels) is next to another vowel. [4]

5 of the 9 letters of the word MINCEMEAT are selected.

(iii) Find the number of possible selections which contain exactly 1 M and exactly 1 E. [2]

(iv) Find the number of possible selections which contain at least 1 M and at least 1 E. [3]

66. [9709/s18/62/q6]

(a) Find the number of ways in which all 9 letters of the word AUSTRALIA can be arranged in each of the following cases.

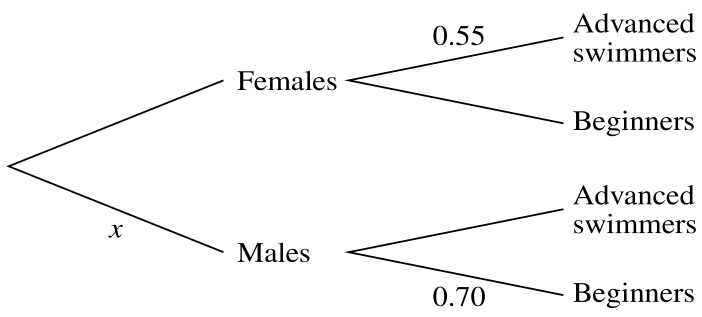
(i) All the vowels (A, I, U are vowels) are together. [3]

(ii) The letter T is in the central position and each end position is occupied by one of the other consonants (R, S, L). [3]

(b) Donna has 2 necklaces, 8 rings and 4 bracelets, all different. She chooses 4 pieces of jewellery. How many possible selections can she make if she chooses at least 1 necklace and at least 1 bracelet? [4]

67. [9709/s18/63/q3]

The members of a swimming club are classified either as ‘Advanced swimmers’ or ‘Beginners’. The proportion of members who are male is x, and the proportion of males who are Beginners is 0.7. The proportion of females who are Advanced swimmers is 0.55. This information is shown in the tree diagram.



For a randomly chosen member, the probability of being an Advanced swimmer is the same as the probability of being a Beginner.

(i) Find x. [3]

(ii) Given that a randomly chosen member is an Advanced swimmer, find the probability that the member is male. [3]

68. [9709/s18/63/q7]

Find the number of ways the 9 letters of the word SEVENTEEN can be arranged in each of the following cases.

(i) One of the letter Es is in the centre with 4 letters on either side. [2]

(ii) No E is next to another E. [3]

5 letters are chosen from the 9 letters of the word SEVENTEEN.

(iii) Find the number of possible selections which contain exactly 2 Es and exactly 2 Ns. [1]

(iv) Find the number of possible selections which contain at least 2 Es. [4]

69. [9709/w18/61/q1]

9 people are to be divided into a group of 4, a group of 3 and a group of 2. In how many different ways can this be done? [3]

70. [9709/w18/61/q3]

In an orchestra, there are 11 violinists, 5 cellists and 4 double bass players. A small group of 6 musicians is to be selected from these 20.

(i) How many different selections of 6 musicians can be made if there must be at least 4 violinists, at least 1 cellist and no more than 1 double bass player? [4]

The small group that is selected contains 4 violinists, 1 cellist and 1 double bass player. They sit in a line to perform a concert.

(ii) How many different arrangements are there of these 6 musicians if the violinists must sit together? [3]

71. [9709/w18/62/q4]

(i) Find the number of different ways that 5 boys and 6 girls can stand in a row if all the boys stand together and all the girls stand together. [3]

(ii) Find the number of different ways that 5 boys and 6 girls can stand in a row if no boy stands next to another boy. [3]

72. [9709/w18/63/q1]

A group consists of 5 men and 2 women. Find the number of different ways that the group can stand in a line if the women are not next to each other. [3]

73. [9709/m17/62/q5]

(i) A plate of cakes holds 12 different cakes. Find the number of ways these cakes can be shared between Alex and James if each receives an odd number of cakes. [3]

(ii) Another plate holds 7 cup cakes, each with a different colour icing, and 4 brownies, each of a different size. Find the number of different ways these 11 cakes can be arranged in a row if no brownie is next to another brownie. [3]

(iii) A plate of biscuits holds 4 identical chocolate biscuits, 6 identical shortbread biscuits and 2 identical gingerbread biscuits. These biscuits are all placed in a row. Find how many different arrangements are possible if the chocolate biscuits are all kept together. [3]

74. [9709/s17/61/q7]

(a) Eight children of different ages stand in a random order in a line. Find the number of different ways this can be done if none of the three youngest children stand next to each other. [3]

(b) David chooses 5 chocolates from 6 different dark chocolates, 4 different white chocolates and 1 milk chocolate. He must choose at least one of each type. Find the number of different selections he can make. [4]

(c) A password for Chelsea's computer consists of 4 characters in a particular order. The characters are chosen from the following.

- The 26 capital letters A to Z
- The 9 digits 1 to 9
- The 5 symbols # ~ * ? !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Chelsea can make. [4]

75. [9709/s17/62/q6]

A library contains 4 identical copies of book A, 2 identical copies of book B and 5 identical copies of book C. These 11 books are arranged on a shelf in the library.

(i) Calculate the number of different arrangements if the end books are either both book A or both book B. [4]

(ii) Calculate the number of different arrangements if all the books A are next to each other and none of the books B are next to each other. [5]

76. [9709/s17/63/q6]

(a) Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5,

(i) if digits are not repeated, [2]

(ii) if digits can be repeated and the number formed is odd. [3]

(b) A box of 20 biscuits contains 4 different chocolate biscuits, 2 different oatmeal biscuits and 14 different ginger biscuits. 6 biscuits are selected from the box at random.

(i) Find the number of different selections that include the 2 oatmeal biscuits. [2]

(ii) Find the probability that fewer than 3 chocolate biscuits are selected. [4]

77. [9709/w17/61/q6]

(a) A village hall has seats for 40 people, consisting of 8 rows with 5 seats in each row. Mary, Ahmad, Wayne, Elsie and John are the first to arrive in the village hall and no seats are taken before they arrive.

(i) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John assuming there are no restrictions? [2]

(ii) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John if Mary and Ahmad sit together in the front row and the other three sit together in one of the other rows? [4]

(b) In how many ways can a team of 4 people be chosen from 10 people if 2 of the people, Ross and Lionel, refuse to be in the team together? [4]

78. [9709/w17/62/q6]

(a) Find the number of different 3-digit numbers greater than 300 that can be made from the digits 1, 2, 3, 4, 6, 8 if

(i) no digit can be repeated, [3]

(ii) a digit can be repeated and the number made is even. [3]

(b) A team of 5 is chosen from 6 boys and 4 girls. Find the number of ways the team can be chosen if

(i) there are no restrictions, [1]

(ii) the team contains more boys than girls. [3]

79. [9709/m16/62/q6]

Hannah chooses 5 singers from 15 applicants to appear in a concert. She lists the 5 singers in the order in which they will perform.

(i) How many different lists can Hannah make? [2]

Of the 15 applicants, 10 are female and 5 are male.

(ii) Find the number of lists in which the first performer is male, the second is female, the third is male, the fourth is female and the fifth is male. [2]

Hannah's friend Ami would like the group of 5 performers to include more males than females. The order in which they perform is no longer relevant.

(iii) Find the number of different selections of 5 performers with more males than females. [3]

(iv) Two of the applicants are Mr and Mrs Blake. Find the number of different selections that include Mr and Mrs Blake and also fulfil Ami's requirement. [3]

80. [9709/s16/61/q6]

(a) **(i)** Find how many numbers there are between 100 and 999 in which all three digits are different. [3]

(ii) Find how many of the numbers in part **(i)** are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

81. [9709/s16/62/q7]

(a) Find the number of different arrangements which can be made of all 10 letters of the word WALLFLOWER if

(i) there are no restrictions, [1]

(ii) there are exactly six letters between the two Ws. [4]

(b) A team of 6 people is to be chosen from 5 swimmers, 7 athletes and 4 cyclists. There must be at least 1 from each activity and there must be more athletes than cyclists. Find the number of different ways in which the team can be chosen. [4]

82. [9709/s16/63/q6]

Find the number of ways all 9 letters of the word EVERGREEN can be arranged if

(i) there are no restrictions, [1]

(ii) the first letter is R and the last letter is G, [2]

(iii) the Es are all together. [2]

Three letters from the 9 letters of the word EVERGREEN are selected.

(iv) Find the number of selections which contain no Es and exactly 1 R. [1]

(v) Find the number of selections which contain no Es. [3]

83. [9709/w16/61/q5]

(a) Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]

(b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.

(i) Find the number of different teams the manager can choose. [2]

(ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

84. [9709/w16/62/q6]

Find the number of ways all 10 letters of the word COPENHAGEN can be arranged so that

(i) the vowels (A, E, O) are together and the consonants (C, G, H, N, P) are together, [3]

(ii) the Es are not next to each other. [4]

Four letters are selected from the 10 letters of the word COPENHAGEN.

(iii) Find the number of different selections if the four letters must contain the same number of Es and Ns with at least one of each. [5]

85. [9709/w16/63/q1]

A committee of 5 people is to be chosen from 4 men and 6 women. William is one of the 4 men and Mary is one of the 6 women. Find the number of different committees that can be chosen if William and Mary refuse to be on the committee together. [3]

86. [9709/s15/61/q7]

(a) Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if

(i) there are no restrictions, [2]

(ii) the number made is an even number. [4]

(b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

87. [9709/s15/62/q6]

(a) Find the number of different ways the 7 letters of the word BANANAS can be arranged

(i) if the first letter is N and the last letter is B, [3]

(ii) if all the letters A are next to each other. [3]

(b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cannot both be in the group together. [3]

88. [9709/s15/63/q7]

Rachel has 3 types of ornament. She has 6 different wooden animals, 4 different sea-shells and 3 different pottery ducks.

(i) She lets her daughter Cherry choose 5 ornaments to play with. Cherry chooses at least 1 of each type of ornament. How many different selections can Cherry make? [5]

Rachel displays 10 of the 13 ornaments in a row on her window-sill. Find the number of different arrangements that are possible if

(ii) she has a duck at each end of the row and no ducks anywhere else, [3]

(iii) she has a duck at each end of the row and wooden animals and sea-shells are placed alternately in the positions in between. [3]

89. [9709/w15/61/q5]

(a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged

(i) if all the letters E are together, [3]

(ii) if the T is at one end and there is an S at the other end. [3]

(b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

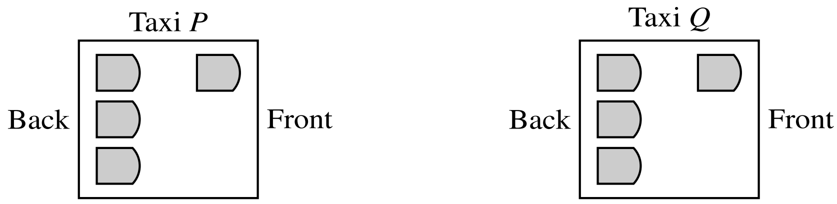
90. [9709/w15/62/q2]

A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

91. [9709/w15/62/q4]

A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

92. [9709/w15/63/q5]

(a) Find the number of different ways that the 13 letters of the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other. [3]

(b) There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. [5]

Chapter 3

Probability

1. [9709/m25/52/q4]

Eddie has 16 toy cars, of which 8 are white, 5 are black and 3 are silver. He places all the cars in a bag and selects three of them at random, without replacement.

(a) Find the probability that all three cars are the same colour. [3]

(b) Find the probability that, when the 3 cars are selected, at least one car is white and at least one car is black. [4]

2. [9709/s25/51/q2]

(a) Find the number of different arrangements of the 8 letters in the word KANGAROO in which the two As are together and the two Os are **not** together. [3]

A fair 8-sided dice has faces labelled K, A, N, G, A, R, O, O. The dice is rolled repeatedly.

(b) Find the probability that fewer than 6 rolls of this dice are required to obtain an A. [2]

(c) Find the probability that the second A is obtained on the 6th roll of the dice. [2]

3. [9709/s25/52/q3]

A bag contains 4 blue marbles and 12 red marbles. One marble is selected at random from the bag. If this marble is blue, it is replaced in the bag, but if it is red, it is not replaced. A second marble is now selected at random from the bag.

(a) Find the probability that both marbles selected are the same colour. [2]

(b) Find the probability that the first marble is blue given that the second marble is red. [3]

4. [9709/s25/53/q5]

Bag A contains 6 red marbles, 5 blue marbles and 1 green marble.

Bag B contains 5 red marbles and 3 blue marbles.

A marble is chosen at random from bag A and placed in bag B.

A marble is now chosen at random from bag B.

(a) Draw a tree diagram to represent this information, giving the probability on each branch. [3]

(b) Find the probability that both marbles chosen are the same colour. [2]

(c) Find the probability that the marble chosen from bag A is blue, given that the marble chosen from bag B is blue. [3]

5. [9709/s25/55/q4]

Students applying to Drydale College take an entrance test. A student is either accepted or rejected or required to take another test with probabilities 0.3, 0.2 and 0.5 respectively. When a student takes a second test the outcomes and probabilities are exactly the same as for the first test. A student who has to take a third test is accepted with probability 0.25 and rejected with probability 0.75.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that a randomly chosen student who applies to Drydale College is accepted. [2]

(c) Find the probability that a randomly chosen student who applies to Drydale College takes at least two tests given that the student is accepted. [3]

Three friends apply to Drydale College.

(d) Find the probability that all three are rejected. [2]

6. [9709/w25/51/q2]

A fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly until a 6 is obtained.

(a) Find the probability that a 6 is obtained for the first time on the 8th throw. [1]

(b) Find the probability that a 6 is obtained for the third time on the 7th throw. [3]

7. [9709/w25/51/q4]

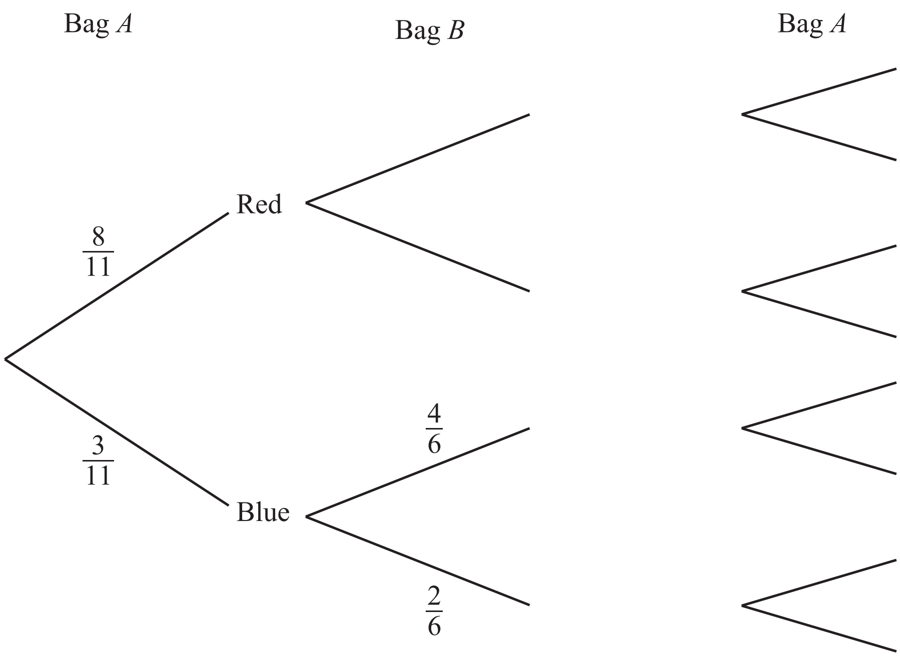
Bag A contains 8 red marbles and 3 blue marbles. Bag B contains 4 red marbles and 1 blue marble.

A marble is chosen at random from bag A. If the marble chosen is red, it is discarded. If the marble chosen is blue, it is placed in bag B.

A marble is then chosen at random from bag B. If the marble chosen is red, it is discarded. If the marble chosen is blue, it is placed in bag A.

A marble is now chosen at random from bag A.

(a) Complete the tree diagram below by entering all the remaining outcomes and probabilities. [3]



(b) Find the probability that all three marbles chosen are the same colour. [2]

8. [9709/w25/52/q1]

A coin is biased so that the probability of obtaining a head when it is thrown is 0.4. The coin is thrown repeatedly until the first head is obtained.

(a) Find the probability that the first head is obtained on the 5th throw. [1]

(b) Find the probability that the first head is obtained after the 6th throw. [2]

9. [9709/w25/52/q4]

Gio has a pack of 18 cards. Ivy has a pack of x cards.

Each card has a picture of a bus or a car or a train. The number of cards with each picture in the two packs is shown in the table.

	Bus	Car	Train
Gio's pack	6	10	2
Ivy's pack	$x-12$	9	3

One card is chosen at random from each pack. The probability that the two cards have pictures of buses on them is equal to twice the probability that the two cards have pictures of cars on them.

(a) Write down an equation in terms of x and hence find the value of x. [4]

(b) Find the probability that the two cards have pictures of the same type of vehicle on them. [3]

10. [9709/w25/53/q5]

Chen has three boxes. Box A contains 5 counters, of which 3 are white and 2 are yellow. Box W contains 4 red marbles and 3 blue marbles. Box Y contains 5 red marbles and 3 blue marbles.

Chen chooses one counter at random from box A. If the counter is white, he chooses two marbles from box W, at random and without replacement. If the counter is yellow, he chooses two marbles from box Y, at random and without replacement.

(a) Draw a fully labelled tree diagram to illustrate this information, including all the probabilities. [3]

(b) Find the probability that one red marble and one blue marble are obtained. [3]

(c) Find the probability that a white counter is chosen from box A, given that one red marble and one blue marble are obtained. [2]

11. [9709/w25/55/q3]

Priti has two bags of discs, X and Y.

Bag X contains 8 red discs and 7 blue discs.

Bag Y contains 6 red discs and 9 blue discs.

Priti tosses a fair coin.

If she obtains a head, she chooses at random and **without** replacement two discs from bag X.

If she obtains a tail, she chooses at random and **with** replacement two discs from bag Y.

(a) Draw a tree diagram to represent this information, showing all the probabilities. [2]

(b) Find the probability that the two discs that Priti chooses are blue. [2]

(c) Find the probability that the two discs that Priti chooses come from bag X given that at least one of the discs is red. [4]

12. [9709/m24/52/q1]

A bag contains 9 blue marbles and 3 red marbles. One marble is chosen at random from the bag. If this marble is blue, it is replaced back into the bag. If this marble is red, it is **not** returned to the bag. A second marble is now chosen at random from the bag.

(a) Find the probability that both the marbles chosen are red. [1]

(b) Find the probability that the first marble chosen is blue given that the second marble chosen is red. [3]

13. [9709/s24/51/q4]

A game for two players is played using a fair 4-sided dice with sides numbered 1, 2, 3 and 4. One turn consists of throwing the dice repeatedly up to a maximum of three times. When a 4 is obtained, no further throws are made during that turn. A player who obtains a 4 in their turn scores 1 point.

(a) Show that the probability that a player obtains a 4 in one turn is $\frac{37}{64}$. [2]

Xeno and Yao play this game.

(b) Find the probability that neither Xeno nor Yao score any points in their first two turns. [1]

(c) Xeno and Yao each have three turns.

Find the probability that Xeno scores 2 more points than Yao. [3]

14. [9709/s24/52/q2]

Seva has a coin which is biased so that when it is thrown the probability of obtaining a head is $\frac{1}{3}$. He also has a bag containing 4 red marbles and 5 blue marbles.

Seva throws the coin. If he obtains a head, he selects one marble from the bag at random. If he obtains a tail, he selects two marbles from the bag at random and without replacement.

(a) Find the probability that Seva selects at least one red marble. [3]

(b) Find the probability that Seva obtains a head given that he selects no red marbles. [2]

15. [9709/s24/53/q3]

Box A contains 6 green balls and 3 yellow balls.

Box B contains 4 green balls and x yellow balls.

A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Draw a tree diagram to represent this information, showing the probability on each of the branches. [4]

The probability that both the balls chosen are the same colour is $\frac{8}{15}$.

(b) Find the value of x. [3]

16. [9709/w24/51/q4]

Rahul has two bags, X and Y. Bag X contains 4 red marbles and 2 blue marbles. Bag Y contains 3 red marbles and 4 blue marbles. Rahul also has a coin which is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.

Rahul throws the coin.

- If he obtains a head, he chooses at random a marble from bag X. He notes the colour and replaces the marble in bag X. He then chooses at random a second marble from bag X.
- If he obtains a tail, he chooses at random a marble from bag Y. He notes the colour and discards the marble. He then chooses at random a second marble from bag Y.

(a) Find the probability that the two marbles that Rahul chooses are the same colour. [3]

(b) Find the probability that the two marbles that Rahul chooses are both from bag Y given that both marbles are blue. [3]

17. [9709/w24/52/q1]

At a college, the students choose exactly one of tennis, hockey or netball to play. The table shows the numbers of students in Year 1 and Year 2 at the college playing each of these sports.

	Tennis	Hockey	Netball
Year 1	16	22	12
Year 2	24	18	28

One student is chosen at random from the 120 students. Events X and N are defined as follows:

X: the student is in Year 1

N: the student plays netball.

(a) Find $\mathrm{P}(X \mid N)$. [1]

(b) Find $\mathrm{P}(N \mid X)$. [1]

(c) Determine whether or not X and N are independent events. [1]

One of the students who plays netball takes 8 shots at goal. On each shot, the probability that she will succeed is 0.15, independently of all other shots.

(d) Find the probability that she succeeds on fewer than 3 of these shots. [3]

18. [9709/w24/53/q5]

A factory produces chocolates. 30% of the chocolates are wrapped in gold foil, 25% are wrapped in red foil and the remainder are unwrapped.

Indigo chooses 8 chocolates at random from the production line.

(a) Find the probability that she obtains no more than 2 chocolates that are wrapped in red foil. [3]

Jake chooses chocolates one at a time at random from the production line.

(b) Find the probability that the first time he obtains a chocolate that is wrapped in red foil is before the 7th choice. [2]

Keifa chooses chocolates one at a time at random from the production line.

(c) Find the probability that the second chocolate chosen is the first one wrapped in gold foil given that the fifth chocolate chosen is the first unwrapped chocolate. [5]

19. [9709/m23/52/q4]

The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining, the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day it is not raining and Aran is not wearing a hat or a scarf is 0.36.

Find the value of x. [3]

20. [9709/m23/52/q5]

Marco has four boxes labelled K, L, M and N. He places them in a straight line in the order K, L, M, N with K on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows.

A: The white marble is in either box L or box M.

B: The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent. [3]

21. [9709/s23/51/q7]

A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

(a) Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

(b) Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

(c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

22. [9709/s23/52/q4]

A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable X denotes the number of spins required.

(a) Find $P(X = 4)$. [1]

(b) Find $P(X < 6)$. [2]

Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5, are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the probability that the score is greater than 0 given that the score is **not** equal to 2. [3]

The two spinners are spun at the same time repeatedly .

(d) For 9 randomly chosen spins of the two spinners, find the probability that the score is greater than 2 on at least 3 occasions. [3]

23. [9709/s23/53/q5]

Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events A and B are defined as follows.

A: The sum of the two numbers is less than 6.

B: The difference between the two numbers is at most 2.

(a) Determine whether or not the events A and B are independent. [4]

(b) Find $\mathrm{P}(B \mid A')$. [3]

24. [9709/w23/52/q6]

Freddie has two bags of marbles.

Bag X contains 7 red marbles and 3 blue marbles.

Bag Y contains 4 red marbles and 1 blue marble.

Freddie chooses one of the bags at random. A marble is removed at random from that bag and not replaced. A new red marble is now added to each bag. A second marble is then removed at random from the same bag that the first marble had been removed from.

(a) Draw a tree diagram to represent this information, showing the probability on each of the branches. [3]

(b) Find the probability that both of the marbles removed from the bag are the same colour. [4]

(c) Find the probability that bag Y is chosen given that the marbles removed are **not** both the same colour. [2]

25. [9709/w23/53/q3]

Tim has two bags of marbles, A and B.

Bag A contains 8 white, 4 red and 3 yellow marbles.

Bag B contains 6 white, 7 red and 2 yellow marbles.

Tim also has an ordinary fair 6-sided dice. He rolls the dice. If he obtains a 1 or 2, he chooses two marbles at random from bag A, without replacement. If he obtains a 3, 4, 5 or 6, he chooses two marbles at random from bag B, without replacement.

(a) Find the probability that both marbles are white. [3]

(b) Find the probability that the two marbles come from bag B given that one is white and one is red. [4]

26. [9709/m22/52/q6]

A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

(d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

(e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

27. [9709/s22/51/q6]

Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

(b) Find the probability that Janice finishes the game. [3]

(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

28. [9709/s22/52/q7]

Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

(b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

(c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

29. [9709/s22/53/q6]

Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

30. [9709/w22/51/q5]

A game is played with an ordinary fair 6-sided die. A player throws the die once. If the result is 2, 3, 4 or 5, that result is the player's score and the player does not throw the die again. If the result is 1 or 6, the player throws the die a second time and the player's score is the sum of the two numbers from the two throws.

(a) Draw a fully labelled tree diagram to represent this information. [2]

Events A and B are defined as follows.

A: the player's score is 5, 6, 7, 8 or 9

B: the player has two throws

(b) Show that $P(A) = \frac{1}{3}$. [3]

(c) Determine whether or not events A and B are independent. [2]

(d) Find $P(B \mid A')$. [3]

31. [9709/w22/52/q1]

On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x. [3]

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late. [2]

32. [9709/w22/52/q5]

Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $P(B) = \frac{7}{16}$. [2]

(b) Find $P(A \mid B)$. [2]

The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw up the probability distribution table for X. [3]

33. [9709/w22/53/q7]

Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game.

Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on his first turn. [2]

(b) Find the probability that Tom wins the game on his second turn. [4]

(c) Find the probability that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. [2]

34. [9709/m21/52/q2]

Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The probabilities for the three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears a blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf, she wears a hat with probability 0.3.

(a) Find the probability that on a randomly chosen day Georgie wears a hat. [2]

(b) Find the probability that on a randomly chosen day Georgie wears a yellow scarf given that she does not wear a hat. [3]

35. [9709/m21/52/q7]

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

36. [9709/s21/51/q4]

To gain a place at a science college, students first have to pass a written test and then a practical test.

Each student is allowed a maximum of two attempts at the written test. A student is only allowed a second attempt if they fail the first attempt. No student is allowed more than one attempt at the practical test. If a student fails both attempts at the written test, then they cannot attempt the practical test.

The probability that a student will pass the written test at the first attempt is 0.8. If a student fails the first attempt at the written test, the probability that they will pass at the second attempt is 0.6. The probability that a student will pass the practical test is always 0.3.

(a) Draw a tree diagram to represent this information, showing the probabilities on the branches. [3]

(b) Find the probability that a randomly chosen student will succeed in gaining a place at the college. [2]

(c) Find the probability that a randomly chosen student passes the written test at the first attempt given that the student succeeds in gaining a place at the college. [2]

37. [9709/s21/52/q3]

On each day that Alexa goes to work, the probabilities that she travels by bus, by train or by car are 0.4, 0.35 and 0.25 respectively. When she travels by bus, the probability that she arrives late is 0.55. When she travels by train, the probability that she arrives late is 0.7. When she travels by car, the probability that she arrives late is x.

On a randomly chosen day when Alexa goes to work, the probability that she does not arrive late is 0.48.

(a) Find the value of x. [3]

(b) Find the probability that Alexa travels to work by train given that she arrives late. [3]

38. [9709/s21/53/q4]

Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

(a) Find the probability that the score is 4 on a single throw of the three dice. [3]

(b) Find the probability that a score of 18 is obtained for the first time on the 5th throw of the three dice. [3]

39. [9709/s21/53/q7]

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

40. [9709/w21/51/q3]

For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.2 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the probability that she has a biscuit is 0.6. When she has milk, she never has a biscuit.

Find the probability that Suki has tea given that she does not have a biscuit. [5]

41. [9709/w21/52/q1]

Each of the 180 students at a college plays exactly one of the piano, the guitar and the drums. The numbers of male and female students who play the piano, the guitar and the drums are given in the following table.

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the probability that the student plays the guitar. [1]

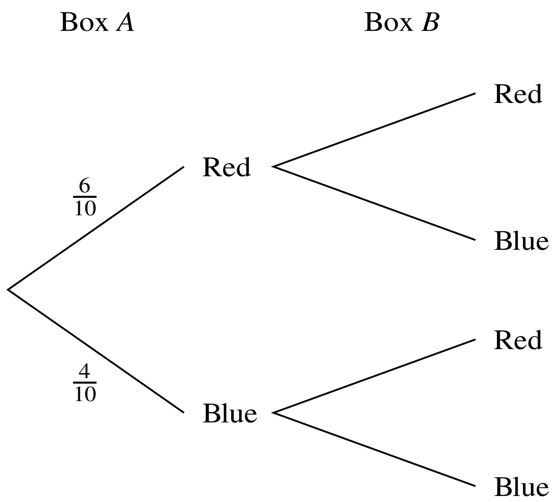
(b) Find the probability that the student is male given that the student plays the drums. [2]

(c) Determine whether the events 'the student plays the guitar' and 'the student is female' are independent, justifying your answer. [2]

42. [9709/w21/53/q7]

Box A contains 6 red balls and 4 blue balls. Box B contains x red balls and 9 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Complete the tree diagram below, giving the remaining four probabilities in terms of x. [3]



(b) Show that the probability that both balls chosen are blue is $\dfrac{4}{x+10}$. [2]

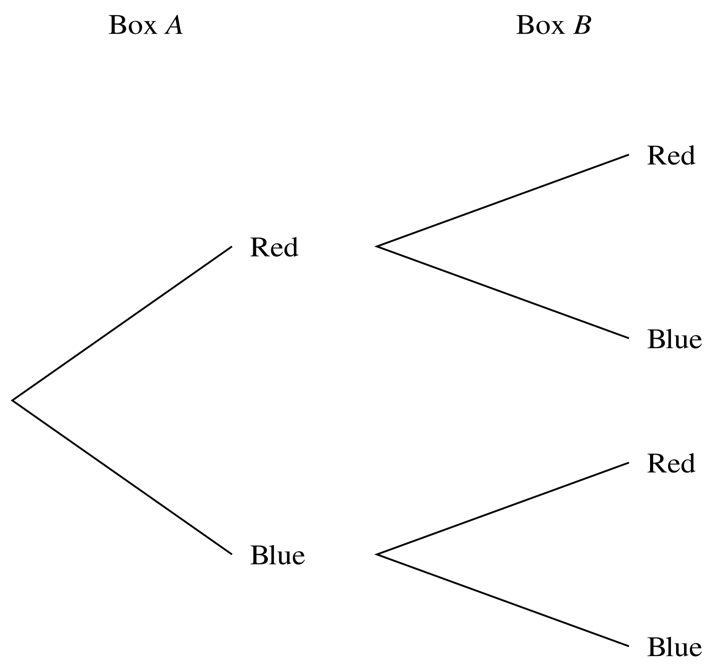
It is given that the probability that both balls chosen are blue is $\frac{1}{6}$.

(c) Find the probability, correct to 3 significant figures, that the ball chosen from box A is red given that the ball chosen from box B is red. [5]

43. [9709/m20/52/q6]

Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. [3]



(b) Find the probability that the two balls chosen are not the same colour. [2]

(c) Find the probability that the ball chosen from box A is blue given that the ball chosen from box B is blue. [4]

44. [9709/s20/51/q5]

On Mondays, Rani cooks her evening meal. She has a pizza, a burger or a curry with probabilities 0.35, 0.44, 0.21 respectively. When she cooks a pizza, Rani has some fruit with probability 0.3. When she cooks a burger, she has some fruit with probability 0.8. When she cooks a curry, she never has any fruit.

(a) Draw a fully labelled tree diagram to represent this information. [2]

(b) Find the probability that Rani has some fruit. [2]

(c) Find the probability that Rani does not have a burger given that she does not have any fruit. [4]

45. [9709/s20/52/q2]

A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

46. [9709/s20/53/q1]

Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the probability that he goes by bus is 0.45 and the probability that he walks is 0.35. When Juan goes by car, the probability that he arrives early is 0.6. When he goes by bus, the probability that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. [2]

47. [9709/w20/51/q1]

Two ordinary fair dice, one red and the other blue, are thrown.

Event A is 'the score on the red die is divisible by 3'.

Event B is 'the sum of the two scores is at least 9'.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Hence determine whether or not the events A and B are independent. [2]

48. [9709/w20/51/q2]

The probability that a student at a large music college plays in the band is 0.6. For a student who plays in the band, the probability that she also sings in the choir is 0.3. For a student who does not play in the band, the probability that she sings in the choir is x. The probability that a randomly chosen student from the college does not sing in the choir is 0.58.

(a) Find the value of x. [3]

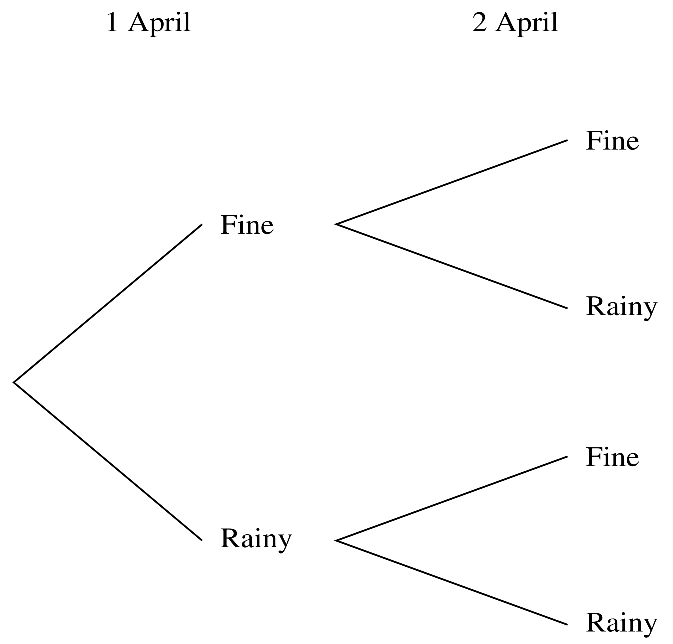
Two students from the college are chosen at random.

(b) Find the probability that both students play in the band and both sing in the choir. [2]

49. [9709/w20/52/q4]

In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$. [3]

(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

50. [9709/m19/62/q1]

On each day that Tamar goes to work, he wears either a blue suit with probability 0.6 or a grey suit with probability 0.4. If he wears a blue suit then the probability that he wears red socks is 0.2. If he wears a grey suit then the probability that he wears red socks is 0.32.

(i) Find the probability that Tamar wears red socks on any particular day that he is at work. [2]

(ii) Given that Tamar is not wearing red socks at work, find the probability that he is wearing a grey suit. [3]

51. [9709/s19/61/q2]

Jameel has 5 plums and 3 apricots in a box. Rosa has x plums and 6 apricots in a box. One fruit is chosen at random from Jameel's box and one fruit is chosen at random from Rosa's box. The probability that both fruits chosen are plums is $\frac{1}{4}$. Write down an equation in x and hence find x. [3]

52. [9709/s19/61/q3]

A fair six-sided die is thrown twice and the scores are noted. Event X is defined as 'The total of the two scores is 4'. Event Y is defined as 'The first score is 2 or 5'. Are events X and Y independent? Justify your answer. [4]

53. [9709/s19/62/q1]

Two ordinary fair dice are thrown and the numbers obtained are noted. Event S is 'The sum of the numbers is even'. Event T is 'The sum of the numbers is either less than 6 or a multiple of 4 or both'. Showing your working, determine whether the events S and T are independent. [4]

54. [9709/s19/63/q2]

Megan sends messages to her friends in one of 3 different ways: text, email or social media. For each message, the probability that she uses text is 0.3 and the probability that she uses email is 0.2. She receives an immediate reply from a text message with probability 0.4, from an email with probability 0.15 and from social media with probability 0.6.

(i) Draw a fully labelled tree diagram to represent this information. [2]

(ii) Given that Megan does not receive an immediate reply to a message, find the probability that the message was an email. [4]

55. [9709/w19/61/q1]

When Shona goes to college she either catches the bus with probability 0.8 or she cycles with probability 0.2. If she catches the bus, the probability that she is late is 0.4. If she cycles, the probability that she is late is x. The probability that Shona is not late for college on a randomly chosen day is 0.63. Find the value of x. [3]

56. [9709/w19/62/q2]

Benju cycles to work each morning and he has two possible routes. He chooses the hilly route with probability 0.4 and the busy route with probability 0.6. If he chooses the hilly route, the probability that he will be late for work is x and if he chooses the busy route the probability that he will be late for work is $2x$. The probability that Benju is late for work on any day is 0.36.

(i) Show that $x = 0.225$. [2]

(ii) Given that Benju is not late for work, find the probability that he chooses the hilly route. [3]

57. [9709/w19/62/q7]

(i) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that all three Os are together and both Ts are together. [1]

(ii) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that the Ts are not together. [4]

(iii) Find the probability that a randomly chosen arrangement of the 9 letters of the word TOADSTOOL has a T at the beginning and a T at the end. [2]

(iv) Five letters are selected from the 9 letters of the word TOADSTOOL. Find the number of different selections if the five letters include at least 2 Os and at least 1 T. [4]

58. [9709/w19/63/q1]

There are 300 students at a music college. All students play exactly one of the guitar, the piano or the flute. The numbers of male and female students that play each of the instruments are given in the following table.

	Guitar	Piano	Flute
Female students	62	35	43
Male students	78	40	42

(i) Find the probability that a randomly chosen student at the college is a male who does not play the piano. [1]

(ii) Determine whether the events ‘a randomly chosen student is male’ and ‘a randomly chosen student does not play the piano’ are independent, justifying your answer. [2]

59. [9709/m18/62/q3]

Last Saturday, Sarah recorded the colour and type of 160 cars in a car park. All the cars that were not red or silver in colour were grouped together as 'other'. Her results are shown in the following table.

		Type of car		
		Saloon	Hatchback	Estate
Colour of car	Red	20	40	12
	Silver	14	26	10
	Other	6	24	8

(i) Find the probability that a randomly chosen car in the car park is a silver estate car. [1]

(ii) Find the probability that a randomly chosen car in the car park is a hatchback car. [1]

(iii) Find the probability that a randomly chosen car in the car park is red, given that it is a hatchback car. [2]

(iv) One of the cars in the car park is chosen at random. Determine whether the events 'the car is a hatchback car' and 'the car is red' are independent, justifying your answer. [2]

60. [9709/s18/61/q6]

Vehicles approaching a certain road junction from town A can either turn left, turn right or go straight on. Over time it has been noted that of the vehicles approaching this particular junction from town A, 55% turn left, 15% turn right and 30% go straight on. The direction a vehicle takes at the junction is independent of the direction any other vehicle takes at the junction.

(i) Find the probability that, of the next three vehicles approaching the junction from town A, one goes straight on and the other two either both turn left or both turn right. [4]

(ii) Three vehicles approach the junction from town A. Given that all three drivers choose the same direction at the junction, find the probability that they all go straight on. [4]

61. [9709/s18/62/q2]

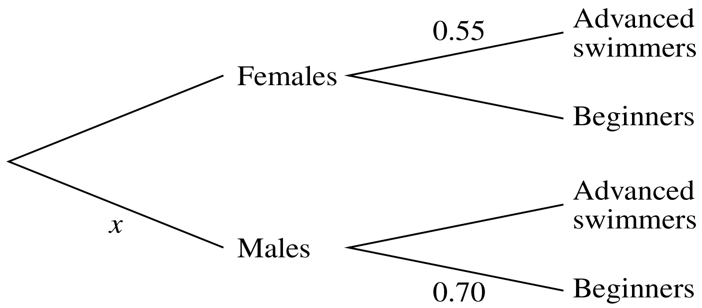
In a group of students, $\frac{3}{4}$ are male. The proportion of male students who like their curry hot is $\frac{3}{5}$ and the proportion of female students who like their curry hot is $\frac{4}{5}$. One student is chosen at random.

(i) Find the probability that the student chosen is either female, or likes their curry hot, or is both female and likes their curry hot. [4]

(ii) Showing your working, determine whether the events 'the student chosen is male' and 'the student chosen likes their curry hot' are independent. [2]

62. [9709/s18/63/q3]

The members of a swimming club are classified either as 'Advanced swimmers' or 'Beginners'. The proportion of members who are male is x, and the proportion of males who are Beginners is 0.7. The proportion of females who are Advanced swimmers is 0.55. This information is shown in the tree diagram.



For a randomly chosen member, the probability of being an Advanced swimmer is the same as the probability of being a Beginner.

(i) Find x. [3]

(ii) Given that a randomly chosen member is an Advanced swimmer, find the probability that the member is male. [3]

63. [9709/w18/61/q7]

In a group of students, the numbers of boys and girls studying Art, Music and Drama are given in the following table. Each of these 160 students is studying exactly one of these subjects.

	Art	Music	Drama
Boys	24	40	32
Girls	15	12	37

(i) Find the probability that a randomly chosen student is studying Music. [1]

(ii) Determine whether the events 'a randomly chosen student is a boy' and 'a randomly chosen student is studying Music' are independent, justifying your answer. [2]

(iii) Find the probability that a randomly chosen student is not studying Drama, given that the student is a girl. [2]

(iv) Three students are chosen at random. Find the probability that exactly 1 is studying Music and exactly 2 are boys. [5]

64. [9709/w18/62/q1]

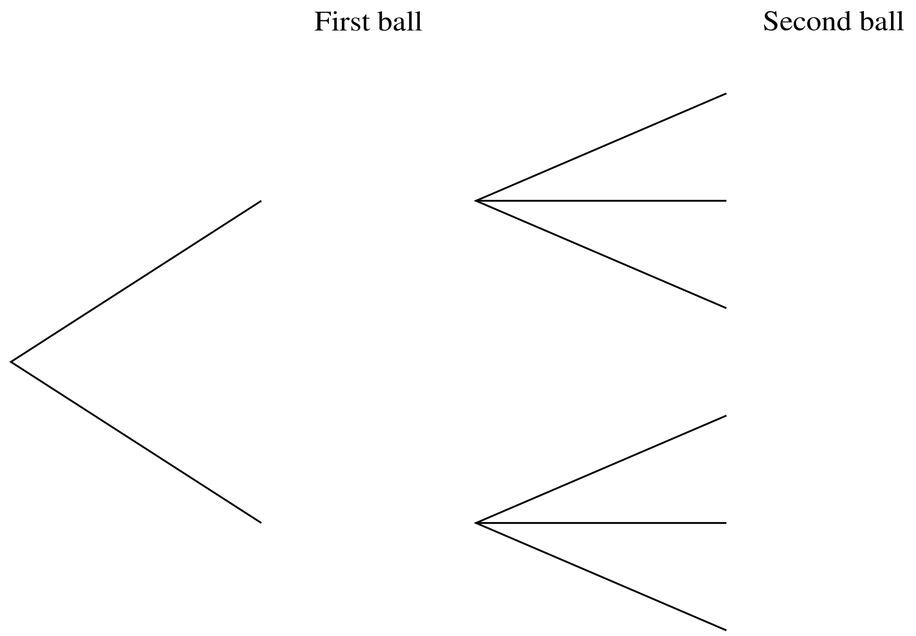
(i) How many different arrangements are there of the 11 letters in the word MISSISSIPPI? [2]

(ii) Two letters are chosen at random from the 11 letters in the word MISSISSIPPI. Find the probability that these two letters are the same. [3]

65. [9709/w18/63/q3]

A box contains 3 red balls and 5 blue balls. One ball is taken at random from the box and not replaced. A yellow ball is then put into the box. A second ball is now taken at random from the box.

(i) Complete the tree diagram to show all the outcomes and the probability for each branch. [2]



(ii) Find the probability that the two balls taken are the same colour. [2]

(iii) Find the probability that the first ball taken is red, given that the second ball taken is blue. [3]

66. [9709/w18/63/q4]

Out of a class of 8 boys and 4 girls, a group of 7 people is chosen at random.

(i) Find the probability that the group of 7 includes one particular boy. [3]

(ii) Find the probability that the group of 7 includes at least 2 girls. [4]

67. [9709/m17/62/q2]

A bag contains 10 pink balloons, 9 yellow balloons, 12 green balloons and 9 white balloons. 7 balloons are selected at random without replacement. Find the probability that exactly 3 of them are green. [3]

68. [9709/s17/61/q2]

Ashfaq throws two fair dice and notes the numbers obtained. R is the event 'The product of the two numbers is 12'. T is the event 'One of the numbers is odd and one of the numbers is even'. By finding appropriate probabilities, determine whether events R and T are independent. [5]

69. [9709/s17/61/q3]

Redbury United soccer team play a match every week. Each match can be won, drawn or lost. At the beginning of the soccer season the probability that Redbury United win their first match is $\frac{3}{5}$, with equal probabilities of losing or drawing. If they win the first match, the probability that they win the second match is $\frac{7}{10}$ and the probability that they lose the second match is $\frac{1}{10}$. If they draw the first match they are equally likely to win, draw or lose the second match. If they lose the first match, the probability that they win the second match is $\frac{3}{10}$ and the probability that they draw the second match is $\frac{1}{20}$.

(i) Draw a fully labelled tree diagram to represent the first two matches played by Redbury United in the soccer season. [2]

(ii) Given that Redbury United win the second match, find the probability that they lose the first match. [4]

70. [9709/s17/62/q4]

Two identical biased triangular spinners with sides marked 1, 2 and 3 are spun. For each spinner, the probabilities of landing on the sides marked 1, 2 and 3 are p, q and r respectively. The score is the sum of the numbers on the sides on which the spinners land. You are given that P(score is 6) = $\frac{1}{36}$ and P(score is 5) = $\frac{1}{9}$. Find the values of p, q and r. [6]

71. [9709/s17/62/q7]

During the school holidays, each day Khalid either rides on his bicycle with probability 0.6, or on his skateboard with probability 0.4. Khalid does not ride on both on the same day. If he rides on his bicycle then the probability that he hurts himself is 0.05. If he rides on his skateboard the probability that he hurts himself is 0.75.

(i) Find the probability that Khalid hurts himself on any particular day. [2]

(ii) Given that Khalid hurts himself on a particular day, find the probability that he is riding on his skateboard. [2]

(iii) There are 45 days of school holidays. Show that the variance of the number of days Khalid rides on his skateboard is the same as the variance of the number of days that Khalid rides on his bicycle. [2]

(iv) Find the probability that Khalid rides on his skateboard on at least 2 of 10 randomly chosen days in the school holidays. [3]

72. [9709/s17/63/q1]

A biased die has faces numbered 1 to 6. The probabilities of the die landing on 1, 3 or 5 are each equal to 0.1. The probabilities of the die landing on 2 or 4 are each equal to 0.2. The die is thrown twice. Find the probability that the sum of the numbers it lands on is 9. [4]

73. [9709/s17/63/q3]

A shop sells two makes of coffee, Café Premium and Café Standard. Both coffees come in two sizes, large jars and small jars. Of the jars on sale, 65% are Café Premium and 35% are Café Standard. Of the Café Premium, 40% of the jars are large and of the Café Standard, 25% of the jars are large. A jar is chosen at random.

(i) Find the probability that the jar is small. [2]

(ii) Find the probability that the jar is Café Standard given that it is large. [3]

74. [9709/w17/61/q5]

Over a period of time Julian finds that on long-distance flights he flies economy class on 82% of flights. On the rest of the flights he flies first class. When he flies economy class, the probability that he gets a good night's sleep is x. When he flies first class, the probability that he gets a good night's sleep is 0.9.

(i) Draw a fully labelled tree diagram to illustrate this situation. [2]

The probability that Julian gets a good night's sleep on a randomly chosen flight is 0.285.

(ii) Find the value of x. [2]

(iii) Given that on a particular flight Julian does not get a good night's sleep, find the probability that he is flying economy class. [3]

75. [9709/w17/63/q3]

At the end of a revision course in mathematics, students have to pass a test to gain a certificate. The probability of any student passing the test at the first attempt is 0.85. Those students who fail are allowed to retake the test once, and the probability of any student passing the retake test is 0.65.

(i) Draw a fully labelled tree diagram to show all the outcomes. [2]

(ii) Given that a student gains the certificate, find the probability that this student fails the test on the first attempt. [4]

76. [9709/w17/63/q6]

A car park has spaces for 18 cars, arranged in a line. On one day there are 5 cars, of different makes, parked in randomly chosen positions and 13 empty spaces.

(i) Find the number of possible arrangements of the 5 cars in the car park. [2]

(ii) Find the probability that the 5 cars are not all next to each other. [5]

On another day, 12 cars of different makes are parked in the car park. 5 of these cars are red, 4 are white and 3 are black. Elizabeth selects 3 of these cars.

(iii) Find the number of selections Elizabeth can make that include cars of at least 2 different colours. [5]

77. [9709/m16/62/q3]

A fair eight-sided die has faces marked 1, 2, 3, 4, 5, 6, 7, 8. The score when the die is thrown is the number on the face the die lands on. The die is thrown twice.

- Event R is 'one of the scores is exactly 3 greater than the other score'.
- Event S is 'the product of the scores is more than 19'.

(i) Find the probability of R. [2]

(ii) Find the probability of S. [2]

(iii) Determine whether events R and S are independent. Justify your answer. [3]

78. [9709/m16/62/q5]

In a certain town, 35% of the people take a holiday abroad and 65% take a holiday in their own country. Of those going abroad 80% go to the seaside, 15% go camping and 5% take a city break. Of those taking a holiday in their own country, 20% go to the seaside and the rest are divided equally between camping and a city break.

(i) A person is chosen at random. Given that the person chosen goes camping, find the probability that the person goes abroad. [5]

(ii) A group of n people is chosen randomly. The probability of all the people in the group taking a holiday in their own country is less than 0.002. Find the smallest possible value of n. [3]

79. [9709/s16/61/q3]

The probability that the school bus is on time on any particular day is 0.6. If the bus is on time the probability that Sam the driver gets a cup of coffee is 0.9. If the bus is not on time the probability that Sam gets a cup of coffee is 0.3.

(i) Find the probability that Sam gets a cup of coffee. [2]

(ii) Given that Sam does not get a cup of coffee, find the probability that the bus is not on time. [3]

80. [9709/s16/62/q1]

Ayman's breakfast drink is tea, coffee or hot chocolate with probabilities 0.65, 0.28, 0.07 respectively. When he drinks tea, the probability that he has milk in it is 0.8. When he drinks coffee, the probability that he has milk in it is 0.5. When he drinks hot chocolate he always has milk in it.

(i) Draw a fully labelled tree diagram to represent this information. [2]

(ii) Find the probability that Ayman's breakfast drink is coffee, given that his drink has milk in it. [3]

81. [9709/s16/63/q1]

In a group of 30 adults, 25 are right-handed and 8 wear spectacles. The number who are right-handed and do not wear spectacles is 19.

(i) Copy and complete the following table to show the number of adults in each category. [2]

	Wears spectacles	Does not wear spectacles	Total
Right-handed			
Not right-handed			
Total			30

An adult is chosen at random from the group. Event X is 'the adult chosen is right-handed'; event Y is 'the adult chosen wears spectacles'.

(ii) Determine whether X and Y are independent events, justifying your answer. [3]

82. [9709/w16/61/q6]

Deeti has 3 red pens and 1 blue pen in her left pocket and 3 red pens and 1 blue pen in her right pocket. 'Operation T' consists of Deeti taking one pen at random from her left pocket and placing it in her right pocket, then taking one pen at random from her right pocket and placing it in her left pocket.

(i) Find the probability that, when Deeti carries out operation T, she takes a blue pen from her left pocket and then a blue pen from her right pocket. [2]

The random variable X is the number of blue pens in Deeti's left pocket after carrying out operation T.

(ii) Find $P(X = 1)$. [3]

(iii) Given that the pen taken from Deeti's right pocket is blue, find the probability that the pen taken from Deeti's left pocket is blue. [4]

83. [9709/w16/62/q1]

When Anya goes to school, the probability that she walks is 0.3 and the probability that she cycles is 0.65; if she does not walk or cycle she takes the bus. When Anya walks the probability that she is late is 0.15. When she cycles the probability that she is late is 0.1 and when she takes the bus the probability that she is late is 0.6. Given that Anya is late, find the probability that she cycles. [5]

84. [9709/w16/63/q4]

For a group of 250 cars the numbers, classified by colour and country of manufacture, are shown in the table.

	Germany	Japan	Korea
Silver	40	26	34
White	32	22	26
Red	28	12	30

One car is selected at random from this group. Find the probability that the selected car is

(i) a red or silver car manufactured in Korea, [1]

(ii) not manufactured in Japan. [1]

X is the event that the selected car is white. Y is the event that the selected car is manufactured in Germany.

(iii) By using appropriate probabilities, determine whether events X and Y are independent. [5]

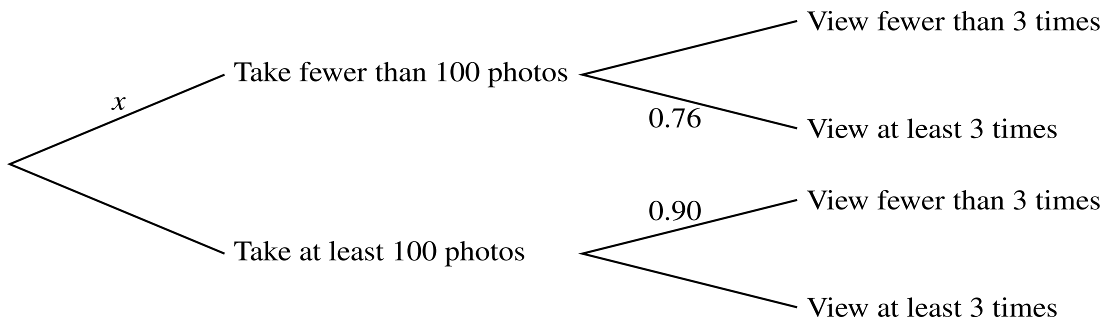
85. [9709/s15/61/q3]

Jason throws two fair dice, each with faces numbered 1 to 6. Event A is 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B is 'the product of the two numbers obtained is even'.

(i) Determine whether events A and B are independent, showing your working. [5]

(ii) Are events A and B mutually exclusive? Justify your answer. [1]

86. [9709/s15/61/q4]

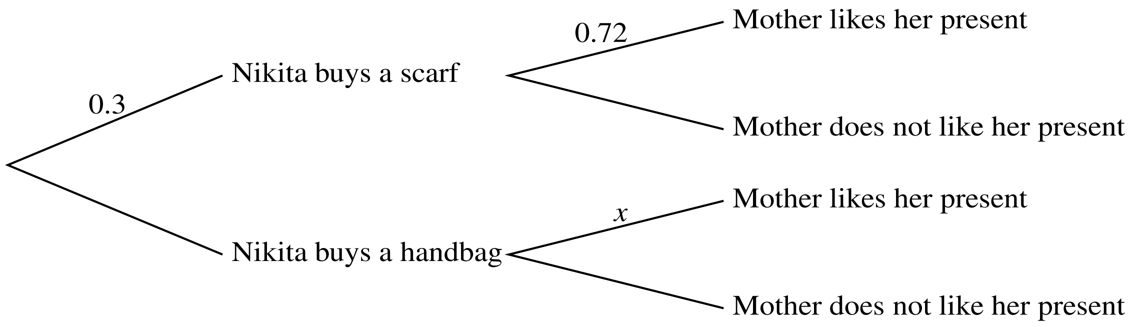


A survey is undertaken to investigate how many photos people take on a one-week holiday and also how many times they view past photos. For a randomly chosen person, the probability of taking fewer than 100 photos is x. The probability that these people view past photos at least 3 times is 0.76. For those who take at least 100 photos, the probability that they view past photos fewer than 3 times is 0.90. This information is shown in the tree diagram. The probability that a randomly chosen person views past photos fewer than 3 times is 0.801.

(i) Find x. [3]

(ii) Given that a person views past photos at least 3 times, find the probability that this person takes at least 100 photos. [4]

87. [9709/s15/62/q4]



Nikita goes shopping to buy a birthday present for her mother. She buys either a scarf, with probability 0.3, or a handbag. The probability that her mother will like the choice of scarf is 0.72. The probability that her mother will like the choice of handbag is x. This information is shown on the tree diagram. The probability that Nikita's mother likes the present that Nikita buys is 0.783.

(i) Find x. [3]

(ii) Given that Nikita's mother does not like her present, find the probability that the present is a scarf. [4]

88. [9709/s15/63/q2]

When Joanna cooks, the probability that the meal is served on time is $\frac{1}{5}$. The probability that the kitchen is left in a mess is $\frac{3}{5}$. The probability that the meal is not served on time and the kitchen is not left in a mess is $\frac{3}{10}$. Some of this information is shown in the following table.

	Kitchen left in a mess	Kitchen not left in a mess	Total
Meal served on time			$\frac{1}{5}$
Meal not served on time		$\frac{3}{10}$	
Total			1

(i) Copy and complete the table. [3]

(ii) Given that the kitchen is left in a mess, find the probability that the meal is not served on time. [2]

89. [9709/w15/61/q7]

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

90. [9709/w15/62/q3]

One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

91. [9709/w15/63/q2]

In country X, 25% of people have fair hair. In country Y, 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y. A person is chosen at random from these 28 million people.

(i) Find the probability that the person chosen is from country X. [1]

(ii) Find the probability that the person chosen has fair hair. [2]

(iii) Find the probability that the person chosen is from country X, given that the person has fair hair. [2]

92. [9709/w15/63/q3]

Ellie throws two fair tetrahedral dice, each with faces numbered 1, 2, 3 and 4. She notes the numbers on the faces that the dice land on. Event S is 'the sum of the two numbers is 4'. Event T is 'the product of the two numbers is an odd number'.

(i) Determine whether events S and T are independent, showing your working. [5]

(ii) Are events S and T exclusive? Justify your answer. [1]

Chapter 4

Probability distribution

1. [9709/m25/52/q1]

Jacob throws three coins at the same time.
The first coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$.
The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.
The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$.
The random variable X is the number of heads obtained.

(a) Show that $\mathrm{P}(X = 2) = \frac{3}{20}$. [1]

(b) Draw up the probability distribution table for X. [3]

(c) Given that $\mathrm{E}(X) = \frac{47}{60}$, find $\mathrm{Var}(X)$. [2]

2. [9709/s25/51/q6]

A bag contains 10 marbles, of which 4 are red and 6 are blue. Four marbles are selected from the bag at random, without replacement. The random variable X denotes the number of blue marbles selected.

(a) Show that $P(X = 2) = \frac{3}{7}$. [2]

(b) Draw up the probability distribution table for X. [4]

(c) Find the probability that at least 2 of the marbles chosen are blue, given that at least 1 red marble and at least 1 blue marble are chosen. [3]

3. [9709/s25/52/q1]

Rachel has three coins. The first coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$. The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$. The third coin is fair.

Rachel throws the three coins at the same time. The random variable X is the number of **tails** that she obtains.

Draw up the probability distribution table for X. [3]

4. [9709/s25/55/q1]

Two fair 6-sided dice with faces labelled 1, 2, 3, 4, 5, 6 are thrown. The two scores are noted. The random variable X is defined as follows.

- If the two scores are equal, $X = 0$
- If the scores are not equal, X is the larger score minus the smaller score

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

5. [9709/w25/51/q1]

The random variable X takes the value x with probability kx^2, where k is a constant and x takes the values -2, 1, 2, 3 only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $\mathrm{E}(X)$. [1]

(c) Find $\mathrm{P}(X \neq 2 \mid X > 0)$. [2]

6. [9709/w25/52/q2]

Kayla has a bag containing 3 red marbles, 1 blue marble and 2 green marbles. She selects one marble from the bag at random and does not replace it in the bag. She repeats this process until she obtains a green marble. The random variable X is the number of marbles that she needs to select until she obtains a green marble.

(a) Draw up the probability distribution table for X. [4]

(b) Find $\text{Var}(X)$. [3]

7. [9709/w25/53/q4]

A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 4 sides, numbered 0, 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The two spinners are spun at the same time.

The random variable X denotes the higher of the two scores obtained. If the two scores are equal, then the value of X is 0.

(a) Draw up the probability distribution table for X. [3]

(b) Find Var(X). [3]

The red spinner, with sides numbered 1, 2, 3, 4, is spun repeatedly.

(c) Find the probability that it lands on 4 for the first time before the 8th spin. [2]

8. [9709/w25/55/q2]

Kai has a spinner with four sides, labelled 1, 2, 3, 4. When the spinner is spun, the score is the number on the side on which the spinner lands. The random variable X denotes this score. The probability distribution table for X is given below.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	0.4	$2p$	p

(a) Find the numerical value of $\mathrm{Var}(X)$. [4]

Kai spins his spinner 10 times.

(b) Find the probability that a score of 2 is obtained fewer than 8 times. [3]

9. [9709/m24/52/q5]

Anil is taking part in a tournament. In each game in this tournament, players are awarded 2 points for a win, 1 point for a draw and 0 points for a loss. For each of Anil's games, the probabilities that he will win, draw or lose are 0.5, 0.3 and 0.2 respectively. The results of the games are all independent of each other.

The random variable X is the total number of points that Anil scores in his first 3 games in the tournament.

(a) Show that $P(X = 2) = 0.114$. [2]

(b) Complete the probability distribution table for X. [3]

x	0	1	2	3	4	5	6
$P(X = x)$			0.114	0.207	0.285		0.125

(c) Find the value of $\text{Var}(X)$. [3]

10. [9709/s24/51/q6]

Harry has three coins:

- One coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$.
- The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.
- The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$.

Harry throws the three coins. The random variable X is the number of heads that he obtains.

(a) Draw up the probability distribution table for X. [4]

Harry has two other coins, each of which is biased so that the probability of obtaining a head when it is thrown is p. He throws all five coins at the same time. The random variable Y is the number of heads that he obtains.

(b) Given that $\mathrm{P}(Y = 0) = 6\mathrm{P}(Y = 5)$, find the value of p. [3]

11. [9709/s24/52/q5]

Jasmine has one \$5 coin, two \$2 coins and two \$1 coins. She selects two of these coins at random. The random variable X is the total value, in dollars, of these two coins.

(a) Show that $\mathrm{P}(X = 7) = 0.2$. [1]

(b) Draw up the probability distribution table for X. [3]

(c) Find the value of $\mathrm{Var}(X)$. [3]

12. [9709/s24/53/q1]

The numbers on the faces of a fair six-sided dice are 1, 2, 2, 3, 3, 3. The random variable X is the total score when the dice is rolled twice.

(a) Draw up the probability distribution table for X. [3]

(b) Find the value of Var(X). [3]

(c) Find the probability that X is even given that $X > 3$. [2]

13. [9709/w24/51/q2]

The random variable X takes the values $-2, -1, 0, 2, 3$. It is given that $P(X = x) = k(x^2 + 2)$, where k is a positive constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find the value of $\mathrm{Var}(X)$. [3]

14. [9709/w24/52/q3]

A fair coin and an ordinary fair six-sided dice are thrown at the same time. The random variable X is defined as follows.

- If the coin shows a tail, X is twice the score on the dice.
- If the coin shows a head, X is the score on the dice if the score is even and X is 0 otherwise.

(a) Draw up the probability distribution table for X. [3]

(b) Find $\mathrm{Var}(X)$. [3]

15. [9709/w24/53/q2]

A red fair six-sided dice has faces labelled 1, 1, 1, 2, 2, 2. A blue fair six-sided dice has faces labelled 1, 1, 2, 2, 3, 3. Both dice are thrown. The random variable X is the product of the scores on the two dice.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$. [1]

16. [9709/m23/52/q2]

Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. [3]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

(c) Given that $E(X) = 2.1$, find the value of $Var(X)$. [2]

17. [9709/s23/52/q1]

The random variable X takes the values -2, 2 and 3. It is given that

$$P(X = x) = k(x^2 - 1),$$

where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

18. [9709/s23/53/q3]

The random variable X takes the values 1, 2, 3, 4. It is given that $P(X = x) = kx(x + a)$, where k and a are constants.

(a) Given that $P(X = 4) = 3P(X = 2)$, find the value of a and the value of k. [4]

(b) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [1]

(c) Given that $E(X) = 3.2$, find $Var(X)$. [2]

19. [9709/w23/51/q5]

A red spinner has four sides labelled 1, 2, 3, 4. When the spinner is spun, the score is the number on the side on which it lands. The random variable X denotes this score. The probability distribution table for X is given below.

x	1	2	3	4
$P(X = x)$	0.28	p	$2p$	$3p$

(a) Show that $p = 0.12$. [1]

A fair blue spinner and a fair green spinner each have four sides labelled 1, 2, 3, 4. All three spinners (red, blue and green) are spun at the same time.

(b) Find the probability that the sum of the three scores is 4 or less. [3]

(c) Find the probability that the product of the three scores is 4 or less given that X is odd. [5]

20. [9709/w23/52/q1]

A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below.

x	0	1	2	3
$P(X = x)$	0.4	p	r	0.15

(a) Given that $E(X) = 1.1$, find the value of p and the value of r. [3]

(b) Find the numerical value of $\mathrm{Var}(X)$. [2]

21. [9709/w23/53/q1]

Becky sometimes works in an office and sometimes works at home. The random variable X denotes the number of days that she works at home in any given week. It is given that

$$P(X = x) = kx(x + 1),$$

where k is a constant and $x = 1$, 2, 3 or 4 only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

22. [9709/m22/52/q1]

A fair red spinner has edges numbered $1, 2, 2, 3$. A fair blue spinner has edges numbered $-3, -2, -1, -1$. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

(b) Given that $\mathrm{E}(X) = 0.25$, find the value of $\mathrm{Var}(X)$. [2]

23. [9709/s22/51/q4]

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

(b) Find $\mathrm{E}(X)$. [1]

Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

24. [9709/s22/52/q2]

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

25. [9709/s22/53/q3]

The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

26. [9709/w22/51/q1]

The probability distribution table for a random variable X is shown below.

x	-2	-1	0.5	1	2
$P(X = x)$	0.12	p	q	0.16	0.3

Given that $E(X) = 0.28$, find the value of p and the value of q. [4]

27. [9709/w22/52/q3]

Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces.

(a) Find the probability that a score of 17 or more is first obtained on the 6th throw. [3]

(b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws. [2]

28. [9709/w22/52/q5]

Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $P(B) = \frac{7}{16}$. [2]

(b) Find $P(A \mid B)$. [2]

The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw up the probability distribution table for X. [3]

29. [9709/w22/53/q4]

Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which each spinner lands is recorded. The random variable X denotes the highest number recorded.

(a) Show that $P(X = 2) = \frac{7}{64}$. [3]

(b) Complete the probability distribution table for X. [2]

x	1	2	3	4
$P(X = x)$		$\frac{7}{64}$	$\frac{19}{64}$	

On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) Find $P(Y = 6)$. [1]

(d) Find $P(Y > 4)$. [2]

30. [9709/m21/52/q4]

The random variable X takes the values 1, 2, 3, 4 only. The probability that X takes the value x is $kx(5-x)$, where k is a constant.

(a) Draw up the probability distribution table for X, in terms of k. [2]

(b) Show that $\mathrm{Var}(X) = 1.05$. [4]

31. [9709/s21/51/q7]

Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable X is the number of tins that she needs to open.

(a) Show that $P(X = 3) = \frac{6}{35}$. [2]

(b) Draw up the probability distribution table for X. [4]

(c) Find $\mathrm{Var}(X)$. [3]

32. [9709/s21/52/q4]

A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered −2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

33. [9709/s21/53/q2]

The random variable X can take only the values $-2, -1, 0, 1, 2$. The probability distribution of X is given in the following table.

x	-2	-1	0	1	2
$\mathrm{P}(X = x)$	p	p	0.1	q	q

Given that $\mathrm{P}(X \geqslant 0) = 3\mathrm{P}(X < 0)$, find the values of p and q. [4]

34. [9709/w21/51/q4]

A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered -1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $\text{Var}(X)$. [3]

35. [9709/w21/52/q3]

A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement.

(a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$. [3]

The random variable X is the number of yellow marbles selected.

(b) Draw up the probability distribution table for X. [3]

(c) Find $E(X)$. [1]

36. [9709/w21/53/q6]

In a game, Jim throws three darts at a board. This is called a 'turn'. The centre of the board is called the bull's-eye.

The random variable X is the number of darts in a turn that hit the bull's-eye. The probability distribution of X is given in the following table.

x	0	1	2	3
$P(X = x)$	0.6	p	q	0.05

It is given that $E(X) = 0.55$.

(a) Find the values of p and q. [4]

(b) Find $\text{Var}(X)$. [2]

Jim is practising for a competition and he repeatedly throws three darts at the board.

(c) Find the probability that $X = 1$ in at least 3 of 12 randomly chosen turns. [3]

(d) Find the probability that Jim first succeeds in hitting the bull's-eye with all three darts on his 9th turn. [1]

37. [9709/m20/52/q2]

An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained.

(a) Find the probability that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6. [3]

On another occasion, this die is thrown 3 times. The random variable X is the number of times that a 1 or a 6 is obtained.

(b) Draw up the probability distribution table for X. [3]

(c) Find $\mathrm{E}(X)$. [2]

38. [9709/s20/51/q3]

A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

(a) Draw up the probability distribution table for the number of jellies that Jemeel chooses. [4]

The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

39. [9709/s20/52/q5]

A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Find $E(X)$ and $Var(X)$. [3]

40. [9709/s20/53/q4]

A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered -2, -1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

(b) Find $\text{Var}(X)$. [3]

41. [9709/w20/51/q4]

The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

(b) Find the probability that $Y = 2$ given that Y is even. [2]

42. [9709/w20/52/q2]

A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Given that $\mathrm{E}(X) = \frac{15}{8}$, find $\mathrm{Var}(X)$. [2]

43. [9709/w20/53/q6]

Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

(c) Given that $E(X) = \frac{32}{15}$, find $\text{Var}(X)$. [2]

44. [9709/m19/62/q4]

The random variable X takes the values -1, 1, 2, 3 only. The probability that X takes the value x is kx^2, where k is a constant.

(i) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

45. [9709/s19/61/q6]

At a funfair, Amy pays \$1 for two attempts to make a bell ring by shooting at it with a water pistol.

- If she makes the bell ring on her first attempt, she receives \$3 and stops playing. This means that overall she has gained \$2.
- If she makes the bell ring on her second attempt, she receives \$1.50 and stops playing. This means that overall she has gained \$0.50.
- If she does not make the bell ring in the two attempts, she has lost her original \$1.

The probability that Amy makes the bell ring on any attempt is 0.2, independently of other attempts.

(i) Show that the probability that Amy loses her original \$1 is 0.64. [2]

(ii) Complete the probability distribution table for the amount that Amy gains. [4]

Amy's gain (\$)			
Probability	0.64		

(iii) Calculate Amy's expected gain. [1]

46. [9709/s19/62/q5]

Maryam has 7 sweets in a tin; 6 are toffees and 1 is a chocolate. She chooses one sweet at random and takes it out. Her friend adds 3 chocolates to the tin. Then Maryam takes another sweet at random out of the tin.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Draw up the probability distribution table for the number of toffees taken. [3]

(iii) Find the mean number of toffees taken. [1]

(iv) Find the probability that the first sweet taken is a chocolate, given that the second sweet taken is a toffee. [4]

47. [9709/s19/63/q6]

A fair five-sided spinner has sides numbered 1, 1, 1, 2, 3. A fair three-sided spinner has sides numbered 1, 2, 3. Both spinners are spun once and the score is the product of the numbers on the sides the spinners land on.

(i) Draw up the probability distribution table for the score. [4]

(ii) Find the mean and the variance of the score. [3]

(iii) Find the probability that the score is greater than the mean score. [2]

48. [9709/w19/61/q4]

In a probability distribution the random variable X takes the values -1, 0, 1, 2, 4. The probability distribution table for X is as follows.

x	-1	0	1	2	4
$P(X = x)$	$\frac{1}{4}$	p	p	$\frac{3}{8}$	$4p$

(i) Find the value of p. [2]

(ii) Find $E(X)$ and $Var(X)$. [3]

(iii) Given that X is greater than zero, find the probability that X is equal to 2. [2]

49. [9709/w19/62/q5]

A fair red spinner has four sides, numbered 1, 2, 3, 3. A fair blue spinner has three sides, numbered -1, 0, 2. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [4]

(ii) Find $\mathrm{Var}(X)$. [3]

50. [9709/w19/63/q6]

A box contains 3 red balls and 5 white balls. One ball is chosen at random from the box and is not returned to the box. A second ball is now chosen at random from the box.

(i) Find the probability that both balls chosen are red. [1]

(ii) Show that the probability that the balls chosen are of different colours is $\frac{15}{28}$. [2]

(iii) Given that the second ball chosen is red, find the probability that the first ball chosen is red. [2]

The random variable X denotes the number of red balls chosen.

(iv) Draw up the probability distribution table for X. [2]

(v) Find $\text{Var}(X)$. [3]

51. [9709/m18/62/q4]

The discrete random variable X has the following probability distribution.

x	-2	0	1	3	4
$P(X = x)$	0.2	0.1	p	0.1	q

(i) Given that $E(X) = 1.7$, find the values of p and q. [4]

(ii) Find $\mathrm{Var}(X)$. [2]

52. [9709/s18/61/q3]

Andy has 4 red socks and 8 black socks in his drawer. He takes 2 socks at random from his drawer.

(i) Find the probability that the socks taken are of different colours. [2]

The random variable X is the number of red socks taken.

(ii) Draw up the probability distribution table for X. [3]

(iii) Find $\mathrm{E}(X)$. [1]

53. [9709/s18/62/q4]

Mrs Rupal chooses 3 animals at random from 5 dogs and 2 cats. The random variable X is the number of cats chosen.

(i) Draw up the probability distribution table for X. [4]

(ii) You are given that $\mathrm{E}(X) = \frac{6}{7}$. Find the value of $\mathrm{Var}(X)$. [2]

54. [9709/s18/63/q5]

A game is played with 3 coins, A, B and C. Coins A and B are biased so that the probability of obtaining a head is 0.4 for coin A and 0.75 for coin B. Coin C is not biased. The 3 coins are thrown once.

(i) Draw up the probability distribution table for the number of heads obtained. [5]

(ii) Hence calculate the mean and variance of the number of heads obtained. [3]

55. [9709/w18/61/q2]

A random variable X has the probability distribution shown in the following table, where p is a constant.

x	-1	0	1	2	4
$P(X = x)$	p	p	$2p$	$2p$	0.1

(i) Find the value of p. [1]

(ii) Given that $E(X) = 1.15$, find $\mathrm{Var}(X)$. [2]

56. [9709/w18/62/q6]

A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 3 sides, numbered 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [3]

(ii) Find Var(X). [3]

(iii) Find the probability that X is equal to 1, given that X is non-zero. [3]

57. [9709/w18/63/q2]

A fair 6-sided die has the numbers -1, -1, 0, 0, 1, 2 on its faces. A fair 3-sided spinner has edges numbered -1, 0, 1. The die is thrown and the spinner is spun. The number on the uppermost face of the die and the number on the edge on which the spinner comes to rest are noted. The sum of these two numbers is denoted by X.

(i) Draw up a table showing the probability distribution of X. [3]

(ii) Find $\mathrm{Var}(X)$. [3]

58. [9709/m17/62/q6]

Pack A consists of ten cards numbered 0, 0, 1, 1, 1, 1, 1, 3, 3, 3. Pack B consists of six cards numbered 0, 0, 2, 2, 2, 2. One card is chosen at random from each pack. The random variable X is defined as the sum of the two numbers on the cards.

(i) Show that $P(X = 2) = \frac{2}{15}$. [2]

(ii) Draw up the probability distribution table for X. [4]

(iii) Given that $X = 3$, find the probability that the card chosen from pack A is a 1. [3]

59. [9709/s17/62/q3]

In a probability distribution the random variable X takes the value x with probability kx^2, where k is a constant and x takes values -2, -1, 2, 4 only.

(i) Show that $\mathrm{P}(X = -2)$ has the same value as $\mathrm{P}(X = 2)$. [1]

(ii) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

(iii) Find $\mathrm{E}(X)$. [2]

60. [9709/w17/61/q1]

The discrete random variable X has the following probability distribution.

x	1	2	3	6
$P(X = x)$	0.15	p	0.4	q

Given that $E(X) = 3.05$, find the values of p and q. [4]

61. [9709/w17/62/q3]

A box contains 6 identical-sized discs, of which 4 are blue and 2 are red. Discs are taken at random from the box in turn and not replaced. Let X be the number of discs taken, up to and including the first blue one.

(i) Show that $P(X = 3) = \frac{1}{15}$. [2]

(ii) Draw up the probability distribution table for X. [3]

62. [9709/w17/63/q4]

A fair die with faces numbered 1, 2, 2, 2, 3, 6 is thrown. The score, X, is found by squaring the number on the face the die shows and then subtracting 4.

(i) Draw up a table to show the probability distribution of X. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

63. [9709/m16/62/q2]

A flower shop has 5 yellow roses, 3 red roses and 2 white roses. Martin chooses 3 roses at random. Draw up the probability distribution table for the number of white roses Martin chooses. [4]

64. [9709/s16/61/q2]

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X.

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

65. [9709/s16/61/q4]

A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [6]

66. [9709/s16/62/q3]

A particular type of bird lays 1, 2, 3 or 4 eggs in a nest each year. The probability of x eggs is equal to kx, where k is a constant.

(i) Draw up a probability distribution table, in terms of k, for the number of eggs laid in a year and find the value of k. [3]

(ii) Find the mean and variance of the number of eggs laid in a year by this type of bird. [3]

67. [9709/s16/63/q3]

Two ordinary fair dice are thrown. The resulting score is found as follows.

- If the two dice show different numbers, the score is the smaller of the two numbers.
- If the two dice show equal numbers, the score is 0.

(i) Draw up the probability distribution table for the score. [4]

(ii) Calculate the expected score. [2]

68. [9709/w16/61/q2]

Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

69. [9709/w16/62/q2]

Noor has 3 T-shirts, 4 blouses and 5 jumpers. She chooses 3 items at random. The random variable X is the number of T-shirts chosen.

(i) Show that the probability that Noor chooses exactly one T-shirt is $\frac{27}{55}$. [3]

(ii) Draw up the probability distribution table for X. [4]

70. [9709/s15/62/q5]

A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

71. [9709/s15/63/q4]

A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

(ii) Construct the probability distribution table for the number of white rabbits in the sample. [3]

(iii) Find the expected value of the number of white rabbits in the sample. [1]

72. [9709/w15/61/q6]

Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X. [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

73. [9709/w15/62/q6]

A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered -3, -2, -1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

		Spinner A			
		1	2	3	3
Spinner B	-3	-2			
	-2			1	
	-1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find Var(X). [3]

(iv) Find the probability that X is even, given that X is positive. [2]

Chapter 5

Binomial and geometric distribution

1. [9709/m25/52/q2]

Last year, an online store sold a large number of computers. 55% of the computers were made by company F, 30% were made by company G and 15% were made by company H.

A random sample of 3 customers who each bought a computer from this store is chosen.

(a) Find the probability that the 3 customers bought computers all made by different companies. [1]

A random sample of 12 customers who each bought a computer from this store is chosen.

(b) Find the probability that fewer than 10 of these customers bought a computer made by company F. [3]

A random sample of 140 customers who each bought a computer from this store is chosen.

(c) Use a suitable approximation to find the probability that more than 24 of these customers bought a computer made by company H. [5]

2. [9709/s25/51/q4]

Every Saturday, a particular community holds a 'Puzzle' event to raise money for a new Leisure Centre. Competitors attempt to solve a puzzle as quickly as possible.

Last Saturday, 600 competitors took part. The times taken to complete the puzzle were normally distributed with mean 32.4 minutes and standard deviation 2.5 minutes.

(a) How many competitors would you expect to have times within 1.2 minutes of the mean time? [4]

In this Saturday's event, 60% of the competitors had times less than 36.0 minutes.

(b) 9 competitors who took part in this Saturday's event are selected at random.

Find the probability that at least 2 and fewer than 8 of these competitors had times less than 36.0 minutes. [3]

(c) 80 competitors who took part in this Saturday's event are selected at random.

Use a suitable approximation to find the probability that more than 50 of these competitors had times less than 36.0 minutes. [5]

3. [9709/s25/52/q4]

Vehicles approaching a certain road junction from Bromley must go either left, right or straight on. Over time, it is known that 30% turn left, 25% turn right and 45% go straight on. The driver of each vehicle chooses a direction independently of all other drivers.

(a) Find the probability that the next three vehicles approaching this junction from Bromley all go in different directions. [2]

(b) Find the probability that, from the vehicles approaching this junction from Bromley today, the 1st vehicle to go left is before the 9th vehicle. [2]

(c) Find the probability that, from the vehicles approaching this junction from Bromley today, the 2nd vehicle to go left is the 7th vehicle. [2]

4. [9709/s25/53/q2]

At a large college, all students who study Science also study exactly one of Art or Drama or Music. 20% of these students study Art, 45% study Drama and 35% study Music.

(a) 3 students are selected at random from the students who study Science.

Find the probability that at least 1 of these students studies Drama. [2]

(b) 10 students are selected at random from the students who study Science.

Find the probability that more than 7 study Art or Music. [3]

5. [9709/s25/53/q3]

A fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly until a 3 is obtained. The number of throws taken is denoted by the random variable X.

(a) Find $P(X = 8)$. [1]

(b) Find $P(X < 9)$. [2]

(c) Find the probability that a 3 is obtained for the second time before the 6th throw. [3]

6. [9709/w25/51/q5]

On any given day, Cooper either wears a blue jumper or he wears a green jumper or he does not wear a jumper. The probability that he wears a blue jumper is 0.6 and the probability that he wears a green jumper is 0.3. Whether Cooper wears a jumper of either colour, or does not wear a jumper, on any day is independent of his choice on any other day.

(a) Find the probability, that in a week (7 days), Cooper wears a blue jumper on at least 5 days. [3]

(b) Use a suitable approximation to find the probability that, in any 150-day period, Cooper does not wear a jumper on fewer than 22 days. [5]

7. [9709/w25/52/q6]

For a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday.

(a) Find the probability that, out of 10 randomly chosen people, none of them will have their next birthday on a Saturday or Sunday. [1]

(b) Find the probability that, out of 10 randomly chosen people, fewer than 3 will have their next birthday on a Wednesday. [3]

(c) Use a suitable approximation to find the probability that, out of 392 randomly chosen people, more than 65 will have their next birthday on a Friday. [5]

8. [9709/w25/53/q1]

There are a large number of students at Greenfield college. Each student travels to college by car, by bus or on foot, independently of any other student. The probability that any student travels by car is 0.4. The probability that any student travels by bus is 0.35.

(a) 3 students from Greenfield college are selected at random.

Find the probability that none of these 3 students travel to college by car. [1]

(b) 11 students from Greenfield college are selected at random.

Find the probability that fewer than 9 of these 11 students travel to college by car or by bus. [3]

9. [9709/w25/55/q1]

A fair 6-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly. The random variable X denotes the number of throws required to obtain a 6.

(a) Find $P(X = 7)$. [1]

(b) Find $P(3 \leqslant X \leqslant 5)$. [2]

10. [9709/m24/52/q2]

Sam is a member of a soccer club. She is practising scoring goals. The probability that Sam will score a goal on any attempt is 0.7, independently of all other attempts.

(a) Sam makes 10 attempts at scoring goals.

Find the probability that Sam will score goals on fewer than 8 of these attempts. [3]

(b) Find the probability that Sam's first successful attempt will be before her 5th attempt. [2]

(c) Wei is a member of the same soccer club. He is also practising scoring goals. The probability that Wei will score a goal on any attempt is 0.6, independently of all other attempts.

Wei is going to keep making attempts until he scores 3 goals.

Find the probability that he scores his third goal on his 7th attempt. [3]

11. [9709/s24/51/q5]

In a certain area in the Arctic the probability that it snows on any given day is 0.7, independent of all other days.

(a) Find the probability that in a week (7 days) it snows on at least five days. [3]

A week in which it snows on at least five days out of seven is called a 'white' week.

(b) Find the probability that in three randomly chosen weeks at least one is a white week. [2]

In a different area in the Arctic, the probability that a week is a white week is 0.8 .

(c) Use a suitable approximation to find the probability that in 60 randomly chosen weeks fewer than 47 are white weeks. [5]

12. [9709/s24/52/q1]

Rajesh applies once every year for a ticket to a music festival. The probability that he is successful in any particular year is 0.3, independently of other years.

(a) Find the probability that Rajesh is successful for the first time on his 7th attempt. [1]

(b) Find the probability that Rajesh is successful for the first time before his 6th attempt. [2]

(c) Find the probability that Rajesh is successful for the second time on his 10th attempt. [2]

13. [9709/s24/53/q5]

Salah decides to attempt the crossword puzzle in his newspaper each day. The probability that he will complete the puzzle on any given day is 0.65, independent of other days.

(a) Find the probability that Salah completes the puzzle for the first time on the 5th day. [1]

(b) Find the probability that Salah completes the puzzle for the second time on the 5th day. [2]

(c) Find the probability that Salah completes the puzzle fewer than 5 times in a week (7 days). [3]

(d) Use a suitable approximation to find the probability that Salah completes the puzzle more than 50 times in a period of 84 days. [5]

14. [9709/w24/51/q1]

Nicola throws an ordinary fair six-sided dice. The random variable X is the number of throws that she takes to obtain a 6.

(a) Find $P(X < 8)$. [2]

(b) Find the probability that Nicola obtains a 6 for the second time on her 8th throw. [2]

15. [9709/w24/52/q7]

In a game, players attempt to score a goal by kicking a ball into a net. The probability that Leno scores a goal is 0.4 on any attempt, independently of all other attempts. The random variable X denotes the number of attempts that it takes Leno to score a goal.

(a) Find $P(X = 5)$. [1]

(b) Find $P(3 \leqslant X \leqslant 7)$. [2]

(c) Find the probability that Leno scores his second goal on or before his 5th attempt. [3]

Leno has 75 attempts to score a goal.

(d) Use a suitable approximation to find the probability that Leno scores more than 28 goals but fewer than 35 goals. [5]

16. [9709/w24/53/q1]

30% of the residents of Wimfield own an electric car. Three residents are chosen at random.

(a) Find the probability that either all three own an electric car or none of them owns an electric car. [2]

A random sample of 125 of the residents of Wimfield is selected.

(b) Use a suitable approximation to find the probability that more than 45 of these residents own an electric car. [5]

17. [9709/m23/52/q3]

80% of the residents of Kinwawa are in favour of a leisure centre being built in the town.

20 residents of Kinwawa are chosen at random and asked, in turn, whether they are in favour of the leisure centre.

(a) Find the probability that more than 17 of these residents are in favour of the leisure centre. [3]

(b) Find the probability that the 5th person asked is the first person who is **not** in favour of the leisure centre. [1]

(c) Find the probability that the 7th person asked is the second person who is **not** in favour of the leisure centre. [2]

18. [9709/s23/51/q6]

Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X = 3) = \frac{3}{64}$. [2]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	$\frac{81}{256}$			$\frac{3}{64}$	$\frac{1}{256}$

(c) Find $E(X)$. [2]

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

19. [9709/s23/52/q2]

A sports event is taking place for 4 days, beginning on Sunday. The probability that it will rain on Sunday is 0.4. On any subsequent day, the probability that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the probability that it does **not** rain on any of the 4 days of the event. [1]

(b) Find the probability that the first day on which it rains during the event is Tuesday. [2]

(c) Find the probability that it rains on exactly one of the 4 days of the event. [3]

20. [9709/s23/53/q1]

Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X.

(a) State the value of $\mathrm{E}(X)$. [1]

(b) Find the probability that exactly 5 throws are required to obtain a pair of heads. [1]

(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads. [2]

21. [9709/w23/51/q2]

Hazeem repeatedly throws two ordinary fair 6-sided dice at the same time. On each occasion, the score is the sum of the two numbers that she obtains.

(a) Find the probability that it takes exactly 5 throws of the two dice for Hazeem to obtain a score of 8 or more. [2]

(b) Find the probability that it takes no more than 4 throws of the two dice for Hazeem to obtain a score of 8 or more. [2]

(c) For 8 randomly chosen throws of the two dice, find the probability that Hazeem obtains a score of 8 or more on fewer than 3 occasions. [3]

22. [9709/w23/52/q2]

George has a fair 5-sided spinner with sides labelled 1, 2, 3, 4, 5. He spins the spinner and notes the number on the side on which the spinner lands.

(a) Find the probability that it takes fewer than 7 spins for George to obtain a 5. [2]

George spins the spinner 10 times.

(b) Find the probability that he obtains a 5 more than 4 times but fewer than 8 times. [3]

23. [9709/w23/53/q5]

The probability that a driver passes an advanced driving test is 0.3 on any given attempt.

(a) Dipak keeps taking the test until he passes. The random variable X denotes the number of attempts required for Dipak to pass the test.

(i) Find $P(2 \leqslant X \leqslant 6)$. [2]

(ii) Find $E(X)$. [1]

Five friends will each take their advanced driving test tomorrow.

(b) Find the probability that at least three of them will pass tomorrow. [3]

75 people will take their advanced driving test next week.

(c) Use an approximation to find the probability that more than 20 of them will pass next week. [5]

24. [9709/m22/52/q2]

In a certain country, the probability of more than 10 cm of rain on any particular day is 0.18, independently of the weather on any other day.

(a) Find the probability that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. [3]

(b) For 3 randomly chosen 7-day periods, find the probability that exactly two of these periods have at least one day with more than 10 cm of rain. [3]

25. [9709/s22/51/q4]

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

(b) Find $\mathrm{E}(X)$. [1]

Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

26. [9709/s22/52/q5]

In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

27. [9709/s22/53/q4]

Ramesh throws an ordinary fair 6-sided die.

(a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]

(b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

Ramesh now repeatedly throws two ordinary fair 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the probability that Ramesh obtains a total of less than 4 on at least three throws. [4]

28. [9709/w22/51/q2]

The residents of Persham were surveyed about the reliability of their internet service. 12% rated the service as 'poor', 36% rated it as 'satisfactory' and 52% rated it as 'good'.

A random sample of 8 residents of Persham is chosen.

(a) Find the probability that more than 2 and fewer than 8 of them rate their internet service as poor or satisfactory. [3]

A random sample of 125 residents of Persham is now chosen.

(b) Use an approximation to find the probability that more than 72 of these residents rate their internet service as good. [5]

29. [9709/w22/52/q6]

At a company's call centre, 90% of callers are connected immediately to a representative.

A random sample of 12 callers is chosen.

(a) Find the probability that fewer than 10 of these callers are connected immediately. [3]

A random sample of 80 callers is chosen.

(b) Use an approximation to find the probability that more than 69 of these callers are connected immediately. [5]

(c) Justify the use of your approximation in part **(b)**. [1]

30. [9709/w22/53/q5]

Company A produces bags of sugar. An inspector finds that on average 10% of the bags are underweight.

10 of the bags are chosen at random.

(a) Find the probability that fewer than 3 of these bags are underweight. [3]

The weights of the bags of sugar produced by company B are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the probability that a randomly chosen bag produced by company B weighs more than 1.11 kg. [3]

81% of the bags of sugar produced by company B weigh less than w kg.

(c) Find the value of w. [3]

31. [9709/m21/52/q1]

A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands.

(a) Find the probability that a score of 3 is obtained for the first time on the 8th spin. [1]

(b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time. [2]

32. [9709/s21/51/q6]

In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

(c) Justify the use of your approximation in part **(b)**. [1]

33. [9709/s21/52/q1]

An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X.

(a) Write down the mean of X. [1]

(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. [2]

(c) Find the probability that a 5 is first obtained in fewer than 10 throws. [2]

34. [9709/s21/52/q5]

Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

35. [9709/w21/51/q1]

Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

(a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]

(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

36. [9709/w21/52/q5]

In a certain region, the probability that any given day in October is wet is 0.16, independently of other days.

(a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet. [3]

(b) Find the probability that the first wet day in October is 8 October. [2]

(c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October. [2]

37. [9709/m20/52/q5]

In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

(a) Find the probability that the number of adults in this sample who own a car is less than 6. [3]

A random sample of 120 adults from Greenton is now chosen.

(b) Use an approximation to find the probability that more than 75 of them own a car. [5]

38. [9709/s20/51/q1]

The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the mean of X. [1]

(c) Find the probability that a score of 4 is first obtained on the 6th throw. [1]

(d) Find $\mathrm{P}(X < 8)$. [2]

39. [9709/s20/52/q7]

On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

(c) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

40. [9709/s20/53/q2]

In a certain large college, 22% of students own a car.

(a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car. [1]

(b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4. [3]

41. [9709/s20/53/q5]

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

42. [9709/w20/51/q3]

Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

(a) Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

(b) Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

43. [9709/w20/52/q1]

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

44. [9709/w20/53/q2]

An ordinary fair die is thrown until a 6 is obtained.

(a) Find the probability that obtaining a 6 takes more than 8 throws. [2]

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the expected value of X. [1]

(c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

45. [9709/w20/53/q4]

The 13 00 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the probability that the train arrives late on fewer than 3 days. [3]

A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [5]

46. [9709/m19/62/q6]

The results of a survey by a large supermarket show that 35% of its customers shop online.

(i) Six customers are chosen at random. Find the probability that more than three of them shop online. [3]

(ii) For a random sample of n customers, the probability that at least one of them shops online is greater than 0.95. Find the least possible value of n. [3]

(iii) For a random sample of 100 customers, use a suitable approximating distribution to find the probability that more than 39 shop online. [5]

47. [9709/s19/62/q3]

The probability that Janice will buy an item online in any week is 0.35. Janice does not buy more than one item online in any week.

(i) Find the probability that, in a 10-week period, Janice buys at most 7 items online. [3]

(ii) The probability that Janice buys at least one item online in a period of n weeks is greater than 0.99. Find the smallest possible value of n. [3]

48. [9709/w19/61/q2]

Annan has designed a new logo for a sportswear company. A survey of a large number of customers found that 42% of customers rated the logo as good.

(i) A random sample of 10 customers is chosen. Find the probability that fewer than 8 of them rate the logo as good. [3]

(ii) On another occasion, a random sample of n customers of the company is chosen. Find the smallest value of n for which the probability that at least one person rates the logo as good is greater than 0.995. [3]

49. [9709/m18/62/q8]

The results of a survey at a certain large college show that the proportion of students who own a car is $\frac{1}{4}$.

(i) Five students at the college are chosen at random. Find the probability that at least four of these students own a car. [3]

(ii) For a random sample of n students at the college, the probability that at least one of the students owns a car is greater than 0.995. Find the least possible value of n. [3]

(iii) For a random sample of 160 students at the college, use a suitable approximate distribution to find the probability that fewer than 50 own a car. [4]

50. [9709/w18/62/q3]

Jake attempts the crossword puzzle in his daily newspaper every day. The probability that he will complete the puzzle on any given day is 0.75, independently of all other days.

(i) Find the probability that he will complete the puzzle at least three times over a period of five days. [3]

Kenny also attempts the puzzle every day. The probability that he will complete the puzzle on a Monday is 0.8. The probability that he will complete it on a Tuesday is 0.9 if he completed it on the previous day and 0.6 if he did not complete it on the previous day.

(ii) Find the probability that Kenny will complete the puzzle on at least one of the two days Monday and Tuesday in a randomly chosen week. [3]

51. [9709/s17/61/q5]

Eggs are sold in boxes of 20. Cracked eggs occur independently and the mean number of cracked eggs in a box is 1.4.

(i) Calculate the probability that a randomly chosen box contains exactly 2 cracked eggs. [3]

(ii) Calculate the probability that a randomly chosen box contains at least 1 cracked egg. [2]

(iii) A shop sells n of these boxes of eggs. Find the smallest value of n such that the probability of there being at least 1 cracked egg in each box sold is less than 0.01. [2]

52. [9709/s17/62/q7]

During the school holidays, each day Khalid either rides on his bicycle with probability 0.6, or on his skateboard with probability 0.4. Khalid does not ride on both on the same day. If he rides on his bicycle then the probability that he hurts himself is 0.05. If he rides on his skateboard the probability that he hurts himself is 0.75.

(i) Find the probability that Khalid hurts himself on any particular day. [2]

(ii) Given that Khalid hurts himself on a particular day, find the probability that he is riding on his skateboard. [2]

(iii) There are 45 days of school holidays. Show that the variance of the number of days Khalid rides on his skateboard is the same as the variance of the number of days that Khalid rides on his bicycle. [2]

(iv) Find the probability that Khalid rides on his skateboard on at least 2 of 10 randomly chosen days in the school holidays. [3]

53. [9709/s17/63/q5]

Hebe attempts a crossword puzzle every day. The number of puzzles she completes in a week (7 days) is denoted by X.

(i) State two conditions that are required for X to have a binomial distribution. [2]

On average, Hebe completes 7 out of 10 of these puzzles.

(ii) Use a binomial distribution to find the probability that Hebe completes at least 5 puzzles in a week. [3]

(iii) Use a binomial distribution to find the probability that, over the next 10 weeks, Hebe completes 4 or fewer puzzles in exactly 3 of the 10 weeks. [3]

54. [9709/w17/61/q3]

An experiment consists of throwing a biased die 30 times and noting the number of 4s obtained. This experiment was repeated many times and the average number of 4s obtained in 30 throws was found to be 6.21.

(i) Estimate the probability of throwing a 4. [1]

Hence

(ii) find the variance of the number of 4s obtained in 30 throws, [1]

(iii) find the probability that in 15 throws the number of 4s obtained is 2 or more. [3]

55. [9709/w17/62/q4]

A fair tetrahedral die has faces numbered 1, 2, 3, 4. A coin is biased so that the probability of showing a head when thrown is $\frac{1}{3}$. The die is thrown once and the number n that it lands on is noted. The biased coin is then thrown n times. So, for example, if the die lands on 3, the coin is thrown 3 times.

(i) Find the probability that the die lands on 4 and the number of times the coin shows heads is 2. [3]

(ii) Find the probability that the die lands on 3 and the number of times the coin shows heads is 3. [1]

(iii) Find the probability that the number the die lands on is the same as the number of times the coin shows heads. [3]

56. [9709/w17/63/q1]

A statistics student asks people to complete a survey. The probability that a randomly chosen person agrees to complete the survey is 0.2. Find the probability that at least one of the first three people asked agrees to complete the survey. [2]

57. [9709/s16/62/q4]

When people visit a certain large shop, on average 34% of them do not buy anything, 53% spend less than \$50 and 13% spend at least \$50.

(i) 15 people visiting the shop are chosen at random. Calculate the probability that at least 14 of them buy something. [3]

(ii) n people visiting the shop are chosen at random. The probability that none of them spends at least \$50 is less than 0.04. Find the smallest possible value of n. [3]

58. [9709/w16/61/q3]

Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

(i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]

(ii) Find the probability that, out of 12 randomly chosen visitors, fewer than 3 see lions. [3]

(iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

59. [9709/w16/63/q2]

A fair triangular spinner has three sides numbered 1, 2, 3. When the spinner is spun, the score is the number of the side on which it lands. The spinner is spun four times.

(i) Find the probability that at least two of the scores are 3. [3]

(ii) Find the probability that the sum of the four scores is 5. [3]

60. [9709/s15/61/q6]

(i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

(ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer. [5]

(iii) Justify your use of the approximation in part **(ii)**. [1]

61. [9709/s15/62/q1]

A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

62. [9709/s15/63/q5]

The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than 29 cm is classified as ‘large’.

(i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large. [6]

(ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n. [3]

63. [9709/w15/61/q1]

In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

64. [9709/w15/63/q7]

A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

(iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

(iv) Justify the use of your approximation in part **(iii)**. [1]

Chapter 6

The normal distribution

1. [9709/m25/52/q2]

Last year, an online store sold a large number of computers. 55% of the computers were made by company *F*, 30% were made by company *G* and 15% were made by company *H*.

A random sample of 3 customers who each bought a computer from this store is chosen.

(a) Find the probability that the 3 customers bought computers all made by different companies. [1]

A random sample of 12 customers who each bought a computer from this store is chosen.

(b) Find the probability that fewer than 10 of these customers bought a computer made by company *F*. [3]

A random sample of 140 customers who each bought a computer from this store is chosen.

(c) Use a suitable approximation to find the probability that more than 24 of these customers bought a computer made by company *H*. [5]

2. [9709/m25/52/q5]

The mass of peaches sold per day in a supermarket is normally distributed with mean 65.8 kg and standard deviation 9.6 kg.

(a) Find the probability that the mass of peaches sold on any given day is between 56 kg and 75 kg. [3]

The mass of cherries sold per day in a supermarket is normally distributed with mean 72.4 kg and standard deviation σ kg. It is known that on 10% of days less than 59.1 kg of cherries are sold.

(b) Find the value of σ. [3]

The supermarket is open 7 days a week.

(c) Find the probability that, in a randomly chosen week, the first day on which less than 59.1 kg of cherries are sold is the fifth day of the week. [1]

(d) Find the probability that, in a randomly chosen week, the first day on which less than 59.1 kg of cherries are sold is before the fifth day of the week. [2]

3. [9709/s25/51/q1]

The masses of the bags of rice made by a company are normally distributed with mean μ kg and standard deviation 0.14 kg. The probability that the mass of a randomly chosen bag of this rice is less than 1.48 kg is 0.22.

Find the value of μ. [3]

4. [9709/s25/51/q4]

Every Saturday, a particular community holds a 'Puzzle' event to raise money for a new Leisure Centre. Competitors attempt to solve a puzzle as quickly as possible.

Last Saturday, 600 competitors took part. The times taken to complete the puzzle were normally distributed with mean 32.4 minutes and standard deviation 2.5 minutes.

(a) How many competitors would you expect to have times within 1.2 minutes of the mean time? [4]

In this Saturday's event, 60% of the competitors had times less than 36.0 minutes.

(b) 9 competitors who took part in this Saturday's event are selected at random.

Find the probability that at least 2 and fewer than 8 of these competitors had times less than 36.0 minutes. [3]

(c) 80 competitors who took part in this Saturday's event are selected at random.

Use a suitable approximation to find the probability that more than 50 of these competitors had times less than 36.0 minutes. [5]

5. [9709/s25/52/q2]

In Millford, 70% of the residents own a bicycle. A random sample of 160 residents is selected.

Use a suitable approximation to find the probability that more than 120 of these residents own a bicycle. [5]

6. [9709/s25/52/q7]

Kestrels are birds whose adult wingspans are normally distributed with mean 74.8 cm and standard deviation 3.2 cm. A random sample of 120 adult kestrels is selected.

(a) How many of these 120 adult kestrels would you expect to have wingspan between 72.4 cm and 76.3 cm? [4]

The masses of adult kestrels are normally distributed with mean μ kg and standard deviation σ kg. It is known that 20% of adult kestrels have mass greater than 0.202 kg and 28% have mass less than 0.185 kg.

(b) Find the value of μ and the value of σ. [5]

10 adult kestrels are selected at random.

(c) Find the probability that fewer than 3 have masses greater than 0.202 kg. [3]

7. [9709/s25/53/q6]

A company sells bags of pasta. The masses of large bags of pasta are normally distributed with mean 2.50 kg and standard deviation 0.12 kg.

(a) Find the probability that the mass of pasta in a randomly chosen large bag is less than 2.65 kg. [2]

A restaurant manager buys 160 of these large bags of pasta.

(b) Find the number of bags for which you would expect the mass of pasta to be more than 1.65 standard deviations above the mean. [3]

The masses of small bags of pasta sold by the company are normally distributed with mean μ kg and standard deviation σ kg. Tests show that 77% of these bags have masses greater than 1.26 kg, and 44% have masses less than 1.35 kg.

(c) Find, in either order, the value of μ and the value of σ. [5]

8. [9709/s25/55/q2]

The heights of trees in a certain forest are classified as tall, medium or small. The heights can be modelled by a normal distribution with mean 20 m and standard deviation 5 m. Trees with a height of less than 14 m are classified as small.

(a) For 150 randomly chosen trees from this forest, how many would you expect to be classified as small? [4]

Trees from this forest are classified as tall if their height is at least h m. 25% of the trees are classified as tall.

(b) Find the value of h. [3]

9. [9709/s25/55/q3]

In a certain large school, on average, two pupils in five have music lessons.

A random sample of 80 pupils from this school is chosen.

(a) Use an approximation to find the probability that fewer than 27 pupils have music lessons. [5]

A random sample of 10 pupils from this school is now chosen.

(b) Find the probability that no more than 2 pupils have music lessons. [3]

10. [9709/w25/51/q5]

On any given day, Cooper either wears a blue jumper or he wears a green jumper or he does not wear a jumper. The probability that he wears a blue jumper is 0.6 and the probability that he wears a green jumper is 0.3. Whether Cooper wears a jumper of either colour, or does not wear a jumper, on any day is independent of his choice on any other day.

(a) Find the probability, that in a week (7 days), Cooper wears a blue jumper on at least 5 days. [3]

(b) Use a suitable approximation to find the probability that, in any 150-day period, Cooper does not wear a jumper on fewer than 22 days. [5]

11. [9709/w25/51/q6]

A factory produces chocolate bars. The weights of the bars are normally distributed with mean 155 g and standard deviation 6 g. A random sample of 350 of these bars is chosen.

(a) How many of these 350 bars would you expect to weigh between 148 g and 160 g? [4]

A second factory also produces chocolate bars. The weights of these bars are normally distributed with mean μ g and standard deviation σ g. Tests show that 8% of the bars weigh more than 114.0 g and 20% weigh less than 99.5 g.

(b) Find the value of μ and the value of σ. [5]

12. [9709/w25/52/q3]

The heights of the 124 Senior members of the Giraffes basketball club are normally distributed with mean 187.4 cm and standard deviation 6.4 cm.

(a) How many members of the club would you expect to have heights within 5 cm of the mean? [4]

The heights of the Junior members of the Giraffes club are normally distributed with mean 172.7 cm and standard deviation σ cm. 23% of these members have height less than 170.3 cm.

(b) Find the value of σ. [3]

13. [9709/w25/52/q6]

For a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday.

(a) Find the probability that, out of 10 randomly chosen people, none of them will have their next birthday on a Saturday or Sunday. [1]

(b) Find the probability that, out of 10 randomly chosen people, fewer than 3 will have their next birthday on a Wednesday. [3]

(c) Use a suitable approximation to find the probability that, out of 392 randomly chosen people, more than 65 will have their next birthday on a Friday. [5]

14. [9709/w25/53/q7]

The times taken by Obi to walk to work each morning are normally distributed with mean 14.8 minutes and standard deviation 1.5 minutes.

(a) Find the probability that, on a randomly chosen day, Obi takes more than 15.6 minutes to walk to work. [3]

(b) On 90% of days, Obi takes more than t minutes to walk to work.

Find the value of t. [3]

Obi walks to work 5 days a week for 45 weeks in a year.

(c) On how many days in a year would you expect Obi to take within one minute of the mean time to walk to work? [4]

15. [9709/w25/55/q6]

A large number of runners took part in two charity runs to raise money for a new community centre.

In the first run, the times to complete the run were normally distributed with mean 46.3 seconds and standard deviation 6.4 seconds.

(a) Find the probability that a randomly chosen runner took more than 55.1 seconds to complete the run. [3]

In the second run, the times to complete the run were normally distributed with mean 39.8 seconds and standard deviation σ seconds. 10% of the runners took more than 48.6 seconds.

(b) Find the value of σ. [3]

150 runners are chosen at random from those who took part in the second run.

(c) Use an approximation to find the probability that fewer than 20 of the 150 runners took more than 48.6 seconds to complete the run. [5]

16. [9709/m24/52/q4]

A company sells small and large bags of rice. The masses of the small bags of rice are normally distributed with mean 1.20 kg and standard deviation 0.16 kg.

(a) In a random sample of 500 of these small bags of rice, how many would you expect to have a mass greater than 1.26 kg? [4]

The masses of the large bags of rice are normally distributed with mean 2.50 kg and standard deviation σ kg. 20% of these large bags of rice have a mass less than 2.40 kg.

(b) Find the value of σ. [3]

A random sample of 80 large bags of rice is chosen.

(c) Use a suitable approximation to find the probability that fewer than 22 of these large bags of rice have a mass less than 2.40 kg. [5]

17. [9709/s24/51/q2]

The lengths of the tails of adult raccoons of a certain species are normally distributed with mean 28 cm and standard deviation 3.3 cm.

(a) Find the probability that a randomly chosen adult raccoon of this species has a tail length between 23 cm and 35 cm. [4]

The masses of adult raccoons of this species are normally distributed with mean 8.5 kg and standard deviation σ kg. 75% of adult raccoons of this species have mass greater than 7.6 kg.

(b) Find the value of σ. [3]

18. [9709/s24/52/q3]

The weights of oranges can be modelled by a normal distribution with mean 131 grams and standard deviation 54 grams. Oranges are classified as small, medium or large. A large orange weighs at least 184 grams and 20% of oranges are classified as small.

(a) Find the percentage of oranges that are classified as large. [3]

(b) Find the greatest possible weight of a small orange. [3]

19. [9709/s24/52/q6]

The residents of Mahjing were asked to classify their local bus service:

- 25% of residents classified their service as good.
- 60% of residents classified their service as satisfactory.
- 15% of residents classified their service as poor.

(a) A random sample of 110 residents of Mahjing is chosen.

Use a suitable approximation to find the probability that fewer than 22 residents classified their bus service as good. [5]

(b) For a random sample of 10 residents of Mahjing, find the probability that fewer than 8 classified their bus service as good or satisfactory. [3]

(c) Three residents of Mahjing are selected at random.

Find the probability that one resident classified the bus service as good, one as satisfactory and one as poor. [2]

20. [9709/s24/53/q2]

In a certain country, the heights of the adult population are normally distributed with mean 1.64 m and standard deviation 0.25 m.

(a) Find the probability that an adult chosen at random from this country will have height greater than 1.93 m. [3]

In another country, the heights of the adult population are also normally distributed. 33% of the adult population have height less than 1.56 m. 25% of the adult population have height greater than 1.86 m.

(b) Find the mean and the standard deviation of this distribution. [5]

21. [9709/w24/51/q5]

The weights of the green apples sold by a shop are normally distributed with mean 90 grams and standard deviation 8 grams.

(a) Find the probability that a randomly chosen green apple weighs between 83 grams and 95 grams. [4]

(b) The shop also sells red apples. 60% of the red apples sold by the shop weigh more than 80 grams. 160 red apples are chosen at random from the shop.

Use a suitable approximation to find the probability that fewer than 105 of the chosen red apples weigh more than 80 grams. [5]

22. [9709/w24/51/q6]

The heights of the female students at Breven college are normally distributed:

- 90% of the female students have heights less than 182.7 cm.
- 40% of the female students have heights less than 162.5 cm.

(a) Find the mean and the standard deviation of the heights of the female students at Breven college. [5]

Ten female students are chosen at random from those at Breven college.

(b) Find the probability that fewer than 8 of these 10 students have heights more than 162.5 cm. [3]

23. [9709/w24/52/q4]

The heights, in metres, of white pine trees are normally distributed with mean 19.8 and standard deviation 2.4 .

In a certain forest there are 450 white pine trees.

(a) How many of these trees would you expect to have height less than 18.2 metres? [4]

The heights, in metres, of red pine trees are normally distributed with mean 23.4 and standard deviation σ. It is known that 26% of red pine trees have height greater than 25.5 metres.

(b) Find the value of σ. [3]

24. [9709/w24/52/q7]

In a game, players attempt to score a goal by kicking a ball into a net. The probability that Leno scores a goal is 0.4 on any attempt, independently of all other attempts. The random variable X denotes the number of attempts that it takes Leno to score a goal.

(a) Find $\mathrm{P}(X = 5)$. [1]

(b) Find $\mathrm{P}(3 \leqslant X \leqslant 7)$. [2]

(c) Find the probability that Leno scores his second goal on or before his 5th attempt. [3]

Leno has 75 attempts to score a goal.

(d) Use a suitable approximation to find the probability that Leno scores more than 28 goals but fewer than 35 goals. [5]

25. [9709/w24/53/q1]

30% of the residents of Wimfield own an electric car. Three residents are chosen at random.

(a) Find the probability that either all three own an electric car or none of them owns an electric car. [2]

A random sample of 125 of the residents of Wimfield is selected.

(b) Use a suitable approximation to find the probability that more than 45 of these residents own an electric car. [5]

26. [9709/w24/53/q3]

In Molimba, the heights, in cm, of adult males are normally distributed with mean 176 cm and standard deviation 4.8 cm.

(a) Find the probability that a randomly chosen adult male in Molimba has a height greater than 170 cm. [3]

60% of adult males in Molimba have a height between 170 cm and k cm, where k is greater than 170.

(b) Find the value of k, giving your answer correct to 1 decimal place. [4]

27. [9709/m23/52/q6]

In a cycling event the times taken to complete a course are modelled by a normal distribution with mean 62.3 minutes and standard deviation 8.4 minutes.

(a) Find the probability that a randomly chosen cyclist has a time less than 74 minutes. [2]

(b) Find the probability that 4 randomly chosen cyclists all have times between 50 and 74 minutes. [4]

In a different cycling event, the times can also be modelled by a normal distribution. 23% of the cyclists have times less than 36 minutes and 10% of the cyclists have times greater than 54 minutes.

(c) Find estimates for the mean and standard deviation of this distribution. [5]

28. [9709/s23/51/q4]

A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and standard deviation 5.2 minutes.

(a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle? [4]

All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean μ and standard deviation σ. It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

(b) Find the value of μ and the value of σ. [5]

29. [9709/s23/51/q6]

Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X = 3) = \frac{3}{64}$. [2]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	$\frac{81}{256}$			$\frac{3}{64}$	$\frac{1}{256}$

(c) Find $E(X)$. [2]

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

30. [9709/s23/52/q5]

The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

(a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm? [4]

The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation σ cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

(b) Find the value of σ. [3]

A random sample of 120 Eastern bluebirds is chosen.

(c) Use an approximation to find the probability that fewer than 80 of these 120 bluebirds have length greater than 17.1 cm. [5]

31. [9709/s23/53/q2]

Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.

Use an approximation to find the probability that, of the 120 voters, between 36 and 54 inclusive voted for Anil. [5]

32. [9709/s23/53/q6]

The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of the mass of grapes sold per day. [3]

The mass of grapes sold on any day is independent of the mass sold on any other day.

(b) 12 days are chosen at random.

Find the probability that less than 16 kg of grapes are sold on more than 2 of these 12 days. [3]

(c) In a random sample of 365 days, on how many days would you expect the mass of grapes sold to be within 1.3 standard deviations of the mean? [4]

33. [9709/w23/51/q3]

A farmer sells eggs. The weights, in grams, of the eggs can be modelled by a normal distribution with mean 80.5 and standard deviation 6.6. Eggs are classified as small, medium or large according to their weight. A small egg weighs less than 76 grams and 40% of the eggs are classified as medium.

(a) Find the percentage of eggs that are classified as small. [3]

(b) Find the least possible weight of an egg classified as large. [3]

150 of the eggs for sale last week were weighed.

(c) Use an approximation to find the probability that more than 68 of these eggs were classified as medium. [5]

34. [9709/w23/52/q3]

A factory produces a certain type of electrical component. It is known that 15% of the components produced are faulty. A random sample of 200 components is chosen.

Use an approximation to find the probability that more than 40 of these components are faulty. [5]

35. [9709/w23/52/q5]

(a) The heights of the members of a club are normally distributed with mean 166 cm and standard deviation 10 cm.

(i) Find the probability that a randomly chosen member of the club has height less than 170 cm. [2]

(ii) Given that 40% of the members have heights greater than h cm, find the value of h correct to 2 decimal places. [3]

(b) The random variable X is normally distributed with mean μ and standard deviation σ.

Given that $\sigma = \frac{2}{3}\mu$, find the probability that a randomly chosen value of X is positive. [3]

36. [9709/w23/53/q2]

The weights of large bags of pasta produced by a company are normally distributed with mean 1.5 kg and standard deviation 0.05 kg.

(a) Find the probability that a randomly chosen large bag of pasta weighs between 1.42 kg and 1.52 kg. [3]

The weights of small bags of pasta produced by the company are normally distributed with mean 0.75 kg and standard deviation σ kg. It is found that 68% of these small bags have weight less than 0.9 kg.

(b) Find the value of σ. [3]

37. [9709/m22/52/q4]

The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

(a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

(b) Find the value of σ. [3]

The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

(c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

38. [9709/s22/51/q5]

The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2,\ 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

(c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? [4]

39. [9709/s22/52/q4]

The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ. [4]

40. [9709/s22/52/q5]

In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

41. [9709/s22/53/q5]

Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 grams to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20 000.

(b) Calculate an estimate for his total income from this year's apples. [3]

Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

42. [9709/w22/51/q2]

The residents of Persham were surveyed about the reliability of their internet service. 12% rated the service as ‘poor’, 36% rated it as ‘satisfactory’ and 52% rated it as ‘good’.

A random sample of 8 residents of Persham is chosen.

(a) Find the probability that more than 2 and fewer than 8 of them rate their internet service as poor or satisfactory. [3]

A random sample of 125 residents of Persham is now chosen.

(b) Use an approximation to find the probability that more than 72 of these residents rate their internet service as good. [5]

43. [9709/w22/51/q4]

In a large population, the systolic blood pressure (SBP) of adults is normally distributed with mean 125.4 and standard deviation 18.6.

(a) Find the probability that the SBP of a randomly chosen adult is less than 132. [2]

The SBP of 12-year-old children in the same population is normally distributed with mean 117. Of these children 88% have SBP more than 108.

(b) Find the standard deviation of this distribution. [3]

Three adults are chosen at random from this population.

(c) Find the probability that each of these three adults has SBP within 1.5 standard deviations of the mean. [4]

44. [9709/w22/52/q2]

The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.

(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm? [4]

(b) Find the probability that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean. [3]

45. [9709/w22/52/q6]

At a company's call centre, 90% of callers are connected immediately to a representative.

A random sample of 12 callers is chosen.

(a) Find the probability that fewer than 10 of these callers are connected immediately. [3]

A random sample of 80 callers is chosen.

(b) Use an approximation to find the probability that more than 69 of these callers are connected immediately. [5]

(c) Justify the use of your approximation in part **(b)**. [1]

46. [9709/w22/53/q2]

In a large college, 32% of the students have blue eyes. A random sample of 80 students is chosen.

Use an approximation to find the probability that fewer than 20 of these students have blue eyes. [5]

47. [9709/w22/53/q5]

Company A produces bags of sugar. An inspector finds that on average 10% of the bags are underweight.

10 of the bags are chosen at random.

(a) Find the probability that fewer than 3 of these bags are underweight. [3]

The weights of the bags of sugar produced by company B are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the probability that a randomly chosen bag produced by company B weighs more than 1.11 kg. [3]

81% of the bags of sugar produced by company B weigh less than w kg.

(c) Find the value of w. [3]

48. [9709/m21/52/q3]

The time spent by shoppers in a large shopping centre has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

(a) Find the probability that a shopper chosen at random spends between 85 and 100 minutes in the shopping centre. [3]

88% of shoppers spend more than t minutes in the shopping centre.

(b) Find the value of t. [3]

49. [9709/m21/52/q7]

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

50. [9709/s21/51/q2]

A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.

How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? [5]

51. [9709/s21/51/q6]

In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

(c) Justify the use of your approximation in part **(b)**. [1]

52. [9709/s21/52/q2]

The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.

Find the value of σ. [4]

53. [9709/s21/52/q5]

Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

54. [9709/s21/53/q5]

The lengths of the leaves of a particular type of tree are modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves from this type of tree and finds that 42 are less than 4 cm long and 100 are more than 10 cm long.

(a) Find estimates for the mean and standard deviation of the lengths of leaves from this type of tree. [5]

The lengths, in cm, of the leaves of a different type of tree have the distribution $N(\mu, \sigma^2)$. The scientist takes a random sample of 800 leaves from this type of tree.

(b) Find how many of these leaves the scientist would expect to have lengths, in cm, between $\mu - 2\sigma$ and $\mu + 2\sigma$. [4]

55. [9709/s21/53/q7]

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

56. [9709/w21/51/q7]

The times, in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) **(i)** On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? [5]

(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. [3]

(b) On 90% of days, Karli spends more than t minutes on social media.

Find the value of t. [3]

57. [9709/w21/52/q6]

The times taken, in minutes, to complete a particular task by employees at a large company are normally distributed with mean 32.2 and standard deviation 9.6.

(a) Find the probability that a randomly chosen employee takes more than 28.6 minutes to complete the task. [3]

(b) 20% of employees take longer than t minutes to complete the task.

Find the value of t. [3]

(c) Find the probability that the time taken to complete the task by a randomly chosen employee differs from the mean by less than 15.0 minutes. [4]

58. [9709/w21/53/q4]

Raj wants to improve his fitness, so every day he goes for a run. The times, in minutes, of his runs have a normal distribution with mean 41.2 and standard deviation 3.6.

(a) Find the probability that on a randomly chosen day Raj runs for more than 43.2 minutes. [3]

(b) Find an estimate for the number of days in a year (365 days) on which Raj runs for less than 43.2 minutes. [2]

(c) On 95% of days, Raj runs for more than t minutes.

Find the value of t. [3]

59. [9709/m20/52/q3]

The weights of apples of a certain variety are normally distributed with mean 82 grams. 22% of these apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples. [3]

(b) Find the probability that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams. [4]

60. [9709/m20/52/q5]

In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

(a) Find the probability that the number of adults in this sample who own a car is less than 6. [3]

A random sample of 120 adults from Greenton is now chosen.

(b) Use an approximation to find the probability that more than 75 of them own a car. [5]

61. [9709/s20/51/q6]

The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

(a) Find the probability that a randomly chosen female snake of this species has length between 50 cm and 60 cm. [4]

The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find estimates for the mean and standard deviation of the lengths of male snakes of this species. [5]

62. [9709/s20/52/q4]

Trees in the Redian forest are classified as tall, medium or short, according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height of less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short. [3]

Of the trees that are classified as tall or medium, one third are tall and two thirds are medium.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 decimal places. [2]

(c) Find the height above which trees are classified as tall. [3]

63. [9709/s20/52/q7]

On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

(c) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

64. [9709/s20/53/q3]

In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. [2]

(b) Find the value of k such that $P(X < k) = 0.75$. [3]

65. [9709/s20/53/q5]

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

66. [9709/w20/51/q5]

The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

(a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

(b) On 90% of days Davin plays on his games machine for more than t hours. Find the value of t. [3]

(c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

67. [9709/w20/52/q3]

Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t. [3]

(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

68. [9709/w20/53/q1]

The times taken to swim 100 metres by members of a large swimming club have a normal distribution with mean 62 seconds and standard deviation 5 seconds.

(a) Find the probability that a randomly chosen member of the club takes between 56 and 66 seconds to swim 100 metres. [3]

(b) 13% of the members of the club take more than t minutes to swim 100 metres. Find the value of t. [3]

69. [9709/w20/53/q4]

The 13 00 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the probability that the train arrives late on fewer than 3 days. [3]

A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [5]

70. [9709/m19/62/q3]

The times taken, in minutes, for trains to travel between Alphaton and Beeton are normally distributed with mean 140 and standard deviation 12.

(i) Find the probability that a randomly chosen train will take less than 132 minutes to travel between Alphaton and Beeton. [3]

(ii) The probability that a randomly chosen train takes more than k minutes to travel between Alphaton and Beeton is 0.675. Find the value of k. [3]

71. [9709/m19/62/q6]

The results of a survey by a large supermarket show that 35% of its customers shop online.

(i) Six customers are chosen at random. Find the probability that more than three of them shop online. [3]

(ii) For a random sample of n customers, the probability that at least one of them shops online is greater than 0.95. Find the least possible value of n. [3]

(iii) For a random sample of 100 customers, use a suitable approximating distribution to find the probability that more than 39 shop online. [5]

72. [9709/s19/61/q5]

In a certain country the probability that a child owns a bicycle is 0.65.

(i) A random sample of 15 children from this country is chosen. Find the probability that more than 12 own a bicycle. [3]

(ii) A random sample of 250 children from this country is chosen. Use a suitable approximation to find the probability that fewer than 179 own a bicycle. [4]

73. [9709/s19/61/q7]

The weight of adult female giraffes has a normal distribution with mean 830 kg and standard deviation 120 kg.

(i) There are 430 adult female giraffes in a particular game reserve. Find the number of these adult female giraffes which can be expected to weigh less than 700 kg. [4]

(ii) Given that 90% of adult female giraffes weigh between $(830 - w)$ kg and $(830 + w)$ kg, find the value of w. [3]

The weight of adult male giraffes has a normal distribution with mean 1190 kg and standard deviation σ kg.

(iii) Given that 83.4% of adult male giraffes weigh more than 950 kg, find the value of σ. [3]

74. [9709/s19/62/q2]

The volume of ink in a certain type of ink cartridge has a normal distribution with mean 30 ml and standard deviation 1.5 ml. People in an office use a total of 8 cartridges of this ink per month. Find the expected number of cartridges per month that contain less than 28.9 ml of this ink. [4]

75. [9709/s19/62/q4]

It is known that 20% of male giant pandas in a certain area weigh more than 121 kg and 71.9% weigh more than 102 kg. Weights of male giant pandas in this area have a normal distribution. Find the mean and standard deviation of the weights of male giant pandas in this area. [5]

76. [9709/s19/63/q1]

The time taken, in minutes, by a ferry to cross a lake has a normal distribution with mean 85 and standard deviation 6.8.

(i) Find the probability that, on a randomly chosen occasion, the time taken by the ferry to cross the lake is between 79 and 91 minutes. [3]

(ii) Over a long period it is found that 96% of ferry crossings take longer than a certain time t minutes. Find the value of t. [3]

77. [9709/s19/63/q5]

On average, 34% of the people who go to a particular theatre are men.

(i) A random sample of 14 people who go to the theatre is chosen. Find the probability that at most 2 people are men. [3]

(ii) Use an approximation to find the probability that, in a random sample of 600 people who go to the theatre, fewer than 190 are men. [5]

78. [9709/w19/61/q7]

The shortest time recorded by an athlete in a 400 m race is called their personal best (PB). The PBs of the athletes in a large athletics club are normally distributed with mean 49.2 seconds and standard deviation 2.8 seconds.

(i) Find the probability that a randomly chosen athlete from this club has a PB between 46 and 53 seconds. [4]

(ii) It is found that 92% of athletes from this club have PBs of more than t seconds. Find the value of t. [3]

Three athletes from the club are chosen at random.

(iii) Find the probability that exactly 2 have PBs of less than 46 seconds. [3]

79. [9709/w19/62/q4]

In Quarendon, 66% of households are satisfied with the speed of their wifi connection.

(i) Find the probability that, out of 10 households chosen at random in Quarendon, at least 8 are satisfied with the speed of their wifi connection. [3]

(ii) A random sample of 150 households in Quarendon is chosen. Use a suitable approximation to find the probability that more than 84 are satisfied with the speed of their wifi connection. [5]

80. [9709/w19/62/q6]

The heights, in metres, of fir trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

(i) Find the probability that a fir tree chosen at random in this forest has a height less than 45 metres. [2]

(ii) Find the probability that a fir tree chosen at random in this forest has a height within 5 metres of the mean. [2]

In another forest, the heights of another type of fir tree are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

(iii) Find the mean and standard deviation of the heights of trees of this type. [5]

81. [9709/w19/63/q4]

The heights of students at the Mainland college are normally distributed with mean 148 cm and standard deviation 8 cm.

(i) The probability that a Mainland student chosen at random has a height less than h cm is 0.67. Find the value of h. [3]

120 Mainland students are chosen at random.

(ii) Find the number of these students that would be expected to have a height within half a standard deviation of the mean. [4]

82. [9709/w19/63/q7]

A competition is taking place between two choirs, the Notes and the Classics. There is a large audience for the competition.

- 30% of the audience are Notes supporters.
- 45% of the audience are Classics supporters.
- The rest of the audience are not supporters of either of these choirs.
- No one in the audience supports both of these choirs.

(i) A random sample of 6 people is chosen from the audience.

(a) Find the probability that no more than 2 of the 6 people are Notes supporters. [3]

(b) Find the probability that none of the 6 people support either of these choirs. [2]

(ii) A random sample of 240 people is chosen from the audience. Use a suitable approximation to find the probability that fewer than 50 do not support either of the choirs. [5]

83. [9709/m18/62/q7]

The weights of packets of a certain type of biscuit are normally distributed with mean 400 grams and standard deviation σ grams.

(i) In a random sample of 6000 packets of this type of biscuit, 225 packets weighed more than 410 grams. Find the value of σ. [4]

(ii) In a random sample of 500 packets of this type of biscuit, how many packets would you expect to find with weights that are more than 1.5 standard deviations from the mean? [4]

84. [9709/m18/62/q8]

The results of a survey at a certain large college show that the proportion of students who own a car is $\frac{1}{4}$.

(i) Five students at the college are chosen at random. Find the probability that at least four of these students own a car. [3]

(ii) For a random sample of n students at the college, the probability that at least one of the students owns a car is greater than 0.995. Find the least possible value of n. [3]

(iii) For a random sample of 160 students at the college, use a suitable approximate distribution to find the probability that fewer than 50 own a car. [4]

85. [9709/s18/61/q4]

(a) The distance that car tyres of a certain make can travel before they need to be replaced has a normal distribution. A survey of a large number of these tyres found that the probability of this distance being more than 36 800 km is 0.0082 and the probability of this distance being more than 31 000 km is 0.6915. Find the mean and standard deviation of the distribution. [5]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$, where $3\sigma = 4\mu$ and $\mu \neq 0$. Find $P(X < 3\mu)$. [3]

86. [9709/s18/61/q5]

In Pelmerdon 22% of families own a dishwasher.

(i) Find the probability that, of 15 families chosen at random from Pelmerdon, between 4 and 6 inclusive own a dishwasher. [3]

(ii) A random sample of 145 families from Pelmerdon is chosen. Use a suitable approximation to find the probability that more than 26 families own a dishwasher. [5]

87. [9709/s18/62/q3]

(i) The volume of soup in Super Soup cartons has a normal distribution with mean μ millilitres and standard deviation 9 millilitres. Tests have shown that 10% of cartons contain less than 440 millilitres of soup. Find the value of μ. [3]

(ii) A food retailer orders 150 Super Soup cartons. Calculate the number of these cartons for which you would expect the volume of soup to be more than 1.8 standard deviations above the mean. [3]

88. [9709/s18/62/q7]

In a certain country, 60% of mobile phones sold are made by Company A, 35% are made by Company B and 5% are made by other companies.

(i) Find the probability that, out of a random sample of 13 people who buy a mobile phone, fewer than 11 choose a mobile phone made by Company A. [3]

(ii) Use a suitable approximation to find the probability that, out of a random sample of 130 people who buy a mobile phone, at least 50 choose a mobile phone made by Company B. [5]

(iii) A random sample of n mobile phones sold is chosen. The probability that at least one of these phones is made by Company B is more than 0.98. Find the least possible value of n. [3]

89. [9709/s18/63/q2]

The random variable X has the distribution $N(-3, \sigma^2)$. The probability that a randomly chosen value of X is positive is 0.25.

(i) Find the value of σ. [3]

(ii) Find the probability that, of 8 random values of X, fewer than 2 will be positive. [3]

90. [9709/s18/63/q6]

The diameters of apples in an orchard have a normal distribution with mean 5.7 cm and standard deviation 0.8 cm. Apples with diameters between 4.1 cm and 5 cm can be used as toffee apples.

(i) Find the probability that an apple selected at random can be used as a toffee apple. [3]

(ii) 250 apples are chosen at random. Use a suitable approximation to find the probability that fewer than 50 can be used as toffee apples. [5]

91. [9709/w18/61/q4]

(a) It is given that $X \sim N(31.4, 3.6)$. Find the probability that a randomly chosen value of X is less than 29.4. [3]

(b) The lengths of fish of a particular species are modelled by a normal distribution. A scientist measures the lengths of 400 randomly chosen fish of this species. He finds that 42 fish are less than 12 cm long and 58 are more than 19 cm long. Find estimates for the mean and standard deviation of the lengths of fish of this species. [5]

92. [9709/w18/61/q5]

At the Nonland Business College, all students sit an accountancy examination at the end of their first year of study. On average, 80% of the students pass this examination.

(i) A random sample of 9 students who will take this examination is chosen. Find the probability that at most 6 of these students will pass the examination. [3]

(ii) A random sample of 200 students who will take this examination is chosen. Use a suitable approximate distribution to find the probability that more than 166 of them will pass the examination. [5]

(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

93. [9709/w18/62/q7]

(a) The time, X hours, for which students use a games machine in any given day has a normal distribution with mean 3.24 hours and standard deviation 0.96 hours.

(i) On how many days of the year (365 days) would you expect a randomly chosen student to use a games machine for less than 4 hours? [3]

(ii) Find the value of k such that $P(X > k) = 0.2$. [3]

(iii) Find the probability that the number of hours for which a randomly chosen student uses a games machine in a day is within 1.5 standard deviations of the mean. [3]

(b) The variable Y is normally distributed with mean μ and standard deviation σ, where $4\sigma = 3\mu$ and $\mu \neq 0$. Find the probability that a randomly chosen value of Y is positive. [3]

94. [9709/w18/63/q5]

The weights of apples sold by a store can be modelled by a normal distribution with mean 120 grams and standard deviation 24 grams. Apples weighing less than 90 grams are graded as ‘small’; apples weighing more than 140 grams are graded as ‘large’; the remainder are graded as ‘medium’.

(i) Show that the probability that an apple chosen at random is graded as medium is 0.692, correct to 3 significant figures. [4]

(ii) Four apples are chosen at random. Find the probability that at least two are graded as medium. [4]

95. [9709/w18/63/q6]

The lifetimes, in hours, of a particular type of light bulb are normally distributed with mean 2000 hours and standard deviation σ hours. The probability that a randomly chosen light bulb of this type has a lifetime of more than 1800 hours is 0.96.

(i) Find the value of σ. [3]

New technology has resulted in a new type of light bulb. It is found that on average one in five of these new light bulbs has a lifetime of more than 2500 hours.

(ii) For a random selection of 300 of these new light bulbs, use a suitable approximate distribution to find the probability that fewer than 70 have a lifetime of more than 2500 hours. [4]

(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

96. [9709/m17/62/q3]

It is found that 10% of the population enjoy watching Historical Drama on television. Use an appropriate approximation to find the probability that, out of 160 people chosen randomly, more than 17 people enjoy watching Historical Drama on television. [5]

97. [9709/m17/62/q7]

(a) The lengths, in centimetres, of middle fingers of women in Raneland have a normal distribution with mean μ and standard deviation σ. It is found that 25% of these women have fingers longer than 8.8 cm and 17.5% have fingers shorter than 7.7 cm.

(i) Find the values of μ and σ. [5]

The lengths, in centimetres, of middle fingers of women in Snoland have a normal distribution with mean 7.9 and standard deviation 0.44. A random sample of 5 women from Snoland is chosen.

(ii) Find the probability that exactly 3 of these women have middle fingers shorter than 8.2 cm. [5]

(b) The random variable X has a normal distribution with mean equal to the standard deviation. Find the probability that a particular value of X is less than 1.5 times the mean. [3]

98. [9709/s17/61/q6]

(a) The random variable X has a normal distribution with mean μ and standard deviation σ. You are given that $\sigma = 0.25\mu$ and $P(X < 6.8) = 0.75$.

(i) Find the value of μ. [4]

(ii) Find $P(X < 4.7)$. [3]

(b) The lengths of metal rods have a normal distribution with mean 16 cm and standard deviation 0.2 cm. Rods which are shorter than 15.75 cm or longer than 16.25 cm are not usable. Find the expected number of usable rods in a batch of 1000 rods. [4]

99. [9709/s17/62/q5]

The lengths of videos of a certain popular song have a normal distribution with mean 3.9 minutes. 18% of these videos last for longer than 4.2 minutes.

(i) Find the standard deviation of the lengths of these videos. [3]

(ii) Find the probability that the length of a randomly chosen video differs from the mean by less than half a minute. [4]

The lengths of videos of another popular song have a normal distribution with the same mean of 3.9 minutes but the standard deviation is twice the standard deviation in part **(i)**. The probability that the length of a randomly chosen video of this song differs from the mean by less than half a minute is denoted by p.

(iii) Without any further calculation, determine whether p is more than, equal to, or less than your answer to part **(ii)**. You must explain your reasoning. [2]

100. [9709/s17/63/q2]

The probability that George goes swimming on any day is $\frac{1}{3}$. Use an approximation to calculate the probability that in 270 days George goes swimming at least 100 times. [5]

101. [9709/s17/63/q4]

(a) The random variable X has the distribution $N(\mu, \sigma^2)$, where $\mu = 1.5\sigma$. A random value of X is chosen. Find the probability that this value of X is greater than 0. [3]

(b) The life of a particular type of torch battery is normally distributed with mean 120 hours and standard deviation s hours. It is known that 87.5% of these batteries last longer than 70 hours. Find the value of s. [3]

102. [9709/w17/61/q7]

The weight, in grams, of pineapples is denoted by the random variable X which has a normal distribution with mean 500 and standard deviation 91.5. Pineapples weighing over 570 grams are classified as 'large'. Those weighing under 390 grams are classified as 'small' and the rest are classified as 'medium'.

(i) Find the proportions of large, small and medium pineapples. [5]

(ii) Find the weight exceeded by the heaviest 5% of pineapples. [3]

(iii) Find the value of k such that $P(k < X < 610) = 0.3$. [5]

103. [9709/w17/62/q5]

Blank CDs are packed in boxes of 30. The probability that a blank CD is faulty is 0.04. A box is rejected if more than 2 of the blank CDs are faulty.

(i) Find the probability that a box is rejected. [3]

(ii) 280 boxes are chosen randomly. Use an approximation to find the probability that at least 30 of these boxes are rejected. [5]

104. [9709/w17/62/q7]

In Jimpuri the weights, in kilograms, of boys aged 16 years have a normal distribution with mean 61.4 and standard deviation 12.3.

(i) Find the probability that a randomly chosen boy aged 16 years in Jimpuri weighs more than 65 kilograms. [3]

(ii) For boys aged 16 years in Jimpuri, 25% have a weight between 65 kilograms and k kilograms, where k is greater than 65. Find k. [4]

In Brigville the weights, in kilograms, of boys aged 16 years have a normal distribution. 99% of the boys weigh less than 97.2 kilograms and 33% of the boys weigh less than 55.2 kilograms.

(iii) Find the mean and standard deviation of the weights of boys aged 16 years in Brigville. [5]

105. [9709/w17/63/q7]

Josie aims to catch a bus which departs at a fixed time every day. Josie arrives at the bus stop T minutes before the bus departs, where $T \sim N(5.3, 2.1^2)$.

(i) Find the probability that Josie has to wait longer than 6 minutes at the bus stop. [3]

On 5% of days Josie has to wait longer than x minutes at the bus stop.

(ii) Find the value of x. [3]

(iii) Find the probability that Josie waits longer than x minutes on fewer than 3 days in 10 days. [3]

(iv) Find the probability that Josie misses the bus. [3]

106. [9709/m16/62/q7]

The times taken by a garage to fit a tow bar onto a car have a normal distribution with mean m hours and standard deviation 0.35 hours. It is found that 95% of times taken are longer than 0.9 hours.

(i) Find the value of m. [3]

(ii) On one day 4 cars have a tow bar fitted. Find the probability that none of them takes more than 2 hours to fit. [5]

The times in hours taken by another garage to fit a tow bar onto a car have the distribution $N(\mu, \sigma^2)$ where $\mu = 3\sigma$.

(iii) Find the probability that it takes more than 0.6μ hours to fit a tow bar onto a randomly chosen car at this garage. [3]

107. [9709/s16/61/q1]

The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

108. [9709/s16/61/q5]

Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

109. [9709/s16/62/q2]

When visiting the dentist the probability of waiting less than 5 minutes is 0.16, and the probability of waiting less than 10 minutes is 0.88.

(i) Find the probability of waiting between 5 and 10 minutes. [1]

A random sample of 180 people who visit the dentist is chosen.

(ii) Use a suitable approximation to find the probability that more than 115 of these people wait between 5 and 10 minutes. [5]

110. [9709/s16/62/q6]

The time in minutes taken by Peter to walk to the shop and buy a newspaper is normally distributed with mean 9.5 and standard deviation 1.3.

(i) Find the probability that on a randomly chosen day Peter takes longer than 10.2 minutes. [3]

(ii) On 90% of days he takes longer than t minutes. Find the value of t. [3]

(iii) Calculate an estimate of the number of days in a year (365 days) on which Peter takes less than 8.8 minutes to walk to the shop and buy a newspaper. [3]

111. [9709/s16/63/q5]

The heights of school desks have a normal distribution with mean 69 cm and standard deviation σ cm. It is known that 15.5% of these desks have a height greater than 70 cm.

(i) Find the value of σ. [3]

When Jodu sits at a desk, his knees are at a height of 58 cm above the floor. A desk is comfortable for Jodu if his knees are at least 9 cm below the top of the desk. Jodu's school has 300 desks.

(ii) Calculate an estimate of the number of these desks that are comfortable for Jodu. [5]

112. [9709/s16/63/q7]

Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

(iii) Find the probability that, of a random sample of 250 full minibuses travelling to Picton, more than 54 will contain exactly 7 passengers carrying backpacks. [6]

113. [9709/w16/61/q1]

The random variable X is such that $X \sim \mathrm{N}(20, 49)$. Given that $\mathrm{P}(X > k) = 0.25$, find the value of k. [3]

114. [9709/w16/61/q4]

Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

(i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]

(ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

115. [9709/w16/62/q3]

On any day at noon, the probabilities that Kersley is asleep or studying are 0.2 and 0.6 respectively.

(i) Find the probability that, in any 7-day period, Kersley is either asleep or studying at noon on at least 6 days. [3]

(ii) Use an approximation to find the probability that, in any period of 100 days, Kersley is asleep at noon on at most 30 days. [5]

116. [9709/w16/62/q4]

The time taken to cook an egg by people living in a certain town has a normal distribution with mean 4.2 minutes and standard deviation 0.6 minutes.

(i) Find the probability that a person chosen at random takes between 3.5 and 4.5 minutes to cook an egg. [3]

12% of people take more than t minutes to cook an egg.

(ii) Find the value of t. [3]

(iii) A random sample of n people is taken. Find the smallest possible value of n if the probability that none of these people takes more than t minutes to cook an egg is less than 0.003. [3]

117. [9709/w16/63/q6]

The weights of bananas in a fruit shop have a normal distribution with mean 150 grams and standard deviation 50 grams. Three sizes of banana are sold.

Small: under 95 grams
Medium: between 95 grams and 205 grams
Large: over 205 grams

(i) Find the proportion of bananas that are small. [3]

(ii) Find the weight exceeded by 10% of bananas. [3]

The prices of bananas are 10 cents for a small banana, 20 cents for a medium banana and 25 cents for a large banana.

(iii) (a) Show that the probability that a randomly chosen banana costs 20 cents is 0.7286. [1]

(b) Calculate the expected total cost of 100 randomly chosen bananas. [3]

118. [9709/w16/63/q7]

Each day Annabel eats rice, potato or pasta. Independently of each other, the probability that she eats rice is 0.75, the probability that she eats potato is 0.15 and the probability that she eats pasta is 0.1.

(i) Find the probability that, in any week of 7 days, Annabel eats pasta on exactly 2 days. [2]

(ii) Find the probability that, in a period of 5 days, Annabel eats rice on 2 days, potato on 1 day and pasta on 2 days. [3]

(iii) Find the probability that Annabel eats potato on more than 44 days in a year of 365 days. [5]

119. [9709/s15/61/q1]

The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ. [4]

120. [9709/s15/61/q6]

(i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

(ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer. [5]

(iii) Justify your use of the approximation in part **(ii)**. [1]

121. [9709/s15/62/q7]

(a) Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

(i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]

(ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t. [3]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ. [5]

122. [9709/s15/63/q1]

The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ. [3]

123. [9709/s15/63/q3]

On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard. [5]

124. [9709/s15/63/q5]

The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than 29 cm is classified as 'large'.

(i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large. [6]

(ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n. [3]

125. [9709/w15/61/q2]

The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ. [4]

126. [9709/w15/61/q7]

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

127. [9709/w15/62/q7]

(a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

128. [9709/w15/63/q4]

The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

(i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

(ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

129. [9709/w15/63/q7]

A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

(iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

(iv) Justify the use of your approximation in part **(iii)**. [1]

Formula Sheet MF19

Cambridge Assessment
International Education

List MF19

List of formulae and statistical tables

Cambridge International AS & A Level
Mathematics (9709) and Further Mathematics (9231)

For use from 2020 in all papers for the above syllabuses.

CST319

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PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3} \times$ base area $\times$ height

Area of curved surface of cone = $\pi r \times$ slant height

Arc length of circle $= r\theta$ (θ in radians)

Area of sector of circle $= \frac{1}{2} r^2 \theta$ (θ in radians)

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \qquad S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \qquad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1), \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \ldots + b^n, \text{ where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots, \text{ where } n \text{ is rational and } |x| < 1$$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \qquad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi, \qquad 0 \leqslant \cos^{-1} x \leqslant \pi, \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$v\frac{du}{dx} + u\frac{dv}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(\boldsymbol{x})$	$\int \mathbf{f}(\boldsymbol{x})\,\mathbf{d}\boldsymbol{x}$	
x^n	$\dfrac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$	$(\lvert x\rvert < a)$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\lvert f(x)\rvert$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$$

FURTHER PURE MATHEMATICS

Algebra

Summations:

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1), \qquad \sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1), \qquad \sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Maclaurin's series:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leqslant x \leqslant 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 < x < 1)$$

Trigonometry

If $t = \tan\tfrac{1}{2}x$ then:

$$\sin x = \frac{2t}{1+t^2} \qquad \text{and} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x \equiv 1, \qquad \sinh 2x \equiv 2\sinh x \cosh x, \qquad \cosh 2x \equiv \cosh^2 x + \sinh^2 x$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \qquad (x \geqslant 1)$$

$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$	
$\sec x$	$\ln\lvert \sec x + \tan x\rvert = \ln\lvert \tan(\frac{1}{2}x + \frac{1}{4}\pi)\rvert$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\operatorname{cosec} x$	$-\ln\lvert \operatorname{cosec} x + \cot x\rvert = \ln\lvert \tan(\frac{1}{2}x)\rvert$	$(0 < x < \pi)$
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	
$\operatorname{sech}^2 x$	$\tanh x$	
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$	$(x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$	

MECHANICS

Uniformly accelerated motion

$$v = u + at\,, \qquad s = \tfrac{1}{2}(u+v)t\,, \qquad s = ut + \tfrac{1}{2}at^2\,, \qquad v^2 = u^2 + 2as$$

FURTHER MECHANICS

Motion of a projectile

Equation of trajectory is:

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}\,, \qquad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \quad \text{or} \quad \frac{v^2}{r}$$

Centres of mass of uniform bodies

Triangular lamina: $\tfrac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\tfrac{3}{8}r$ from centre

Hemispherical shell of radius r: $\tfrac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\tfrac{3}{4}h$ from vertex

PROBABILITY & STATISTICS

Summary statistics

For ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad \text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}, \qquad \text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$$

Discrete random variables

$$\mathrm{E}(X) = \Sigma xp, \qquad \mathrm{Var}(X) = \Sigma x^2 p - \{\mathrm{E}(X)\}^2$$

For the binomial distribution $\mathrm{B}(n, p)$:

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \qquad \mu = np, \qquad \sigma^2 = np(1-p)$$

For the geometric distribution Geo(p):

$$p_r = p(1-p)^{r-1}, \qquad \mu = \frac{1}{p}$$

For the Poisson distribution $\mathrm{Po}(\lambda)$

$$p_r = \mathrm{e}^{-\lambda} \frac{\lambda^r}{r!}, \qquad \mu = \lambda, \qquad \sigma^2 = \lambda$$

Continuous random variables

$$\mathrm{E}(X) = \int x \,\mathrm{f}(x)\,\mathrm{d}x, \qquad \mathrm{Var}(X) = \int x^2 \,\mathrm{f}(x)\,\mathrm{d}x - \{\mathrm{E}(X)\}^2$$

Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Central Limit Theorem:

$$\bar{X} \sim \mathrm{N}\left(\mu, \ \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

$$\mathrm{N}\left(p, \ \frac{p(1-p)}{n}\right)$$

FURTHER PROBABILITY & STATISTICS

Sampling and testing

Two-sample estimate of a common variance:

$$s^2 = \frac{\Sigma(x_1 - \overline{x}_1)^2 + \Sigma(x_2 - \overline{x}_2)^2}{n_1 + n_2 - 2}$$

Probability generating functions

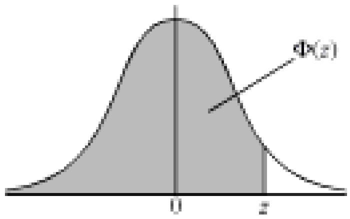
$\mathrm{G}_X(t) = \mathrm{E}(t^X)$, $\mathrm{E}(X) = \mathrm{G}'_X(1)$, $\mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \leqslant z).$$

For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

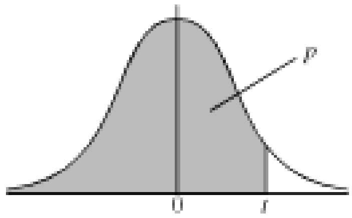
$$P(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom, then, for each pair of values of p and ν, the table gives the value of t such that:

$P(T \leqslant t) = p$.

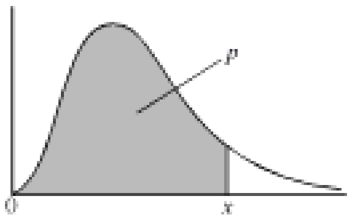


p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2-DISTRIBUTION

If X has a χ^2-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of x such that

$$P(X \leqslant x) = p.$$



p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu = 1$	0.0^31571	0.0^39821	0.0^23932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

WILCOXON SIGNED-RANK TEST

The sample has size n.

P is the sum of the ranks corresponding to the positive differences.

Q is the sum of the ranks corresponding to the negative differences.

T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *T*

	Level of significance			
One-tailed Two-tailed	0.05 0.1	0.025 0.05	0.01 0.02	0.005 0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n, each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

WILCOXON RANK-SUM TEST

The two samples have sizes m and n, where $m \leqslant n$.

R_m is the sum of the ranks of the items in the sample of size m.

W is the smaller of R_m and $m(n + m + 1) - R_m$.

For each pair of values of m and n, the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *W*

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n, the normal distribution with mean $\frac{1}{2}m(m+n+1)$ and variance $\frac{1}{12}mn(m+n+1)$ should be used as an approximation to the distribution of R_m.

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Syllabus 26-27 Probability and Statistics 1

5 Probability & Statistics 1 (for Paper 5)

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1.

Knowledge of the following probability notation is also assumed: $\mathrm{P}(A)$, $\mathrm{P}(A \cup B)$, $P(A \cap B)$, $\mathrm{P}(A|B)$ and the use of A' to denote the complement of A.

5.1 Representation of data

Candidates should be able to:	Notes and examples
• select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have	
• draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs	Including back-to-back stem-and-leaf diagrams.
• understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)	e.g. in comparing and contrasting sets of data.
• use a cumulative frequency graph	e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.
• calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2, or coded totals $\Sigma(x-a)$ and $\Sigma(x-a)^2$, and use such totals in solving problems which may involve up to two data sets.	

5.2 Permutations and combinations

Candidates should be able to:	Notes and examples
• understand the terms permutation and combination, and solve simple problems involving selections	
• solve problems about arrangements of objects in a line, including those involving – repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') – restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).	Questions may include cases such as people sitting in two (or more) rows. Questions about objects arranged in a circle will not be included.

Cambridge International AS & A Level Mathematics 9709 syllabus for 2026 and 2027. Subject content

5 Probability & Statistics 1

5.3 Probability

Candidates should be able to:	Notes and examples
• evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations	e.g. the total score when two fair dice are thrown. e.g. drawing balls at random from a bag containing balls of different colours.
• use addition and multiplication of probabilities, as appropriate, in simple cases	Explicit use of the general formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is not required.
• understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $P(A \cap B)$ and $P(A) \times P(B)$	
• calculate and use conditional probabilities in simple cases.	e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ may be required in simple cases.

5.4 Discrete random variables

Candidates should be able to:	Notes and examples
• draw up a probability distribution table relating to a given situation involving a discrete random variable X, and calculate $E(X)$ and $Var(X)$	
• use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models	Including the notations $B(n, p)$ and $Geo(p)$. $Geo(p)$ denotes the distribution in which $p_r = p(1-p)^{r-1}$ for $r = 1, 2, 3, \ldots$.
• use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.	Proofs of formulae are not required.

5 Probability & Statistics 1

5.5 The normal distribution

Candidates should be able to:	Notes and examples
• understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables	Sketches of normal curves to illustrate distributions or probabilities may be required.
• solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$, including – finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ. – finding a relationship between x_1, μ and σ given the value of $P(X > x_1)$ or a related probability	For calculations involving standardisation, full details of the working should be shown. e.g. $Z = \dfrac{(X - \mu)}{\sigma}$
• recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.	n sufficiently large to ensure that both $np > 5$ and $nq > 5$.