The Question Bank of

Pure Mathematics 1

for CAIE 9709 paper 1.

v1.0

Edited by Thoridal

Instructions for Use

- This question bank is organized by chapter for systematic revision.
- This question bank is complied based on the 26-27 CAIE Pure Mathematics 1 syllabus, which is included as appendix.
- Each question includes its source for reference.
- Mark schemes are provided in the separate answer booklet.
- The formula sheet (MF19) is included as appendix.
- Use this resource for targeted practice and exam preparation.

Contents

Edited by Thoridal

Chapter 1

Quadratics and Functions

1. [9709/m25/12/q11]

Functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 4x^2 - c \quad \text{and} \quad \mathrm{g}(x) = 2x + k,$$

where c and k are positive constants. It is given that $\mathrm{g}^{-1}(3k+1) = c$.

(a) Show that $\mathrm{gf}(x) = 8x^2 - k - 1$. [4]

(b) The curve with equation $y = 8x^2 - k - 1$ is transformed to the curve with equation $y = \mathrm{h}(x)$ by the following sequence of transformations.

Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
Stretch in the y-direction by scale factor k
Reflection in the x-axis

Find an expression for $\mathrm{h}(x)$ in terms of x and k. [3]

(c) The range of h is given by $\mathrm{h}(x) \leqslant 15$.

Find the values of c and k. [3]

2. [9709/s25/11/q6]

The equation of a curve is $2x^2 - kxy + 2 = 0$ and the equation of a line is $y = px + 3$, where k and p are constants.

(a) Given that $k = 2$ and $p = 11$, find the coordinates of the points of intersection of the curve and the line. [4]

(b) Given instead that $p = 4$, find the set of values of k for which the curve and the line do not intersect. [5]

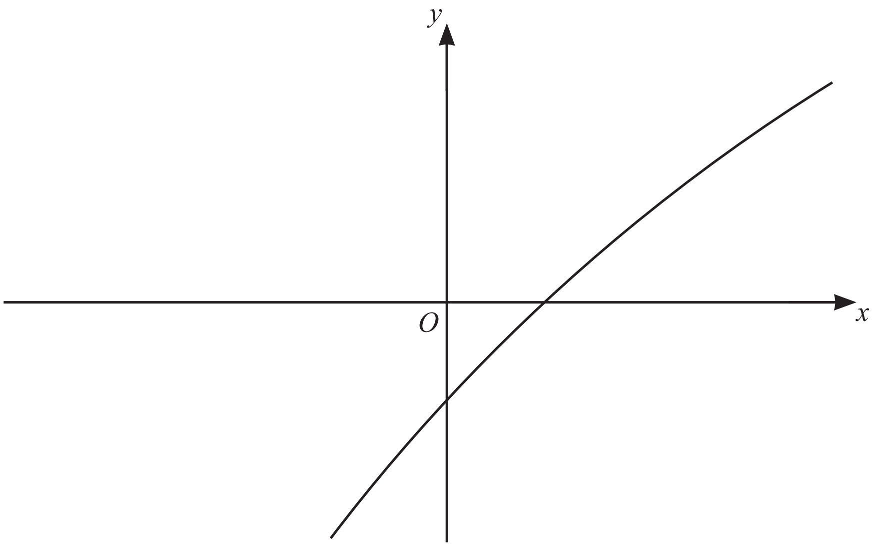
3. [9709/s25/11/q10]

The functions f and g are defined by

$$f(x) = \sqrt{x} \qquad \text{for } x \geqslant 0,$$

$$g(x) = 3\sqrt{x+2} - 5 \qquad \text{for } x \geqslant -2.$$

(a) Describe fully a sequence of transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. You should make clear the order in which the transformations are applied. [5]



The diagram shows the graph of $y = g(x)$.

(b) On the diagram sketch the graph of $y = g^{-1}(x)$ together with any relevant mirror line. [2]

(c) Find an expression for $g^{-1}(x)$. [2]

(d) State the range of g^{-1}. [1]

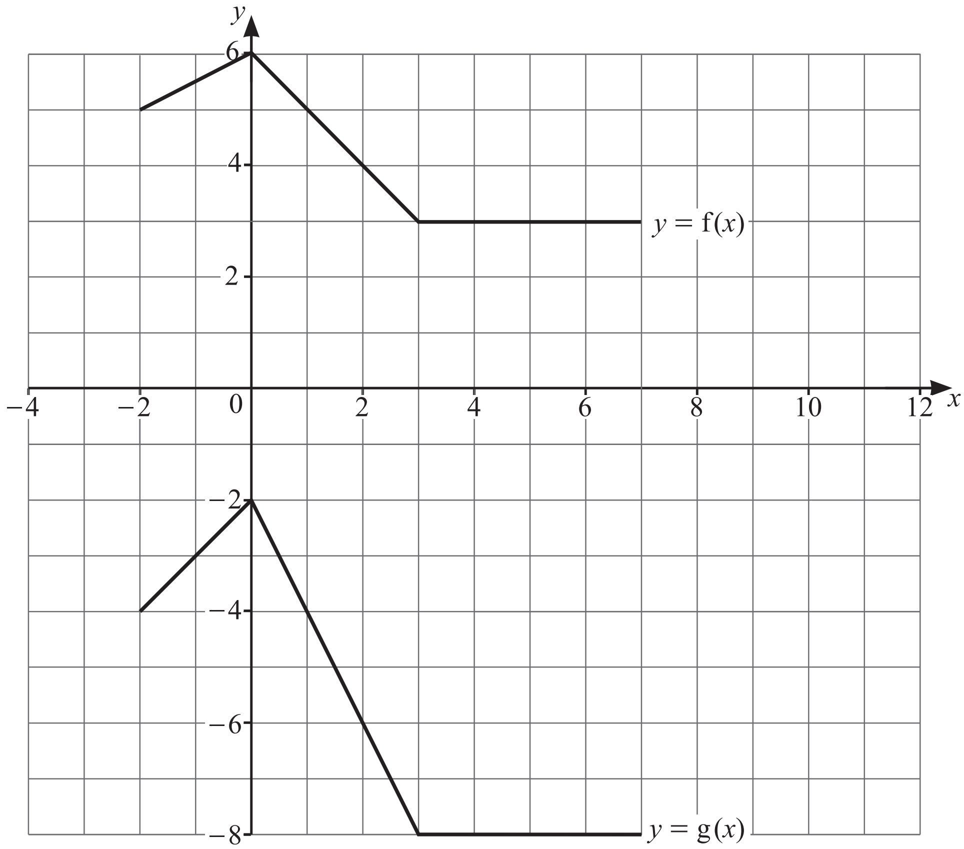
The function h is defined by

$$h(x) = x - 2 \qquad \text{for } x \geqslant 0.$$

(e) Find the value of $g^{-1}h(4)$. [1]

(f) Explain why the composite function hg^{-1} cannot be formed. [1]

4. [9709/s25/12/q1]



The diagram shows the graphs with equations $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{g}(x)$. Make clear the order in which the transformations should be applied. [4]

5. [9709/s25/12/q11]

(a) Express $x^2 + 4x + 2$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

The functions f and g are defined as follows.

$$f(x) = x^2 + 4x + 2 \qquad \text{for } x \leqslant -2$$

$$g(x) = -x - 4 \qquad \text{for } x \geqslant -2$$

(b) **(i)** Find an expression for $f^{-1}(x)$. [3]

(ii) Find an expression for $(gf)^{-1}(x)$. [4]

6. [9709/s25/13/q10]

A curve C has equation $y = \frac{9}{2x-5} + 2x - 5$.

(a) Find the coordinates of the two stationary points. [4]

(b) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of each stationary point. [3]

(c) The curve C is transformed to the curve C_1 using a translation of $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$ followed by reflection in the x-axis.

(i) State the coordinates of the maximum point of C_1. [1]

(ii) Find the equation of C_1 in the form $y = \frac{a}{bx+c} + dx + e$, where a, b, c, d and e are integers. [3]

7. [9709/s25/13/q11]

The function f is defined by $f(x) = x^2 + 4ax + a$ for $x \in \mathbb{R}$, where a is a constant.

The function g is such that $g^{-1}(x) = \sqrt[3]{2x - 4}$ for $x \in \mathbb{R}$.

(a) Given that the range of f is $f(x) \geqslant -33$, find the possible values of a. [4]

(b) Given instead that $fgg(0) = 96$, find the value of a. [6]

8. [9709/s25/15/q7]

In the parallelogram $ABCD$, the coordinates of A are (3, 7), the coordinates of B are (6, p) and the coordinates of D are (1, p). It is given that the gradient of AB is $-\frac{2}{3}$.

(a) Find the value of p. [2]

(b) Find the coordinates of C. [2]

(c) Find the area of the triangle formed by the perpendicular bisector of AB and the x- and y-axes. [5]

9. [9709/w25/11/q1]

Find the set of values of the constant k for which the quadratic equation

$$3kx^2 + (k+8)x + 3 = 0$$

has two distinct real roots. [4]

10. [9709/w25/11/q4]

(a) Express $1-6x-x^2$ in the form $a-(x+b)^2$, where a and b are constants. [3]

(b) The graph of $y=x^2$ is transformed to the graph of $y=1-6x-x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n. [3]

11. [9709/w25/11/q6]

Functions f and g are defined by

$$\mathrm{f}(x) = (x+3)^2 - 12 \quad \text{for } x \geqslant 0,$$

$$\mathrm{g}(x) = 2x - 5 \quad \text{for } x \in \mathbb{R}.$$

(a) State the range of f. [1]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(c) Solve the equation $\mathrm{gf}(x) = 69$. [4]

12. [9709/w25/12/q1]

(a) Express $9x^2 - 36x + 8$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [2]

(b) Hence find the set of values of the constant k for which the equation $9x^2 - 36x + 8 = k$ has no real roots. [1]

(c) Find the exact roots of the equation $9x^2 - 36x + 8 = -15$. [2]

13. [9709/w25/12/q3]

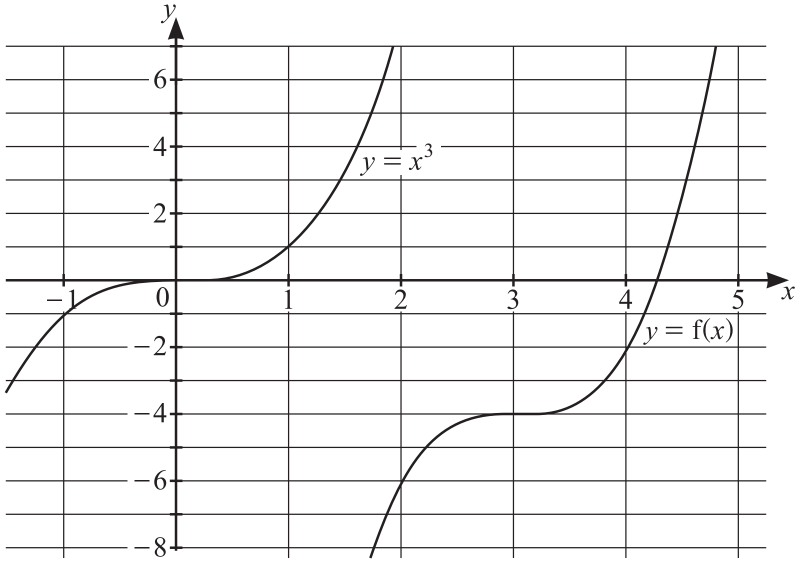
(a) The graph of $y = f(x)$ is transformed to the graph of $y = f(3x) + 2$.

Describe fully the two transformations which have been combined to give the resulting graph. [3]

(b) A different graph has equation $y = g(x)$. This graph is stretched by scale factor 3 in the y-direction and then reflected in the y-axis.

Write down the equation of the transformed graph in terms of the function g. [2]

14. [9709/w25/13/q6]



The diagram shows the graphs of $y = x^3$ and $y = \mathrm{f}(x)$. The graph of $y = x^3$ is transformed to the graph of $y = \mathrm{f}(x)$ by a sequence of transformations.

(a) Describe fully a suitable sequence of transformations. Make clear the order in which the transformations are applied. [5]

(b) You are given that $\mathrm{f}(x) = a(x+b)^3 + c$.

State the values of the constants a, b and c. [3]

15. [9709/w25/13/q7]

The function g is defined by $g(x) = \frac{2}{ax-3} + \frac{1}{2}$ for $x > \frac{3}{a}$, where a is a positive constant.

Find $g^{-1}(x)$ and hence verify that if $a = 6$ then $g^{-1}(x) \equiv g(x)$. [4]

16. [9709/w25/13/q10]

A function f is defined by $\mathrm{f}(x) = px^2 + 4x + q$ for $x \in \mathbb{R}$, where p and q are constants.

(a) It is given that $p = 2$ and $q = 10$.

(i) Express $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(b) It is given instead that $q = -5$ and the roots of $\mathrm{f}(x) = 0$ are $5m$ and $-9m$, where m is a constant.

Find the values of p and m. [5]

17. [9709/w25/15/q3]

(a) Express $4x^2 + 10x + 6$ in the form $a(x+b)^2 + c$, where a, b and c are rational constants to be determined. [2]

(b) The curve with equation $y = 4x^2 + 10x + 6$ and the line $y = k$ have exactly one point of intersection.

Using your answer to part **(a)** or otherwise, state the value of the constant k. [1]

18. [9709/w25/15/q5]

Solve the equation

$$x^3 - 28 + \frac{27}{x^3} = 0. \qquad [3]$$

19. [9709/w25/15/q10]

The function f is defined by

$$f(x) = 3 + \frac{7}{x-2}$$

for $x > 2$.

(a) It is given that $f(a) = 4$.

Find the value of a. [2]

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

(c) The function g is defined by

$$g(x) = \frac{1+4x}{2x-3}$$

for $x > \frac{3}{2}$.

Show that $fg(x) \equiv kx$, where k is a constant to be determined. [3]

20. [9709/m24/12/q9]

The functions f and g are defined for all real values of x by

$$f(x) = (3x-2)^2 + k \quad \text{and} \quad g(x) = 5x-1,$$

where k is a constant.

(a) Given that the range of the function gf is $gf(x) \geqslant 39$, find the value of k. [4]

(b) For this value of k, determine the range of the function fg. [2]

(c) The function h is defined for all real values of x and is such that $gh(x) = 35x + 19$.

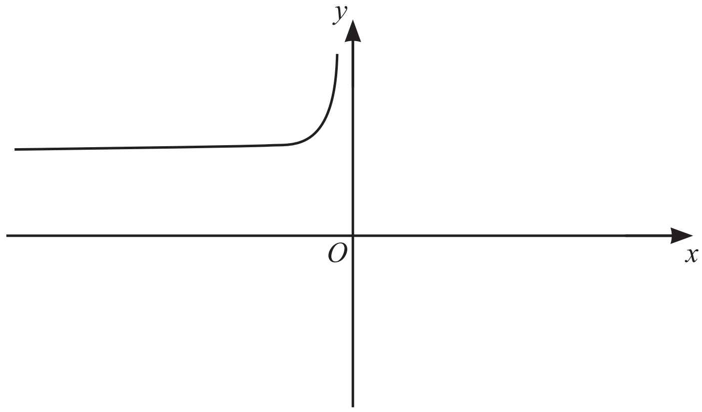
Find an expression for $g^{-1}(x)$ and hence, or otherwise, find an expression for $h(x)$. [3]

21. [9709/s24/11/q1]

(a) Express $3y^2 - 12y - 15$ in the form $3(y+a)^2 + b$, where a and b are constants. [2]

(b) Hence find the exact solutions of the equation $3x^4 - 12x^2 - 15 = 0$. [3]

22. [9709/s24/11/q6]



The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

(a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

(c) Solve the equation $f(x) = 4.5$. [1]

(d) Explain why the equation $f^{-1}(x) = f(x)$ has no solution. [1]

23. [9709/s24/12/q2]

The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

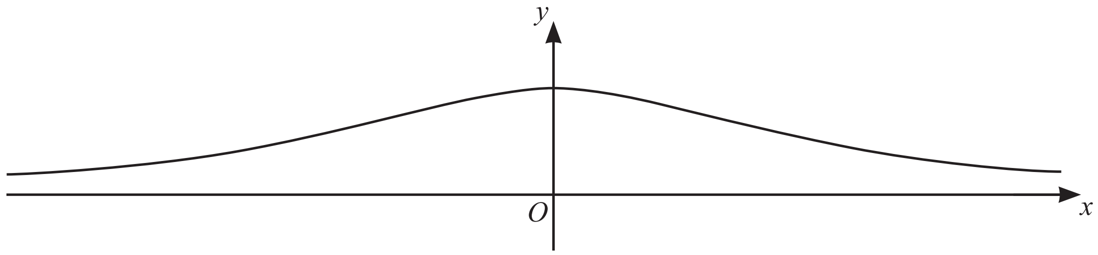
Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

24. [9709/s24/12/q4]

The function f is defined as follows:

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $f^{-1}(x)$. [1]



The diagram shows the graph of $y = g(x)$ where $g(x) = \frac{1}{x^2+2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

The function h is defined by $h(x) = \frac{1}{x^2+2}$ for $x \geqslant 0$.

(c) Solve the equation $hf(x) = f\left(\frac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a, b and c are integers. [4]

25. [9709/s24/13/q6]

A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{dy}{dx} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

26. [9709/s24/13/q11]

The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f. [3]

The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point P.

Determine the coordinates of P. [6]

27. [9709/w24/11/q11]

The function f is defined by $f(x) = 3 + 6x - 2x^2$ for $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants, and state the range of f. [3]

(b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin.

Give details of the reflection and translation involved. [2]

The function g is defined by $g(x) = 3 + 6x - 2x^2$ for $x \leqslant 0$.

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]

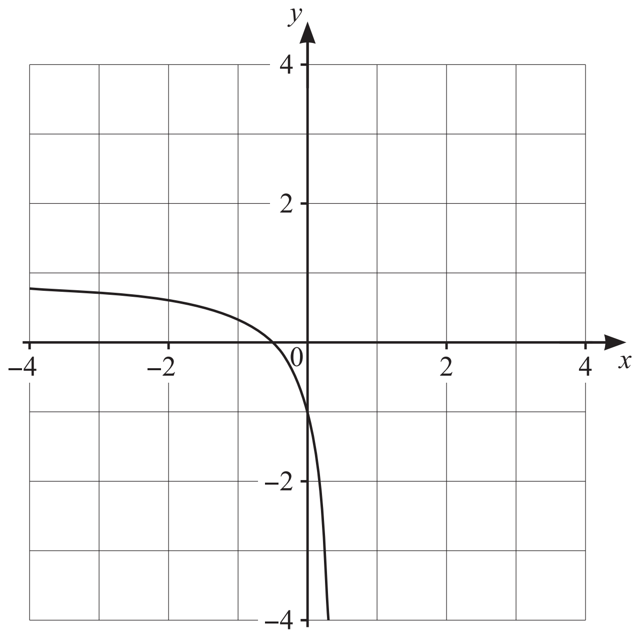
(d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in **(c)**, and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

28. [9709/w24/12/q5]

The function f is defined by $f(x) = \dfrac{2x+1}{2x-1}$ for $x < \dfrac{1}{2}$.

(a) (i) State the value of $f(-1)$. [1]

(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1}. [4]

The function g is defined by $g(x) = 3x+2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

29. [9709/w24/12/q8]

The equation of a circle is $x^2+y^2+px+2y+q=0$, where p and q are constants.

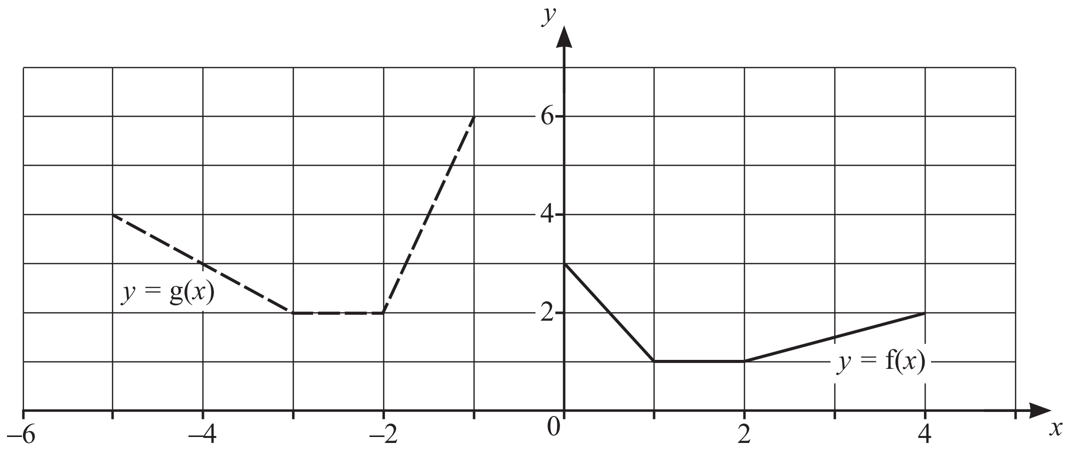
(a) Express the equation in the form $(x-a)^2+(y-b)^2=r^2$, where a is to be given in terms of p and r^2 is to be given in terms of p and q. [2]

The line with equation $x+2y=10$ is the tangent to the circle at the point $A(4, 3)$.

(b) **(i)** Find the equation of the normal to the circle at the point A. [3]

(ii) Find the values of p and q. [5]

30. [9709/w24/13/q5]



In the diagram, the graph with equation $y = \mathrm{f}(x)$ is shown with solid lines and the graph with equation $y = \mathrm{g}(x)$ is shown with broken lines.

(a) Describe fully a sequence of three transformations which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{g}(x)$. [6]

(b) Find an expression for $\mathrm{g}(x)$ in the form $a\mathrm{f}(bx + c)$, where a, b and c are integers. [2]

31. [9709/w24/13/q8]

(a) Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]

32. [9709/m23/12/q2]

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

33. [9709/m23/12/q9]

The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

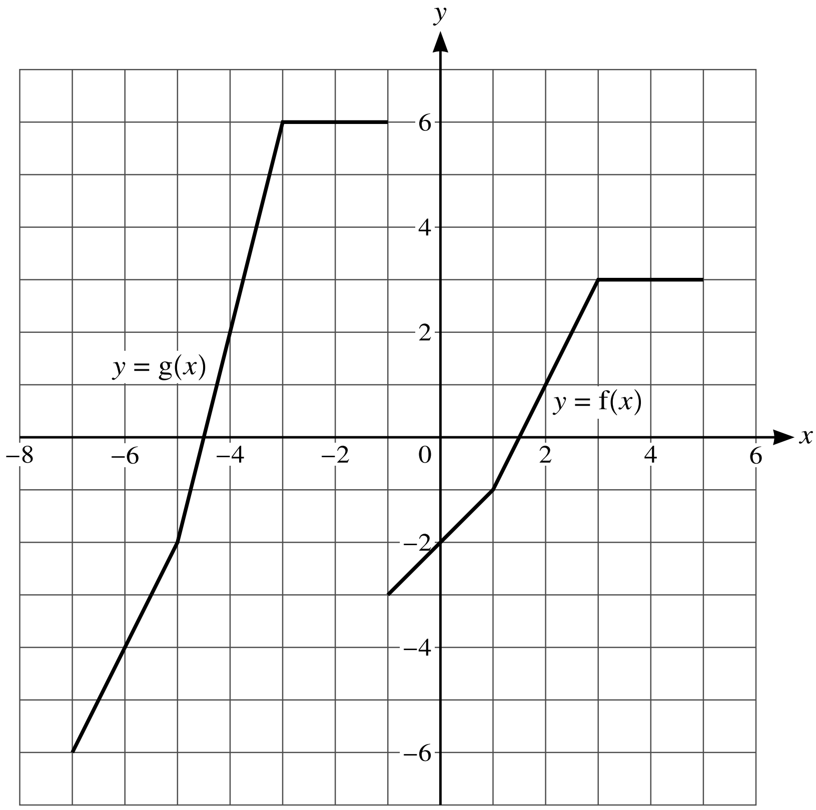
(a) State the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

34. [9709/s23/11/q3]



The diagram shows graphs with equations $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ to $y = \mathrm{g}(x)$. [4]

35. [9709/s23/11/q8]

The functions f and g are defined as follows, where a and b are constants.

$$f(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b. [4]

For the rest of this question, you should use the value of a which you found in **(a)**.

(b) Find the domain of f^{-1}. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

36. [9709/s23/12/q3]

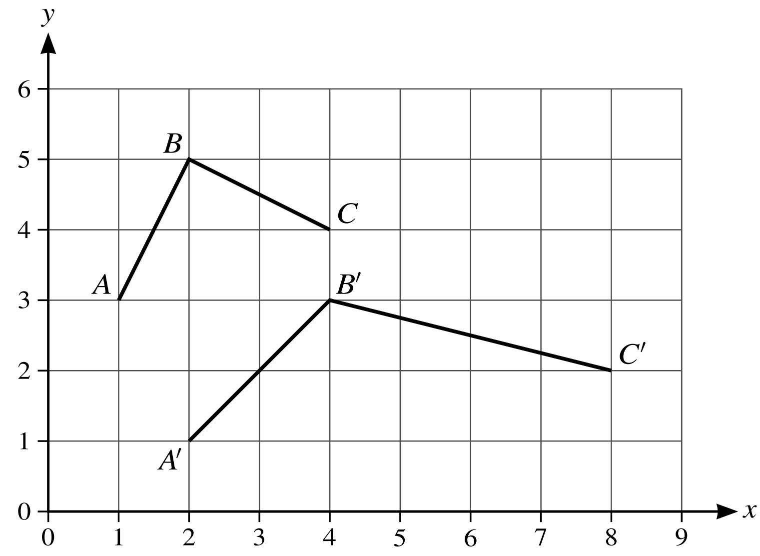
(a) Express $4x^2 - 24x + p$ in the form $a(x+b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

37. [9709/s23/12/q4]

Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

38. [9709/s23/13/q1]



The diagram shows the graph of $y = \mathrm{f}(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = \mathrm{g}(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = \mathrm{f}(x)$.

State fully the two transformations. [4]

39. [9709/s23/13/q2]

The function f is defined for $x \in \mathbb{R}$ by $\mathrm{f}(x) = x^2 - 6x + c$, where c is a constant. It is given that $\mathrm{f}(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

40. [9709/s23/13/q7]

The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

(a) State the range of f. [1]

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a, b, c and d are integers. [3]

41. [9709/w23/11/q4]

The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = \mathrm{g}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

(b) Find the equation, $y = \mathrm{h}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

(c) State fully the transformation that maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{h}(x)$. [2]

42. [9709/w23/11/q9]

(a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

(b) Given that it is possible to form the composite function gf, find the least possible value of p and the greatest possible value of q. [3]

(c) Find an expression for $gf(x)$. [1]

The function h is defined by $h(x) = 4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for $h^{-1}(x)$. [3]

43. [9709/w23/12/q6]

The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve. [2]

The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve. [2]

(c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be found. [4]

44. [9709/w23/12/q8]

Functions f and g are defined by

$$\mathrm{f}(x) = (x + a)^2 - a \text{ for } x \leqslant -a,$$
$$\mathrm{g}(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(b) **(i)** State the domain of the function f^{-1}. [1]

(ii) State the range of the function f^{-1}. [1]

(c) Given that $a = \frac{7}{2}$, solve the equation $\mathrm{gf}(x) = 0$. [3]

45. [9709/w23/13/q7]

The function f is defined by $f(x) = 1 + \dfrac{3}{x-2}$ for $x > 2$.

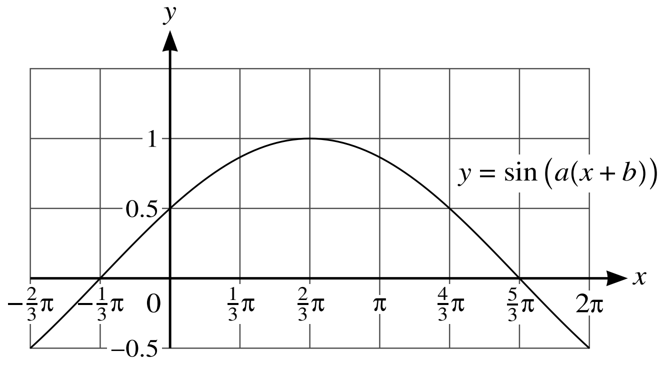
(a) State the range of f. [1]

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

46. [9709/w23/13/q8]



The diagram shows part of the graph of $y = \sin\big(a(x + b)\big)$, where a and b are positive constants.

(a) State the value of a and one possible value of b. [2]

Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q), where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\big(\tfrac{1}{4}(x + 8)\big).$$

(b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q. [3]

47. [9709/m22/12/q2]

A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

48. [9709/m22/12/q5]

(a) Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. [2]

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

(b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

49. [9709/m22/12/q9]

Functions f, g and h are defined as follows:

$$\mathrm{f} : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$\mathrm{g} : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$\mathrm{h} : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $\mathrm{f}(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $\mathrm{f}(x) \equiv \mathrm{gh}(x)$, find the values of m and n. [4]

50. [9709/s22/11/q1]

(a) Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$ where p and q are constants. [2]

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

51. [9709/s22/11/q6]

The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f. [4]

(c) Explain why the composite function ff cannot be formed. [1]

52. [9709/s22/11/q8.a]

(a) The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30^\circ\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30^\circ\right) = 2\sqrt{2}$ for $0^\circ \leqslant x \leqslant 360^\circ$. [3]

53. [9709/s22/12/q5]

The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

(a) Given that the curve and the line intersect at the points with x-coordinates 0 and $\frac{3}{4}$, find the values of k and a. [4]

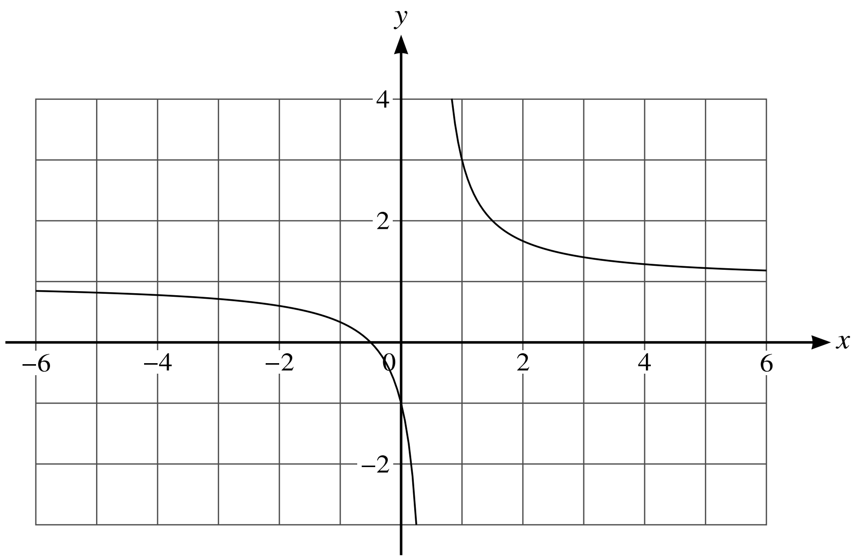
(b) Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

54. [9709/s22/12/q10]

Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$
$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1}. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

(c) Find $gf^{-1}(3)$. [2]

(d) Explain why $g^{-1}(x)$ cannot be found. [1]

(e) Show that $1 + \dfrac{2}{2x-1}$ can be expressed as $\dfrac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

55. [9709/s22/13/q4]

(a) The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

(b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

56. [9709/s22/13/q5]

(a) Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

(b) Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

57. [9709/s22/13/q6]

The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

58. [9709/s22/13/q11]

The point P lies on the line with equation $y = mx + c$, where m and c are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point P on the curve such that the straight line is a tangent to the curve at P.

(a) Find the coordinates of P, giving the y-coordinate in terms of m. [6]

The normal to the curve at P intersects the curve again at the point Q.

(b) Find the coordinates of Q in terms of m. [4]

59. [9709/w22/11/q1]

Solve the equation $3x + 2 = \dfrac{2}{x - 1}$. [3]

60. [9709/w22/11/q8.bc]

The function f is defined by $f(x) = 2 - \dfrac{3}{4x - p}$ for $x > \dfrac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

(b) Express $f^{-1}(x)$ in the form $\dfrac{p}{a} - \dfrac{b}{cx - d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

61. [9709/w22/11/q9]

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]

(b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

(c) Express $g(x)$ in the form $kf(x + h)$, where k and h are integers. [1]

(d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

62. [9709/w22/12/q3]

(a) Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

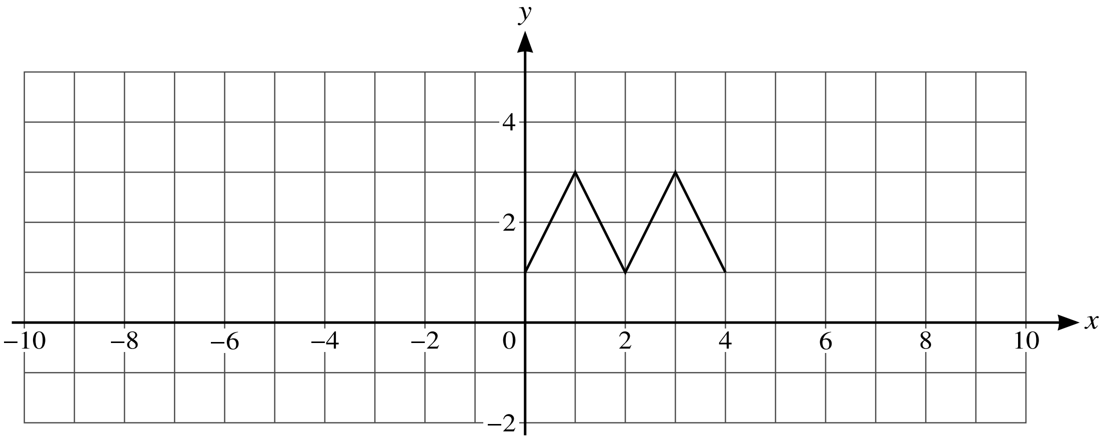
(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

63. [9709/w22/12/q5]

The graph with equation $y = \mathrm{f}(x)$ is transformed to the graph with equation $y = \mathrm{g}(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = \mathrm{f}(x)$.

On the diagram sketch the graph of $y = \mathrm{g}(x)$. [3]



(b) Find an expression for $\mathrm{g}(x)$ in terms of $\mathrm{f}(x)$. [2]

64. [9709/w22/12/q6]

The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3]

(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y-axes. [3]

65. [9709/w22/12/q9]

Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $gf(x)$. [1]

(b) Given that $gf(2) = 11$, find the value of a. [2]

(c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

(e) Explain why the composite function fg cannot be formed. [1]

66. [9709/w22/13/q1]

Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

67. [9709/w22/13/q2]

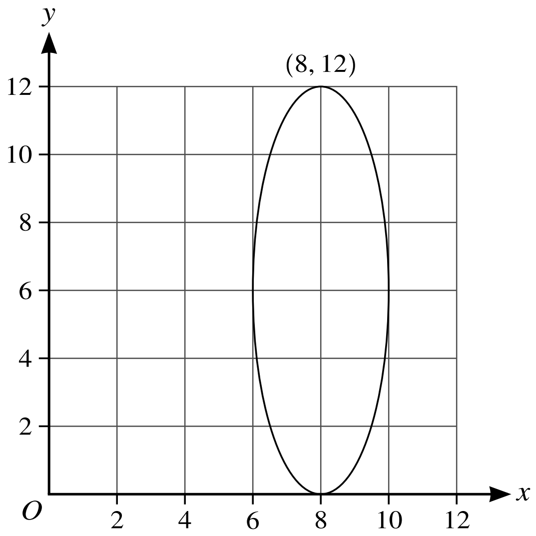
The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

68. [9709/w22/13/q5]



The diagram shows a curve which has a maximum point at $(8,\ 12)$ and a minimum point at $(8,\ 0)$. The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y-direction.

(a) State the scale factor of the stretch. [1]

(b) State the radius of the original circle. [1]

(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

(d) State the coordinates of the centre of the original circle. [2]

69. [9709/m21/12/q2]

By using a suitable substitution, solve the equation

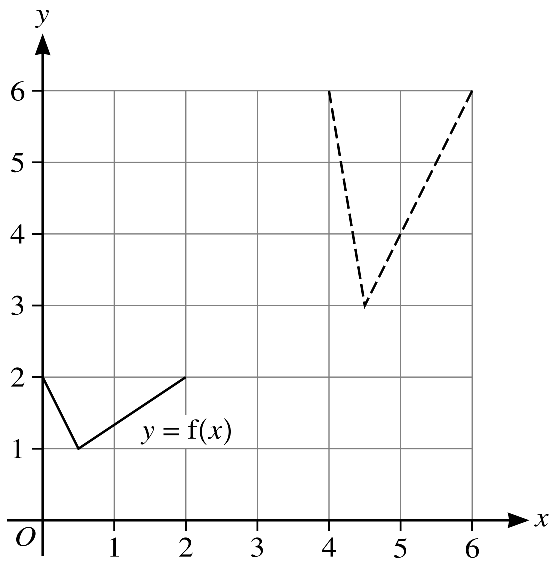
$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0.$$ [4]

70. [9709/m21/12/q4]

A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

71. [9709/m21/12/q5]



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

(a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

72. [9709/m21/12/q7]

Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\mathrm{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(c) Solve the equation $\mathrm{gf}(x) = 13$. [3]

73. [9709/s21/11/q6]

The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

74. [9709/s21/11/q9]

Functions f and g are defined as follows:

$$f(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

(b) Find $f^{-1}(x)$. [2]

(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

(d) Given instead that $ggf^{-1}(12) = 62$, find the possible values of a. [5]

75. [9709/s21/12/q1]

(a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

76. [9709/s21/12/q2]

(a) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 2\mathrm{f}(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

77. [9709/s21/12/q5]

The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

78. [9709/s21/13/q3]

A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

79. [9709/s21/13/q6]

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

80. [9709/s21/13/q8]

Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 1 \ \text{ for } x < 0,$$
$$\mathrm{g} : x \mapsto \frac{1}{2x+1} \ \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $\mathrm{fg}(x) = 3$. [4]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

81. [9709/w21/11/q2]

A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

82. [9709/w21/11/q8]

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x-a)^2 + b$, where a and b are constants. [2]

The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leqslant k$.

(b) State the largest possible value of the constant k. [1]

It is now given that $k = -1$.

(c) State the range of f. [1]

(d) Find an expression for $f^{-1}(x)$. [3]

The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

(e) Express $g(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. [3]

83. [9709/w21/12/q2]

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = \mathrm{f}(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

The point $P\,(5,\ 6)$ lies on the transformed curve $y = \mathrm{f}(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = \mathrm{f}(x)$. [2]

84. [9709/w21/12/q3]

The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of ff(5). [2]

(b) Find an expression for $f^{-1}(x)$. [3]

85. [9709/w21/13/q1]

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 3 - \mathrm{f}(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

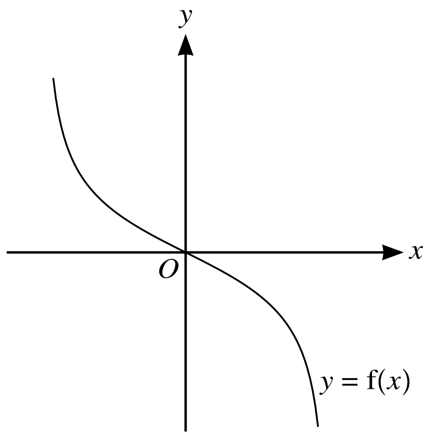
86. [9709/w21/13/q3.a]

(a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

(b) The function f is defined by $\mathrm{f}(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

87. [9709/w21/13/q6]



The diagram shows the graph of $y = \mathrm{f}(x)$.

(a) On this diagram sketch the graph of $y = \mathrm{f}^{-1}(x)$. [1]

It is now given that $\mathrm{f}(x) = -\dfrac{x}{\sqrt{4 - x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [4]

The function g is defined by $\mathrm{g}(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

(d) Assuming that fg can be formed, find and simplify an expression for $\mathrm{fg}(x)$. [2]

88. [9709/m20/12/q2]

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

89. [9709/m20/12/q9]

(a) Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]

90. [9709/s20/11/q5]

The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

91. [9709/s20/11/q6]

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \tfrac{1}{2}x - a,$$
$$\mathrm{g} : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $\mathrm{gg}(2) = 10$ and $\mathrm{f}^{-1}(2) = 14$, find the values of a and b. [4]

(b) Using these values of a and b, find an expression for $\mathrm{gf}(x)$ in the form $cx + d$, where c and d are constants. [2]

92. [9709/s20/12/q5]

The function f is defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $\mathrm{ff}(x)$ and $\mathrm{f}^{-1}(x)$ in terms of a and x. [4]

(b) Given that $\mathrm{ff}(x) = \mathrm{f}^{-1}(x)$, find x in terms of a. [2]

93. [9709/s20/12/q6]

The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

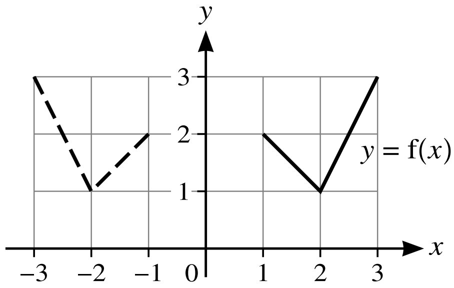
94. [9709/s20/13/q1]

Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

95. [9709/s20/13/q3]

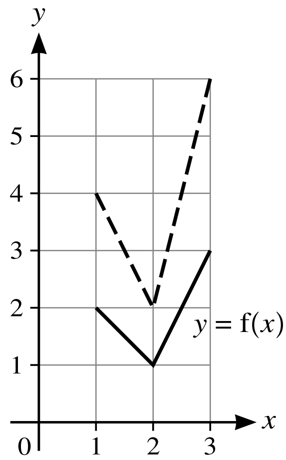
In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)



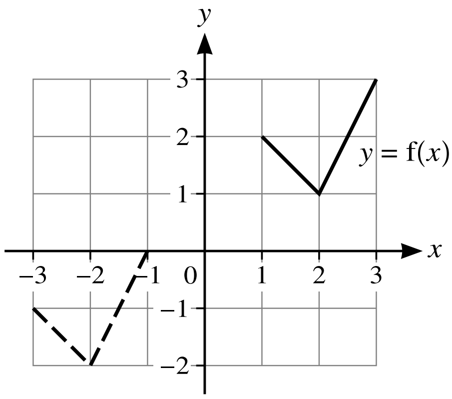
State, in terms of f, the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

96. [9709/s20/13/q9]

The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$
$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(d) Find an expression for $gf(x)$ and state the range of gf. [3]

97. [9709/w20/11/q1]

Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

98. [9709/w20/11/q11]

The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for fg(x). [1]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$ and state the domain of $(\mathrm{fg})^{-1}$. [4]

(c) Solve the equation $\mathrm{fg}(x) - 3 = \mathrm{gf}(x)$. [4]

99. [9709/w20/12/q3]

The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

100. [9709/w20/12/q5]

Functions f and g are defined by

$$\mathrm{f}(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

(b) Find the values of x for which $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

101. [9709/w20/13/q1]

(a) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

102. [9709/w20/13/q4]

A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

103. [9709/w20/13/q6]

The function f is defined by $f(x) = \dfrac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $\dfrac{2}{3} + \dfrac{2}{3(3x-1)}$ can be expressed as $\dfrac{2x}{3x-1}$. [2]

(c) State the range of f. [1]

104. [9709/m19/12/q8]

(i) Express $x^2 - 4x + 7$ in the form $(x + a)^2 + b$. [2]

The function f is defined by $f(x) = x^2 - 4x + 7$ for $x < k$, where k is a constant.

(ii) State the largest value of k for which f is a decreasing function. [1]

The value of k is now given to be 1.

(iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(iv) The function g is defined by $g(x) = \dfrac{2}{x-1}$ for $x > 1$. Find an expression for $gf(x)$ and state the range of gf. [4]

105. [9709/s19/11/q5]

The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the greatest value of $f(x)$. [1]

The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

106. [9709/s19/12/q7]

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{2x+3}{x-1}, \quad x \in \mathbb{R},\ x \neq 1.$$

(i) Obtain expressions for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$, stating the value of x for which $\mathrm{g}^{-1}(x)$ is not defined. [4]

(ii) Solve the equation $\mathrm{fg}(x) = \frac{7}{3}$. [3]

107. [9709/s19/13/q1]

The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.

(i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]

(ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

108. [9709/s19/13/q4]

The function f is defined by $f(x) = \dfrac{48}{x-1}$ for $3 \leqslant x \leqslant 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leqslant x \leqslant b$, where a and b are constants.

(i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf. [2]

It is now given that the conditions for the formation of gf are satisfied.

(ii) Find an expression for $gf(x)$. [1]

(iii) Find an expression for $(gf)^{-1}(x)$. [2]

109. [9709/w19/11/q7]

Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{3}{2x+1} \quad \text{for } x > 0,$$
$$\mathrm{g} : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

(i) Find the range of f and the range of g. [3]

(ii) Find an expression for $\mathrm{fg}(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers. [2]

(iii) Find an expression for $(\mathrm{fg})^{-1}(x)$, giving your answer in the same form as for part **(ii)**. [3]

110. [9709/w19/13/q2]

The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [5]

111. [9709/w19/13/q6]

A line has equation $y = 3kx - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

(i) Find the set of values of k for which the line and curve meet at two distinct points. [4]

(ii) For each of two particular values of k, the line is a tangent to the curve. Show that these two tangents meet on the x-axis. [3]

112. [9709/m18/12/q9]

A curve has equation $y = \frac{1}{x} + c$ and a line has equation $y = cx - 3$, where c is a constant.

(i) Find the set of values of c for which the curve and the line meet. [4]

(ii) The line is a tangent to the curve for two particular values of c. For each of these values find the x-coordinate of the point at which the tangent touches the curve. [4]

113. [9709/m18/12/q10]

Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$
$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

(i) (a) State the range of the function f. [1]

(b) State the range of the function g. [1]

(c) State the range of the function fg. [1]

(ii) Explain why the function gf cannot be formed. [1]

(iii) Find the set of values of x satisfying the inequality $6f'(x) + 2f^{-1}(x) - 5 < 0$. [6]

114. [9709/s18/11/q9]

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \tfrac{1}{2}x - 2,$$
$$\mathrm{g} : x \mapsto 4 + x - \tfrac{1}{2}x^2.$$

(i) Find the points of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. [3]

(ii) Find the set of values of x for which $\mathrm{f}(x) > \mathrm{g}(x)$. [2]

(iii) Find an expression for $\mathrm{fg}(x)$ and deduce the range of fg. [4]

The function h is defined by $\mathrm{h} : x \mapsto 4 + x - \tfrac{1}{2}x^2$ for $x \geqslant k$.

(iv) Find the smallest value of k for which h has an inverse. [2]

115. [9709/s18/12/q7]

The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

(ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

(iv) For this value of k, find $g^{-1}(x)$. [3]

116. [9709/s18/13/q1]

Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

117. [9709/s18/13/q10]

The one-one function f is defined by $f(x) = (x-2)^2 + 2$ for $x \geqslant c$, where c is a constant.

(i) State the smallest possible value of c. [1]

In parts **(ii)** and **(iii)** the value of c is 4.

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) Solve the equation $ff(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [5]

118. [9709/w18/11/q1]

Showing all necessary working, solve the equation $4x - 11x^{\frac{1}{2}} + 6 = 0$. [3]

119. [9709/w18/11/q11]

(a) The one-one function f is defined by $f(x) = (x-3)^2 - 1$ for $x < a$, where a is a constant.

(i) State the greatest possible value of a. [1]

(ii) It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]

(b) The function g is defined by $g(x) = (x-3)^2$ for $x \geqslant 0$.

(i) Show that $gg(2x)$ can be expressed in the form $(2x-3)^4 + b(2x-3)^2 + c$, where b and c are constants to be found. [2]

(ii) Hence expand $gg(2x)$ completely, simplifying your answer. [4]

120. [9709/w18/12/q9]

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the range of f. [1]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leqslant k$.

(iii) State the largest value of k for which g has an inverse. [1]

(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

121. [9709/w18/13/q11]

(i) Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 2x^2 - 12x + 11$ for $x \leqslant k$.

(ii) State the largest value of the constant k for which f is a one-one function. [1]

(iii) For this value of k find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = x + 3$ for $x \leqslant p$.

(iv) With k now taking the value 1, find the largest value of the constant p which allows the composite function fg to be formed, and find an expression for $fg(x)$ whenever this composite function exists. [3]

122. [9709/m17/12/q1]

Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots. [4]

123. [9709/m17/12/q8]

The functions f and g are defined for $x \geqslant 0$ by

$$f : x \mapsto 2x^2 + 3,$$
$$g : x \mapsto 3x + 2.$$

(i) Show that $gf(x) = 6x^2 + 11$ and obtain an unsimplified expression for $fg(x)$. [2]

(ii) Find an expression for $(fg)^{-1}(x)$ and determine the domain of $(fg)^{-1}$. [5]

(iii) Solve the equation $gf(2x) = fg(x)$. [3]

124. [9709/s17/11/q9]

The function f is defined by $f : x \mapsto \dfrac{2}{3 - 2x}$ for $x \in \mathbb{R},\ x \neq \frac{3}{2}$.

(i) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

125. [9709/s17/13/q9]

(i) Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geqslant p$, where p is a constant.

(ii) State the smallest value of p for which f is a one-one function. [1]

(iii) For this value of p, obtain an expression for $f^{-1}(x)$, and state the domain of f^{-1}. [4]

(iv) State the set of values of q for which the equation $f(x) = q$ has no solution. [1]

126. [9709/w17/11/q9]

Functions f and g are defined for $x > 3$ by

$$f : x \mapsto \frac{1}{x^2 - 9},$$
$$g : x \mapsto 2x - 3.$$

(i) Find and simplify an expression for $gg(x)$. [2]

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

(iii) Solve the equation $fg(x) = \frac{1}{7}$. [4]

127. [9709/w17/12/q2]

A function f is defined by $\mathrm{f} : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$ and find the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

128. [9709/w17/13/q6]

The functions f and g are defined by

$$\mathrm{f}(x) = \frac{2}{x^2 - 1} \text{ for } x < -1,$$
$$\mathrm{g}(x) = x^2 + 1 \text{ for } x > 0.$$

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(ii) Solve the equation $\mathrm{gf}(x) = 5$. [4]

129. [9709/m16/12/q8]

The function f is such that $f(x) = a^2x^2 - ax + 3b$ for $x \leqslant \frac{1}{2a}$, where a and b are constants.

(i) For the case where $f(-2) = 4a^2 - b + 8$ and $f(-3) = 7a^2 - b + 14$, find the possible values of a and b. [5]

(ii) For the case where $a = 1$ and $b = -1$, find an expression for $f^{-1}(x)$ and give the domain of f^{-1}. [5]

130. [9709/s16/12/q1]

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \tfrac{3}{2}.$$

Solve the equation $\mathrm{ff}(x) = \mathrm{gf}(2)$. [3]

131. [9709/s16/12/q11]

The function f is defined by $\mathrm{f} : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = \mathrm{f}(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [2]

132. [9709/s16/13/q10]

The function f is such that $f(x) = 2x + 3$ for $x \geqslant 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leqslant q$, where a, b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leqslant q$.

(i) Find the values of a and b. [3]

(ii) Find the greatest possible value of q. [2]

It is now given that $q = -3$.

(iii) Find the range of fg. [1]

(iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

133. [9709/w16/11/q1]

(i) Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

134. [9709/w16/11/q8]

The functions f and g are defined by

$$f(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

$$g(x) = \frac{4}{5x+2} \quad \text{for } x \geqslant 0.$$

(i) Find and simplify an expression for $fg(x)$ and state the range of fg. [3]

(ii) Find an expression for $g^{-1}(x)$ and find the domain of g^{-1}. [5]

135. [9709/w16/13/q8]

(i) Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $f(x) = x^2 + 1$ and $fg(x) = 4x^2 + 12x + 10$. Find $g(x)$. [1]

(iii) Find $(fg)^{-1}(x)$ and give the domain of $(fg)^{-1}$. [4]

136. [9709/s15/11/q8]

The function $f : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ is defined for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 7$, giving your answer correct to 2 decimal places. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Explain why f has an inverse. [1]

(iv) Obtain an expression for $f^{-1}(x)$. [3]

137. [9709/s15/12/q11]

The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $\mathrm{f}(x) = p$ has no real roots. [3]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $\mathrm{g}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of g. [2]

The function h is defined by $\mathrm{h} : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$, where k is a constant.

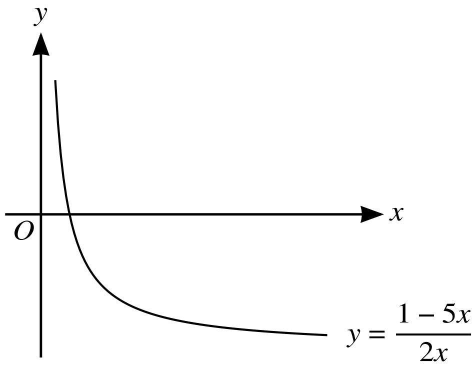
(iv) State the smallest value of k for which h has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{h}^{-1}(x)$. [3]

138. [9709/s15/13/q1]

Express $2x^2 - 12x + 7$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

139. [9709/s15/13/q6]



The diagram shows the graph of $y = \mathrm{f}^{-1}(x)$, where f^{-1} is defined by $\mathrm{f}^{-1}(x) = \dfrac{1-5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $\mathrm{f}(x)$ and state the domain of f. [5]

(ii) The function g is defined by $\mathrm{g}(x) = \dfrac{1}{x}$ for $x \geqslant 1$. Find an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$, giving your answer in the form $ax + b$, where a and b are constants to be found. [2]

140. [9709/w15/11/q6]

A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

141. [9709/w15/11/q9]

(i) Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

The function $f : x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

(ii) State the smallest value of m for which f is one-one. [1]

(iii) For the case where $m = 5$, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

142. [9709/w15/12/q1]

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 4x - 12, \quad x \in \mathbb{R}.$$

Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{gf}(x)$. [4]

143. [9709/w15/12/q8]

The function f is defined, for $x \in \mathbb{R}$, by $\mathrm{f} : x \mapsto x^2 + ax + b$, where a and b are constants.

(i) In the case where $a = 6$ and $b = -8$, find the range of f. [3]

(ii) In the case where $a = 5$, the roots of the equation $\mathrm{f}(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k. [3]

(iii) Show that if the equation $\mathrm{f}(x + a) = a$ has no real roots, then $a^2 < 4(b - a)$. [3]

144. [9709/w15/13/q1]

A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]

145. [9709/w15/13/q8]

The function f is defined by $f(x) = 3x + 1$ for $x \leqslant a$, where a is a constant. The function g is defined by $g(x) = -1 - x^2$ for $x \leqslant -1$.

(i) Find the largest value of a for which the composite function gf can be formed. [2]

For the case where $a = -1$,

(ii) solve the equation $fg(x) + 14 = 0$, [3]

(iii) find the set of values of x which satisfy the inequality $gf(x) \leqslant -50$. [4]

Chapter 2

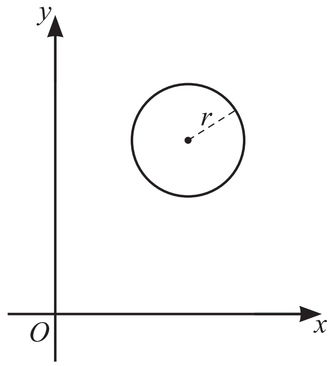
Coordinate geometry

1. [9709/m25/12/q1]

A curve has equation $y = 5 + 3x - 2x^2$ and a straight line has equation $y = kx + 13$, where k is a constant.

Find the set of values of k for which the curve and the line do **not** meet. [4]

2. [9709/m25/12/q6]



The diagram shows a circle C of radius r, where $x > 0$ and $y > 0$ for all points on C. The least distance between any point on C and the x-axis is 8 units, and the least distance between any point on C and the y-axis is 5 units.

(a) State the coordinates of the centre of the circle in terms of r. [1]

(b) Given that the distance between the origin and the centre of the circle is 15 units, find the value of r. [3]

(c) The point on the circle furthest from the origin is denoted by P.

Find the gradient of the tangent to the circle at P. [2]

3. [9709/s25/11/q8]

The circle with equation $x^2 + y^2 - 6x + 10y - 27 = 0$ intersects the line $x = -2$ at the points P and Q.

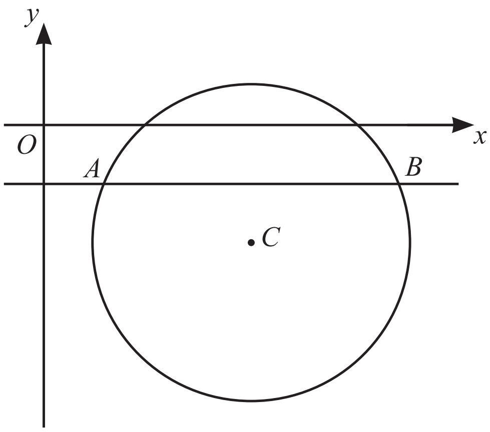
Find the area of the triangle formed by the tangents to the circle at P and Q, and the line $x = -2$. [8]

4. [9709/s25/12/q2]

Find the coordinates of the points of intersection of the curve and the line with equations

$$2xy + 5y^2 = 24 \quad \text{and} \quad 2x + y + 4 = 0.$$ [4]

5. [9709/s25/12/q8]



The diagram shows the circle with equation $x^2+y^2-14x+8y+36=0$ and the line $y=-2$. The line intersects the circle at the points A and B. The centre of the circle is C.

(a) Find the coordinates of A, B and C. [3]

(b) Find the angle ACB in radians. Give your answer correct to 3 significant figures. [2]

(c) The chord AB divides the circle into two segments.

Find the area of the larger segment. [4]

6. [9709/s25/13/q9]

Three points P, Q and R have coordinates $P(-13, 5)$, $Q(5, 1)$ and $R(2, k)$, where k is a constant. It is given that the angle PRQ is a right angle.

(a) Show that one of the possible values of k is 10, and find the other possible value. [4]

(b) It is now given that $k = 10$. A circle passes through the points P, Q and R.

Find the equation of the tangent to the circle at R. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

7. [9709/s25/15/q10]

The equation of a circle is $x^2 + y^2 + 4x - 8y - 12 = 0$.

(a) Find an equation of the tangent to the circle at the point (2, 8), giving your answer in the form $ax + by + c = 0$. [4]

(b) Given that the line $x + 3y = k$ does **not** intersect the circle, show that $k^2 - 20k - 220 > 0$. [5]

8. [9709/w25/11/q10]

A circle has equation $x^2 + y^2 + 4y - 21 = 0$ and a straight line has equation $2x + y - 8 = 0$. The line intersects the circle at two points.

(a) Find the coordinates of these two points of intersection. [4]

(b) The circle has centre C and the two points of intersection are denoted by A and B.

Find the area of the triangle ABC. [3]

9. [9709/w25/12/q7]

The coordinates of the points P and Q are (1, 1) and (7, 11) respectively. The line segment PQ forms a diameter of a circle.

(a) Find the equation of the circle. [4]

(b) Find the equation of the tangent to the circle at the point Q. [3]

(c) The other point on the circle with x-coordinate 7 is R.

Find the coordinates of the point of intersection of the tangent at Q with the tangent at R. [4]

10. [9709/w25/13/q8]

The points (6, 1) and (−2, 7) lie at the opposite ends of a diameter of a circle.

(a) Find the equation of the circle. [3]

(b) There are two tangents to the circle which have gradient $-\frac{1}{2}$.

Find the exact values of the x-coordinates of the points at which these tangents touch the circle. [5]

11. [9709/w25/15/q1]

A circle has centre (2, 6) and radius 10.

(a) State the equation of the circle. [2]

(b) The circle passes through the point (8, k).

Find the two possible values of k. [3]

12. [9709/m24/12/q7]

The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point P.

(a) Find the value of the constant k. [4]

(b) Find the coordinates of P. [2]

13. [9709/s24/11/q10]

The equation of a circle is $(x-3)^2+y^2=18$. The line with equation $y=mx+c$ passes through the point $(0,-9)$ and is a tangent to the circle.

Find the two possible values of m and, for each value of m, find the coordinates of the point at which the tangent touches the circle. [8]

14. [9709/s24/12/q7]

The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

(a) Find the two possible values of the constant a. [5]

(b) For the greater value of a, find the equation of the diameter which is perpendicular to the given tangent. [3]

15. [9709/s24/13/q8]

A circle with equation $x^2 + y^2 - 6x + 2y - 15 = 0$ meets the y-axis at the points A and B. The tangents to the circle at A and B meet at the point P.

Find the coordinates of P. [8]

16. [9709/w24/11/q4]

Show that the curve with equation $x^2 - 3xy - 40 = 0$ and the line with equation $3x + y + k = 0$ meet for all values of the constant k. [5]

17. [9709/w24/11/q6]

Circles C_1 and C_2 have equations

$$x^2 + y^2 + 6x - 10y + 18 = 0 \quad \text{and} \quad (x-9)^2 + (y+4)^2 - 64 = 0$$

respectively.

(a) Find the distance between the centres of the circles. [4]

P and Q are points on C_1 and C_2 respectively. The distance between P and Q is denoted by d.

(b) Find the greatest and least possible values of d. [3]

18. [9709/w24/12/q3]

The equation of a curve is $y = 2x^2 - 3$. Two points A and B with x-coordinates 2 and $(2 + h)$ respectively lie on the curve.

(a) Find and simplify an expression for the gradient of the chord AB in terms of h. [3]

(b) Explain how the gradient of the curve at the point A can be deduced from the answer to part **(a)**, and state the value of this gradient. [2]

19. [9709/w24/12/q9]

The equation of a curve is $y = \frac{1}{2}k^2x^2 - 2kx + 2$ and the equation of a line is $y = kx + p$, where k and p are constants with $0 < k < 1$.

(a) It is given that one of the points of intersection of the curve and the line has coordinates $\left(\frac{5}{2}, \frac{1}{2}\right)$.

Find the values of k and p, and find the coordinates of the other point of intersection. [7]

(b) It is given instead that the line and the curve do **not** intersect.

Find the set of possible values of p. [3]

20. [9709/w24/13/q10]

Points A and B have coordinates (4, 3) and (8, −5) respectively. A circle with radius 10 passes through the points A and B.

(a) Show that the centre of the circle lies on the line $y = \frac{1}{2}x - 4$. [4]

(b) Find the two possible equations of the circle. [5]

21. [9709/m23/12/q1]

A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

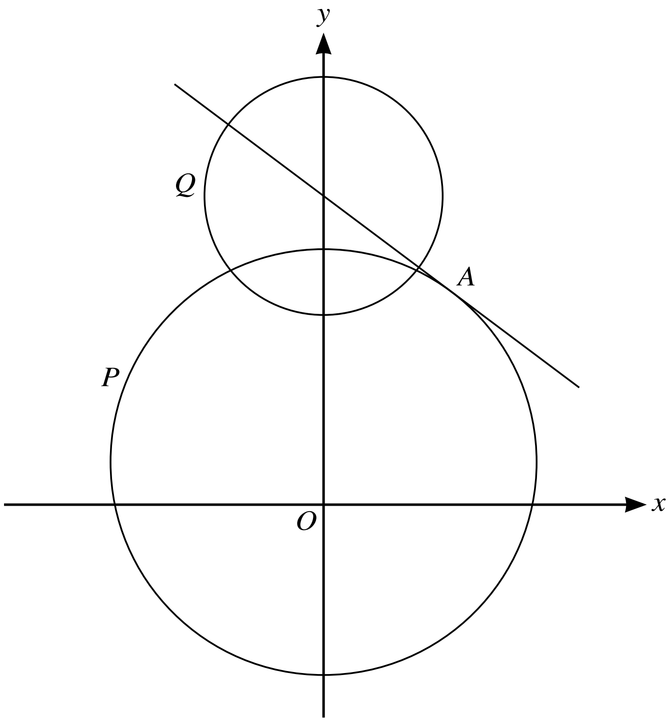
Show that the line and the curve meet for all values of k. [4]

22. [9709/m23/12/q5]

Points $A\,(7,\ 12)$ and B lie on a circle with centre $(-2,\ 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

23. [9709/s23/11/q12]



The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

(b) Find the equation of the tangent to the circle P at A. [2]

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

24. [9709/s23/12/q10]

The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

25. [9709/s23/13/q5]

A circle has equation $(x-1)^2+(y+4)^2=40$. A line with equation $y=x-9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

(b) Find an equation of the circle with diameter AB. [3]

26. [9709/w23/11/q2]

A line has equation $y = 2cx + 3$ and a curve has equation $y = cx^2 + 3x - c$, where c is a constant.

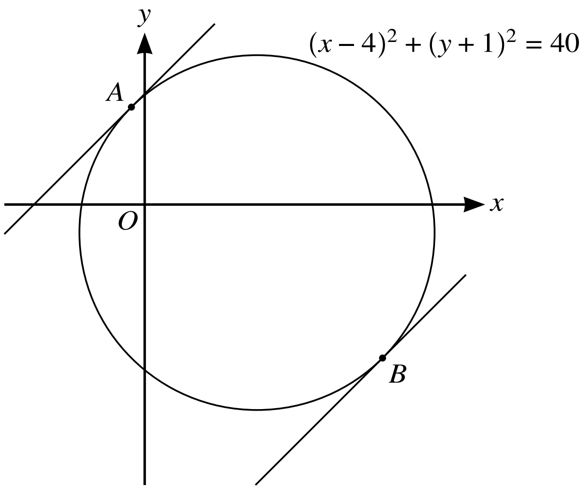
Showing all necessary working, determine which of the following statements is correct.

A The line and curve intersect only for a particular set of values of c.

B The line and curve intersect for all values of c.

C The line and curve do not intersect for any values of c. [4]

27. [9709/w23/11/q11]



The diagram shows the circle with equation $(x-4)^2+(y+1)^2=40$. Parallel tangents, each with gradient 1, touch the circle at points A and B.

(a) Find the equation of the line AB, giving the answer in the form $y = mx + c$. [3]

(b) Find the coordinates of A, giving each coordinate in surd form. [4]

(c) Find the equation of the tangent at A, giving the answer in the form $y = mx + c$, where c is in surd form. [2]

28. [9709/w23/12/q11]

The coordinates of points A, B and C are $(6, 4)$, $(p, 7)$ and $(14, 18)$ respectively, where p is a constant. The line AB is perpendicular to the line BC.

(a) Given that $p < 10$, find the value of p. [4]

A circle passes through the points A, B and C.

(b) Find the equation of the circle. [3]

(c) Find the equation of the tangent to the circle at C, giving the answer in the form $dx + ey + f = 0$, where d, e and f are integers. [3]

29. [9709/w23/13/q2]

The circle with equation $(x-3)^2 + (y-5)^2 = 40$ intersects the y-axis at points A and B.

(a) Find the y-coordinates of A and B, expressing your answers in terms of surds. [2]

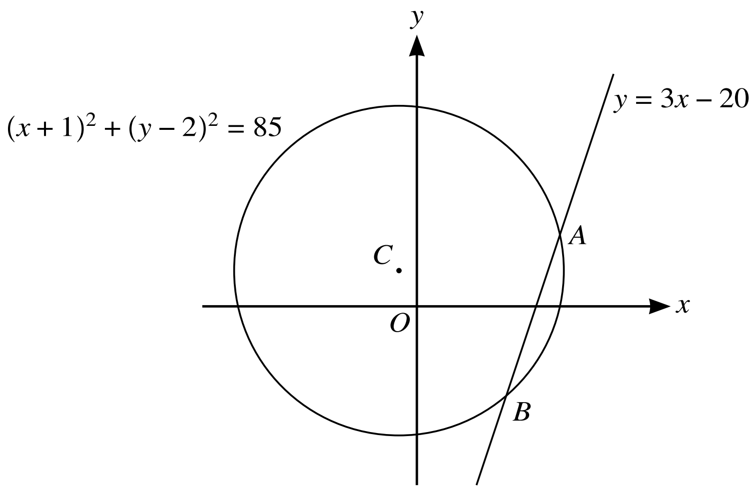
(b) Find the equation of the circle which has AB as its diameter. [2]

30. [9709/w23/13/q6]

A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where c is a constant. The line is a tangent to the curve at point P.

Find the possible values of c and the corresponding coordinates of P. [7]

31. [9709/m22/12/q6]



The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

32. [9709/s22/11/q9]

The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

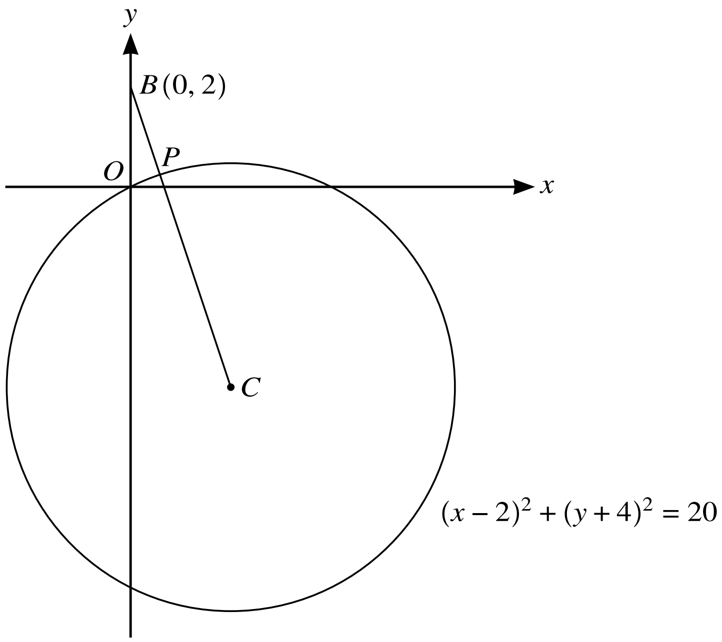
33. [9709/s22/12/q8]

The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A\,(1,\ 1)$ and $B\,(2,\ -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

34. [9709/s22/13/q7]

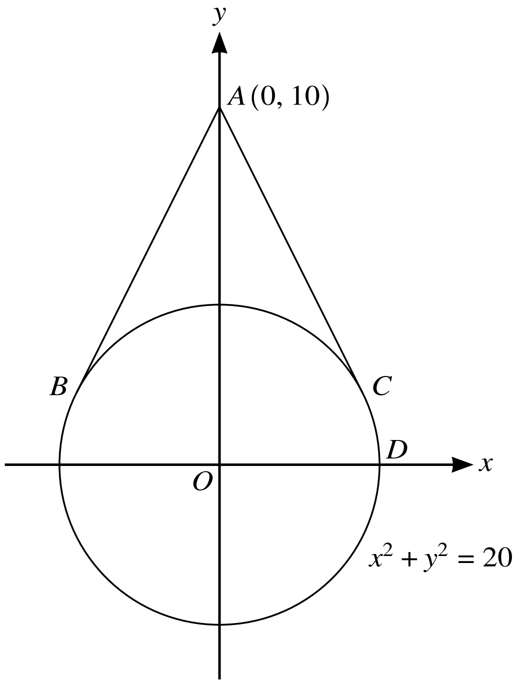


The diagram shows the circle with equation $(x-2)^2+(y+4)^2=20$ and with centre C. The point B has coordinates (0, 2) and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

(b) Hence find the coordinates of P, giving your answer in exact form. [5]

35. [9709/w22/11/q11]



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point $A\,(0,\ 10)$.

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

(b) Find the coordinates of B and C. [3]

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]

36. [9709/w22/12/q1]

Points A and B have coordinates $(5,\ 2)$ and $(10,\ -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

(b) Find the equation of the circle with centre A which passes through B. [3]

37. [9709/w22/13/q11]

The coordinates of points A, B and C are $A\,(5, -2)$, $B\,(10, 3)$ and $C\,(2p, p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]

38. [9709/m21/12/q8]

The points $A\,(7,\ 1)$, $B\,(7,\ 9)$ and $C\,(1,\ 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

39. [9709/s21/11/q10]

The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

40. [9709/s21/12/q6]

Points A and B have coordinates (8, 3) and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

41. [9709/s21/12/q7]

The point A has coordinates $(1,\ 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5,\ 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

42. [9709/s21/13/q10]

Points $A\,(-2,\ 3)$, $B\,(3,\ 0)$ and $C\,(6,\ 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

(b) Hence state the coordinates of D. [1]

(c) Find an equation of the circle. [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

43. [9709/w21/11/q7]

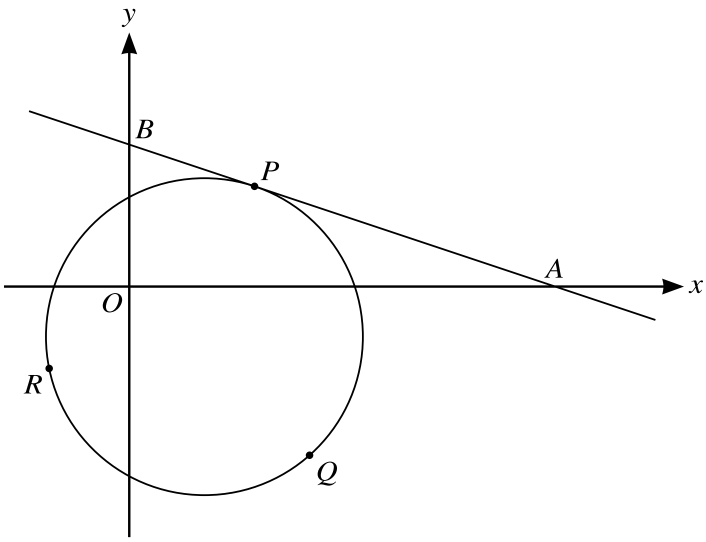
A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle. [2]

The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

44. [9709/w21/12/q12]



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

45. [9709/w21/13/q9]

The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

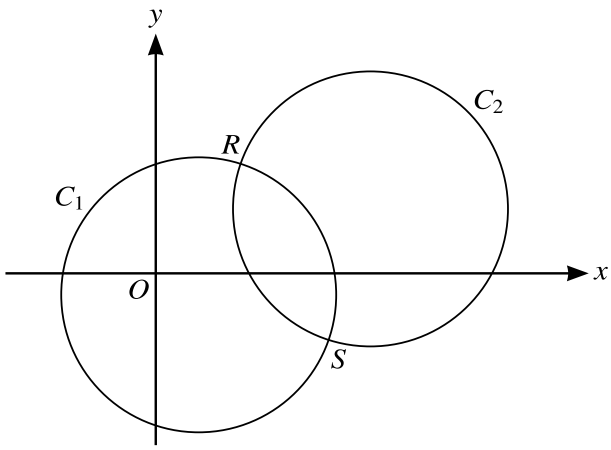
A straight line through the point $(10, 0)$ with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

46. [9709/m20/12/q12]

A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

47. [9709/s20/11/q10]

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

48. [9709/s20/12/q11]

The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

49. [9709/s20/13/q10]

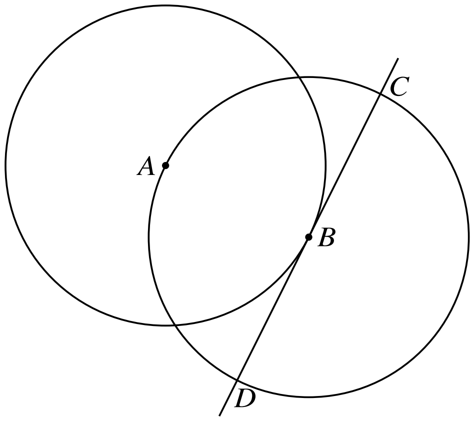
(a) The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

50. [9709/w20/11/q9]



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

(c) Find, by calculation, the x-coordinates of C and D. [3]

51. [9709/w20/12/q9]

A circle has centre at the point $B(5,\ 1)$. The point $A(-1,\ -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5,\ 16)$.

(b) Show that DC is a tangent to the circle. [4]

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

52. [9709/w20/13/q11]

A circle with centre C has equation $(x-8)^2 + (y-4)^2 = 100$.

(a) Show that the point T $(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

(d) Find the x-coordinates of A and B. [3]

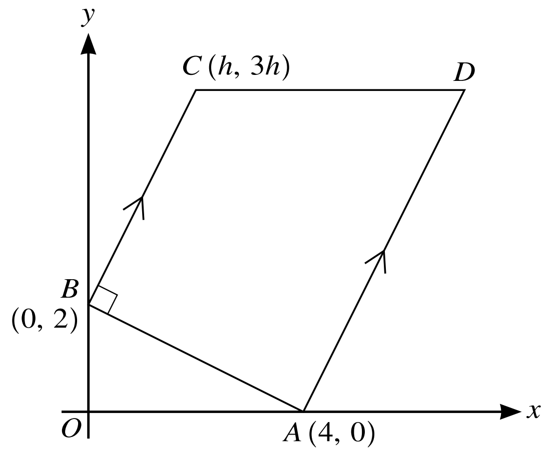
53. [9709/s19/11/q2]

The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

(ii) Find the coordinates of P. [2]

54. [9709/s19/11/q4]



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are (4, 0), (0, 2) and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. [3]

(ii) Hence find the coordinates of D. [3]

55. [9709/s19/12/q2]

Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y-axis at the point C. Find the coordinates of C. [5]

56. [9709/s19/13/q7]

The coordinates of two points A and B are $(1, 3)$ and $(9, -1)$ respectively and D is the mid-point of AB. A point C has coordinates (x, y), where x and y are variables.

(i) State the coordinates of D. [1]

(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. [1]

(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

57. [9709/w19/11/q6]

A straight line has gradient m and passes through the point $(0, -2)$. Find the two values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and, for each value of m, find the coordinates of the point where the line touches the curve. [7]

58. [9709/w19/12/q2]

The point M is the mid-point of the line joining the points $(3,\ 7)$ and $(-1,\ 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

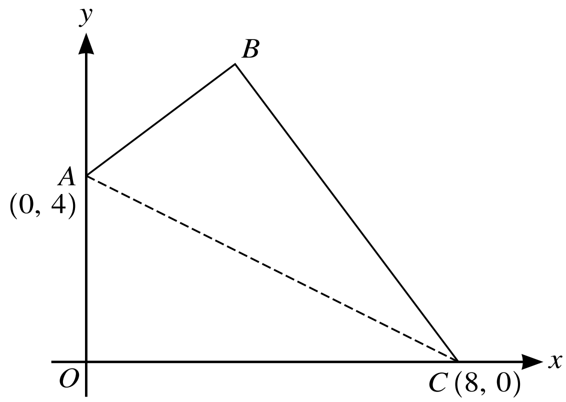
59. [9709/m18/12/q4]

A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0,\ 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]

(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

60. [9709/s18/11/q5]



The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are (0, 4) and (8, 0) respectively and O is the origin.

(i) Find the equations of AC and OB. [4]

(ii) Find, by calculation, the coordinates of B. [3]

61. [9709/s18/12/q2]

The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

62. [9709/s18/12/q8]

Points A and B have coordinates $(h,\ h)$ and $(4h+6,\ 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

63. [9709/s18/13/q6]

The coordinates of points A and B are $(-3k - 1,\ k + 3)$ and $(k + 3,\ 3k + 5)$ respectively, where k is a constant $(k \neq -1)$.

(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

64. [9709/w18/11/q3]

Two points A and B have coordinates $(3a, -a)$ and $(-a, 2a)$ respectively, where a is a positive constant.

(i) Find the equation of the line through the origin parallel to AB. [2]

(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a. [3]

65. [9709/w18/12/q2]

Showing all necessary working, find $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) \mathrm{d}x$. [4]

66. [9709/w18/12/q10]

The equation of a curve is $y = 2x + \dfrac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

67. [9709/w18/13/q4]

Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C. [4]

(ii) Find the distance AC, giving your answer correct to 3 decimal places. [2]

68. [9709/w18/13/q9]

A curve has equation $y = 2x^2 - 3x + 1$ and a line has equation $y = kx + k^2$, where k is a constant.

(i) Show that, for all values of k, the curve and the line meet. [4]

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve. [4]

69. [9709/s17/12/q2]

The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

(ii) Find the coordinates of B. [3]

70. [9709/s17/13/q3]

Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

71. [9709/s17/13/q8]

$A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

72. [9709/w17/11/q6]

The points $A\,(1,\ 1)$ and $B\,(5,\ 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

The perpendicular bisector of AB meets the curve at C and D.

(ii) Find, by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

73. [9709/w17/13/q2]

Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

74. [9709/m16/12/q5]

Two points have coordinates $A\,(5,\ 7)$ and $B\,(9,\ -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C\,(1,\ 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

(ii) Find, by calculation, the distance BX. [5]

75. [9709/s16/11/q6]

(a) Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

76. [9709/s16/12/q8]

Three points have coordinates $A\,(0,\ 7)$, $B\,(8,\ 3)$ and $C\,(3k,\ k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

77. [9709/s16/13/q11]

Triangle ABC has vertices at $A(-2, -1)$, $B(4, 6)$ and $C(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

78. [9709/w16/11/q4]

C is the mid-point of the line joining $A\,(14,\ -7)$ to $B\,(-6,\ 3)$. The line through C perpendicular to AB crosses the y-axis at D.

(i) Find the equation of the line CD, giving your answer in the form $y = mx + c$. [4]

(ii) Find the distance AD. [2]

79. [9709/w16/12/q3]

A curve has equation $y = 2x^2 - 6x + 5$.

(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

80. [9709/w16/12/q5]

The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

81. [9709/w16/13/q1]

Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

82. [9709/w16/13/q6]

Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

83. [9709/s15/11/q6]

The line with gradient -2 passing through the point $P(3t,\ 2t)$ intersects the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

(ii) Show that the mid-point of PC lies on the line $y = x$. [4]

84. [9709/s15/12/q7]

The point C lies on the perpendicular bisector of the line joining the points $A\,(4,\ 6)$ and $B\,(10,\ 2)$. C also lies on the line parallel to AB through $(3,\ 11)$.

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Calculate the coordinates of C. [3]

85. [9709/s15/13/q7]

The point A has coordinates $(p, 1)$ and the point B has coordinates $(9, 3p + 1)$, where p is a constant.

(i) For the case where the distance AB is 13 units, find the possible values of p. [3]

(ii) For the case in which the line with equation $2x + 3y = 9$ is perpendicular to AB, find the value of p. [4]

86. [9709/w15/12/q6]

Points A, B and C have coordinates $A\,(-3,\ 7)$, $B\,(5,\ 1)$ and $C\,(-1,\ k)$, where k is a constant.

(i) Given that $AB = BC$, calculate the possible values of k. [3]

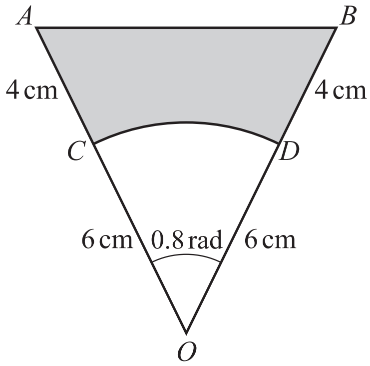
The perpendicular bisector of AB intersects the x-axis at D.

(ii) Calculate the coordinates of D. [5]

Chapter 3

Circular measure

1. [9709/m25/12/q4]

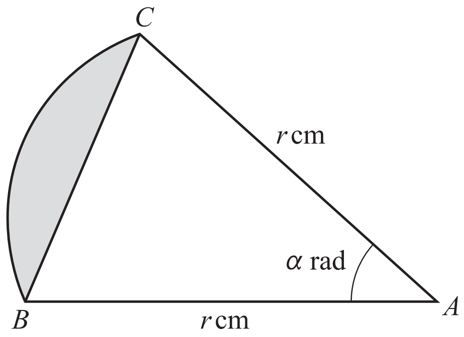


The diagram shows a triangle OAB where $OA = OB = 10$ cm and angle $AOB = 0.8$ radians.
Points C and D on OA and OB respectively are such that the arc CD is part of a circle with centre O and radius 6 cm. The shaded region is bounded by the arc CD and the line segments CA, AB and BD.

(a) Find the perimeter of the shaded region. [3]

(b) Find the area of the shaded region. [3]

2. [9709/s25/11/q9]



The diagram shows a sector ABC of a circle with centre A and radius r cm. The angle BAC is α radians, where $0 < \alpha < \frac{1}{2}\pi$.

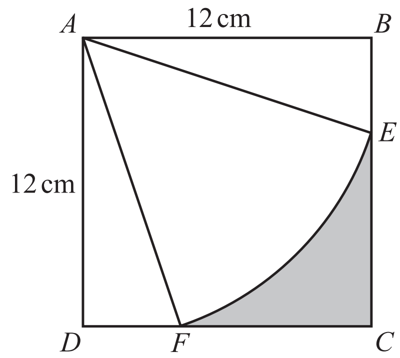
(a) It is given that the area of the triangle ABC is 4 cm^2 and the area of the sector ABC is $8\alpha\text{ cm}^2$.

Find the exact area of the shaded segment. [4]

(b) It is given instead that the length of the chord BC is $\frac{1}{\sqrt{2}}r$ cm but the area of the triangle ABC is still 4 cm^2.

Find the area of the shaded segment. Give your answer correct to 3 significant figures. [4]

3. [9709/s25/13/q8]



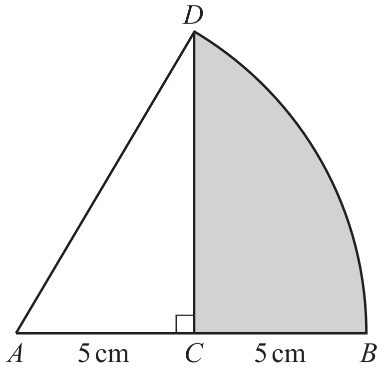
The diagram shows a square $ABCD$ where each side has length 12 cm. Points E and F lie on the sides BC and CD respectively and are such that $BE = \frac{1}{3}BC$ and $DF = \frac{1}{3}DC$. The arc EF is part of a circle with centre A. The shaded region is bounded by the arc EF and the line segments EC and FC.

(a) Show that the size of angle EAF is 0.9273 radians, correct to 4 significant figures. [2]

(b) Find the perimeter of the shaded region. [3]

(c) Find the area of the shaded region. [3]

4. [9709/s25/15/q5]

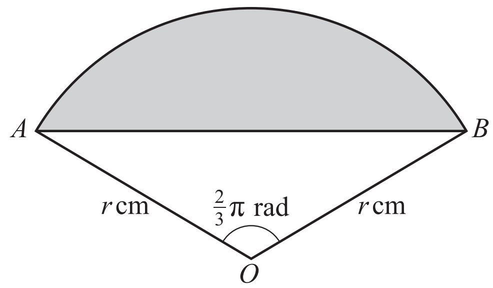


The diagram shows a sector ABD of a circle with centre A and radius 10 cm. The perpendicular bisector of AB passes through D.

(a) Find the perimeter of the shaded region BCD, giving your answer correct to 1 decimal place. [4]

(b) Find the area of the shaded region BCD, giving your answer correct to 1 decimal place. [2]

5. [9709/w25/11/q7]



The diagram shows a sector of a circle with centre O and radius r cm. The shaded region is bounded by the chord AB and the arc AB. The size of angle AOB is $\frac{2}{3}\pi$ radians.

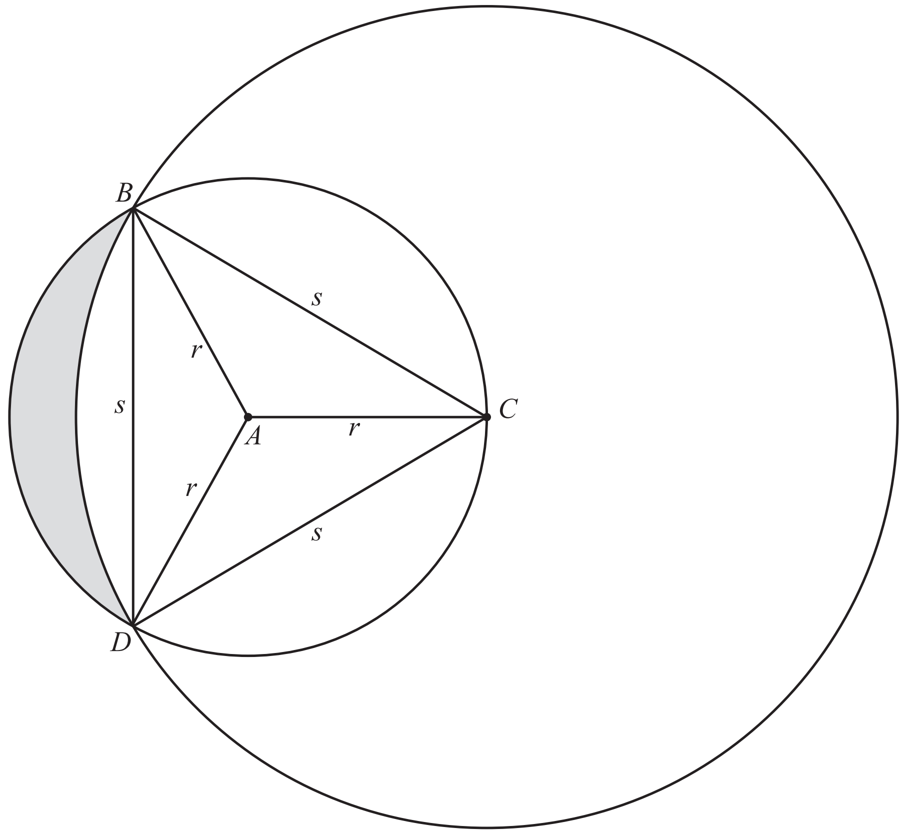
(a) Show that the area of the shaded region is approximately $0.614r^2$ cm^2. [2]

It is given that the radius of the circle is increasing at a rate of $0.4\,\text{cm s}^{-1}$.

(b) **(i)** Find the rate of increase of the area of the shaded region at the instant when $r = 20$. Give your answer correct to 2 significant figures. [3]

(ii) Find the rate of increase of the length of the arc AB. Give your answer correct to 2 significant figures. [3]

6. [9709/w25/12/q10]

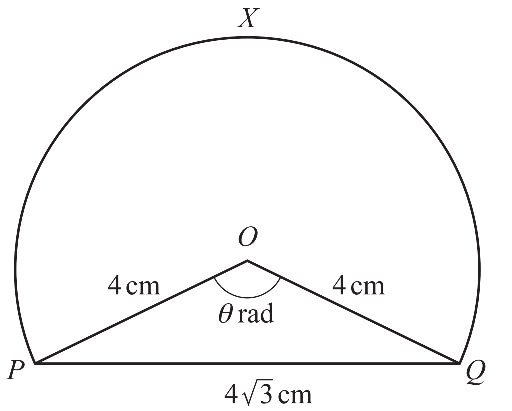


The diagram shows a circle with centre A and radius r passing through points B, C and D. A larger circle of radius s has centre C and passes through B and D. The length BD is also s.

(a) Show that $s = \sqrt{3}r$. [2]

(b) Find an expression for the area of the shaded region. Give your answer in the form $(a + b\pi)r^2$, where a and b are constants to be found. [7]

7. [9709/w25/13/q5]

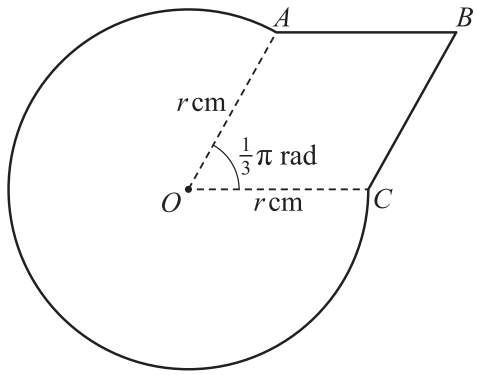


The diagram shows part of a circle with centre O and radius 4 cm. The chord PQ is of length $4\sqrt{3}$ cm and angle $POQ = \theta$ radians. The point X lies on the circle.

(a) Find the exact value of θ. [2]

(b) Find the exact area of the segment PXQ. [3]

8. [9709/w25/15/q4]



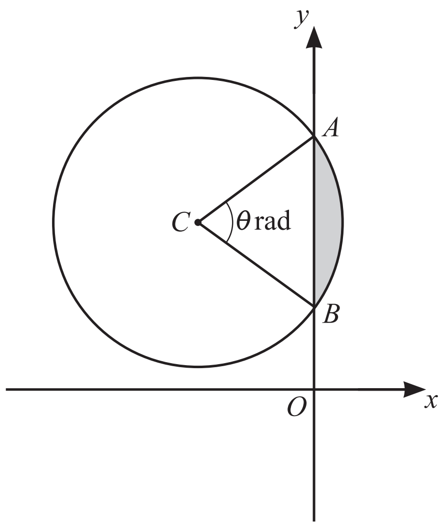
The diagram shows the design for a company's new logo. The sector of the circle, centre O, has radius r cm. The acute angle $AOC = \frac{1}{3}\pi$ radians. The quadrilateral $OABC$ is a rhombus.

(a) Find an expression for the perimeter of the design. Give your answer in terms of π and r. [2]

(b) It is now given that the perimeter of the design is 200 cm.

Find the area of the design. Give your answer to 3 significant figures. [5]

9. [9709/m24/12/q10]



The diagram shows the circle with centre $C(-4, 5)$ and radius $\sqrt{20}$ units. The circle intersects the y-axis at the points A and B. The size of angle ACB is θ radians.

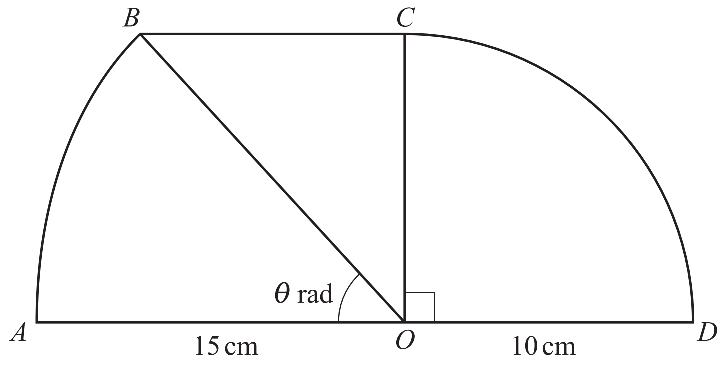
(a) Find the equation of the tangent to the circle at the point $(-6, 9)$. [3]

(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [2]

(c) Find the value of θ correct to 4 significant figures. [3]

(d) Find the perimeter and area of the segment shaded in the diagram. [4]

10. [9709/s24/11/q7]

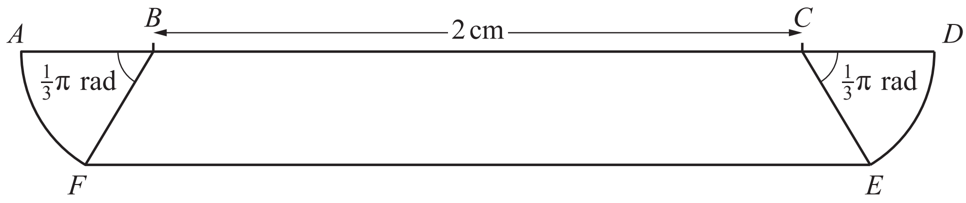


In the diagram, AOD and BC are two parallel straight lines. Arc AB is part of a circle with centre O and radius $15\,\text{cm}$. Angle $BOA = \theta$ radians. Arc CD is part of a circle with centre O and radius $10\,\text{cm}$. Angle $COD = \frac{1}{2}\pi$ radians.

(a) Show that $\theta = 0.7297$, correct to 4 decimal places. [1]

(b) Find the perimeter and the area of the shape $ABCD$. Give your answers correct to 3 significant figures. [7]

11. [9709/s24/12/q8]



The diagram shows a symmetrical plate *ABCDEF*. The line *ABCD* is straight and the length of *BC* is 2 cm. Each of the two sectors *ABF* and *DCE* is of radius *r* cm and each of the angles *ABF* and *DCE* is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

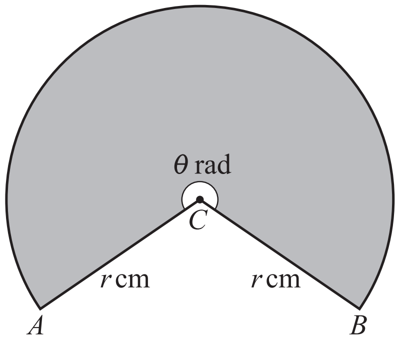
(i) Show that the length $EF = 2.4$ cm. [2]

(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

(b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of *r*. Give your answer correct to 3 significant figures. [4]

12. [9709/s24/13/q3]

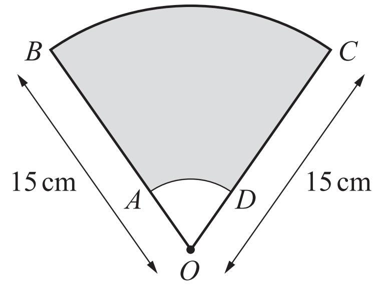


The diagram shows a sector of a circle with centre C. The radii CA and CB each have length r cm and the size of the reflex angle ACB is θ radians. The sector, shaded in the diagram, has a perimeter of 65 cm and an area of 225 cm^2.

(a) Find the values of r and θ. [4]

(b) Find the area of triangle ACB. [2]

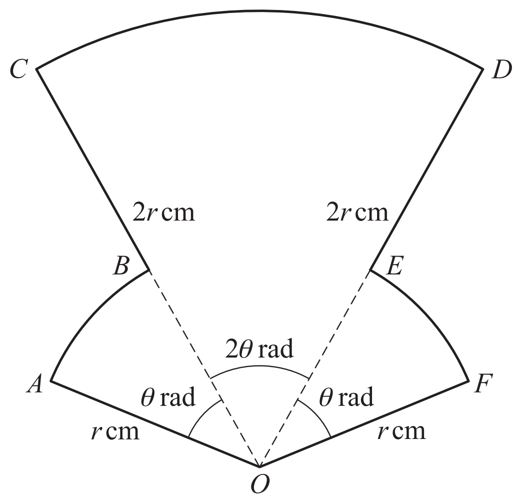
13. [9709/w24/11/q3]



The diagram shows a sector of a circle, centre O, where $OB = OC = 15\,\text{cm}$. The size of angle BOC is $\frac{2}{5}\pi$ radians. Points A and D on the lines OB and OC respectively are joined by an arc AD of a circle with centre O. The shaded region is bounded by the arcs AD and BC and by the straight lines AB and DC. It is given that the area of the shaded region is $\frac{209}{5}\pi\,\text{cm}^2$.

Find the perimeter of the shaded region. Give your answer in terms of π. [5]

14. [9709/w24/12/q6]

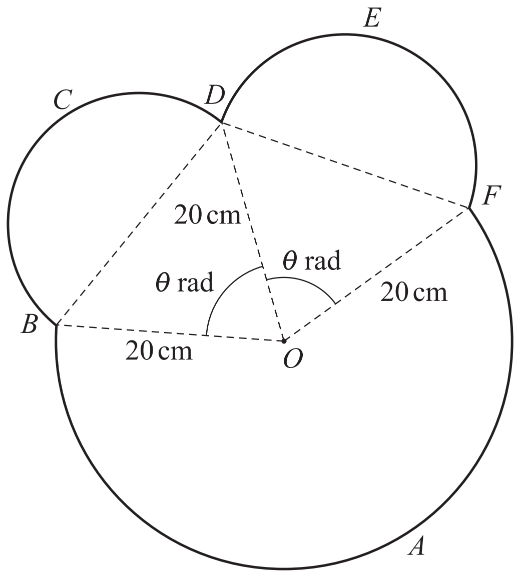


The diagram shows a metal plate $OABCDEF$ consisting of sectors of two circles, each with centre O. The radii of sectors AOB and EOF are r cm and the radius of sector COD is $2r$ cm. Angle AOB = angle $EOF = \theta$ radians and angle $COD = 2\theta$ radians.

It is given that the perimeter of the plate is 14 cm and the area of the plate is 10 cm^2.

Given that $r > \frac{3}{2}$ and $\theta < \frac{3}{4}$, find the values of r and θ. [6]

15. [9709/w24/13/q7]

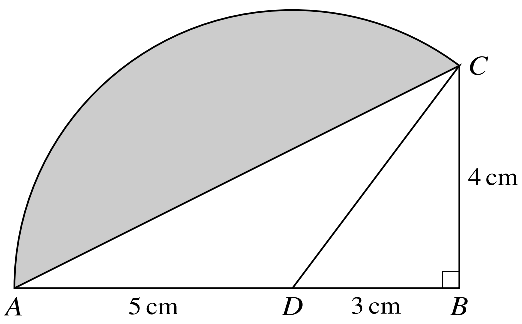


The diagram shows a metal plate *ABCDEF* consisting of five parts. The parts *BCD* and *DEF* are semicircles. The part *BAFO* is a sector of a circle with centre *O* and radius 20 cm, and *D* lies on this circle. The parts *OBD* and *ODF* are triangles. Angles *BOD* and *DOF* are both θ radians.

(a) Given that $\theta = 1.2$, find the area of the metal plate. Give your answer correct to 3 significant figures. [5]

(b) Given instead that the area of each semicircle is $50\pi\ \text{cm}^2$, find the exact perimeter of the metal plate. [5]

16. [9709/m23/12/q8]

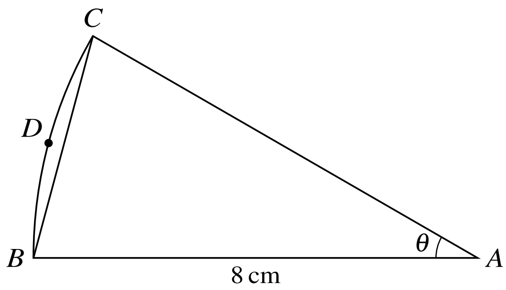


The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

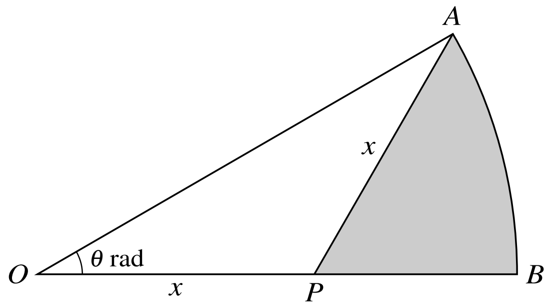
17. [9709/s23/11/q4]



The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi\,\text{cm}^2$. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

18. [9709/s23/12/q6]

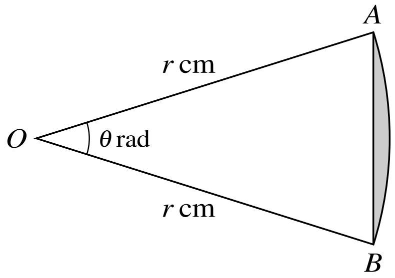


The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

(b) Find the area of the shaded region APB in terms of x and θ. [4]

19. [9709/s23/13/q6]

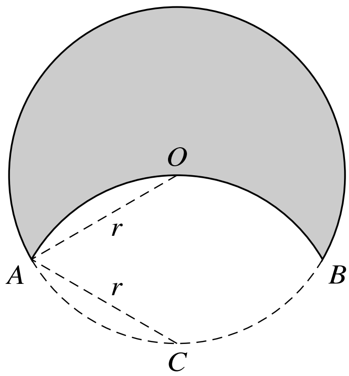


The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2.

(a) Find the area of the shaded region. [5]

(b) Find the perimeter of the shaded region. [2]

20. [9709/w23/11/q6]



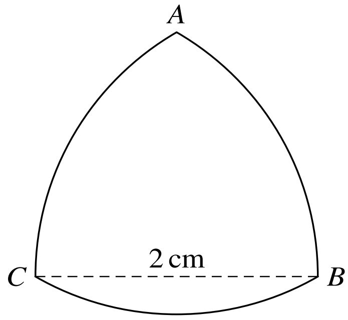
The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O, and the minor arc AOB of a circle, also with radius r but with centre C. The point C lies on the circle with centre O.

(a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k. [1]

(b) State the perimeter of the shaded motif in terms of π and r. [1]

(c) Find the area of the shaded motif, giving your answer in terms of π, r and $\sqrt{3}$. [5]

21. [9709/w23/12/q4]

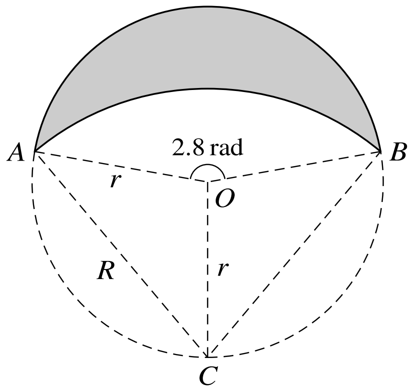


The diagram shows the shape of a coin. The three arcs AB, BC and CA are parts of circles with centres C, A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

(a) Find the perimeter of the coin. [2]

(b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

22. [9709/w23/13/q10]



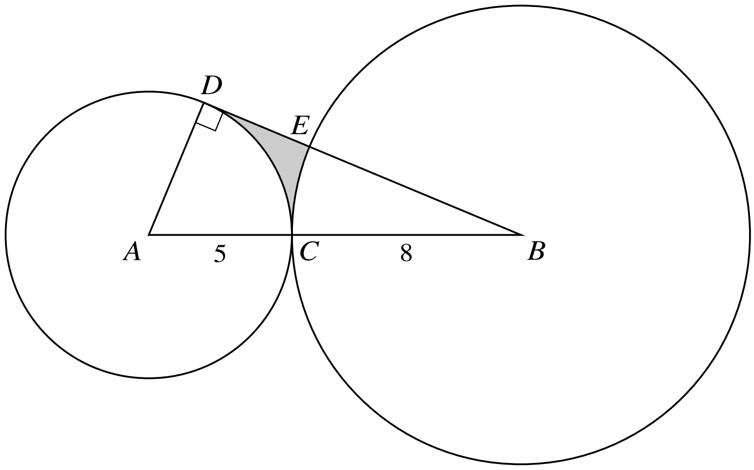
The diagram shows points A, B and C lying on a circle with centre O and radius r. Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r. The lower arc is part of a circle with centre C and radius R.

(a) State the size of angle ACO in radians. [1]

(b) Find R in terms of r. [1]

(c) Find the area of the shaded region in terms of r. [7]

23. [9709/m22/12/q10]

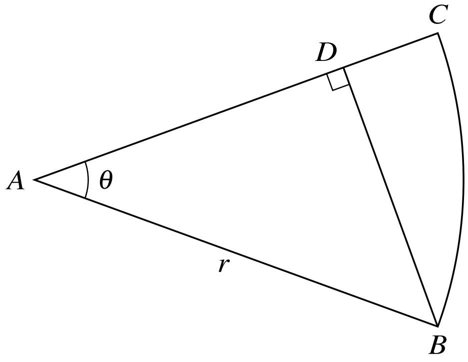


The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

24. [9709/s22/11/q5]

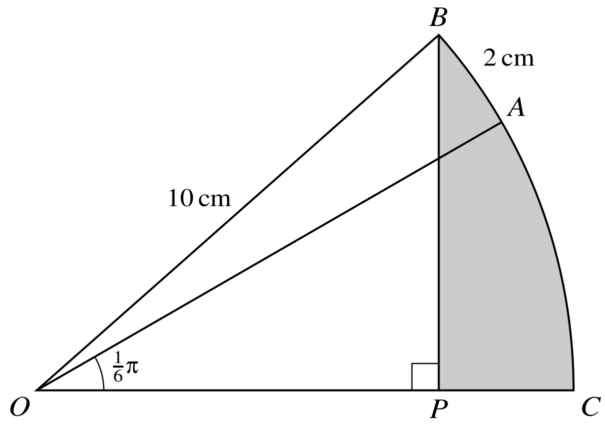


The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

25. [9709/s22/12/q7]

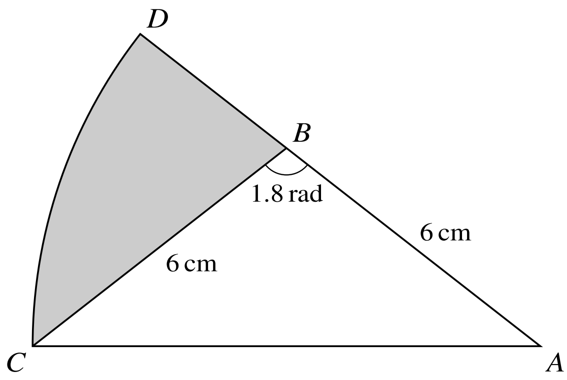


The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC. Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm.

(a) Find the angle BOC. [2]

(b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

26. [9709/s22/13/q9]

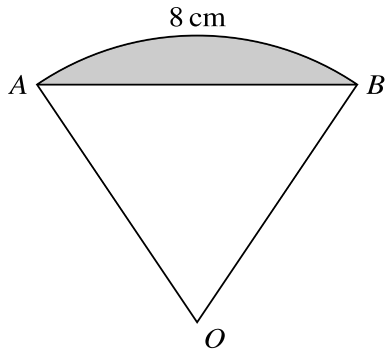


The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

27. [9709/w22/11/q5]

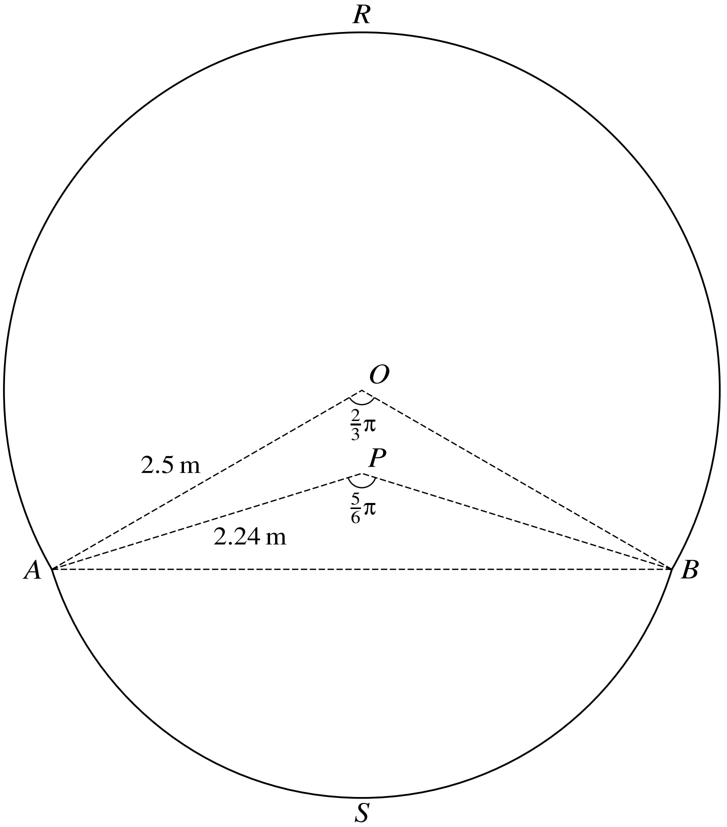


The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

(b) Find the area of the shaded segment. [2]

28. [9709/w22/12/q10]



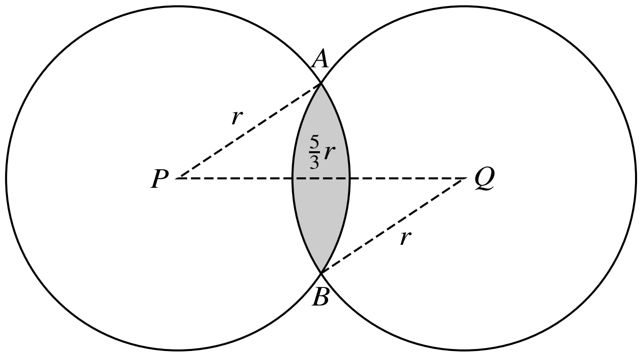
The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O, a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places. [3]

(b) Find the difference in area of the two triangles AOB and APB, giving your answer correct to 2 decimal places. [2]

(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

29. [9709/w22/13/q8]

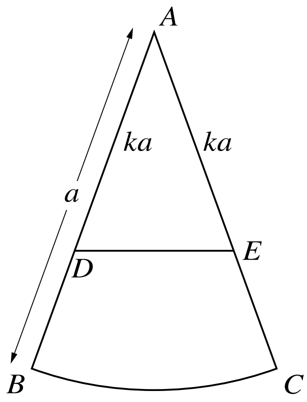


The diagram shows two identical circles intersecting at points A and B and with centres at P and Q. The radius of each circle is r and the distance PQ is $\frac{5}{3}r$.

(a) Find the perimeter of the shaded region in terms of r. [4]

(b) Find the area of the shaded region in terms of r. [3]

30. [9709/m21/12/q10]



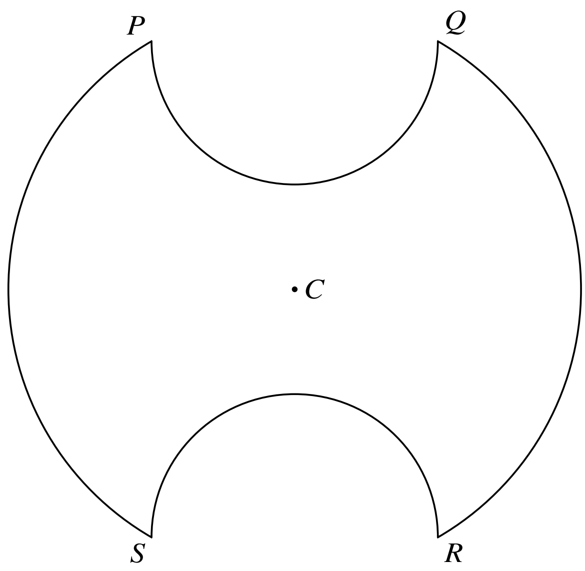
The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

31. [9709/s21/11/q8]



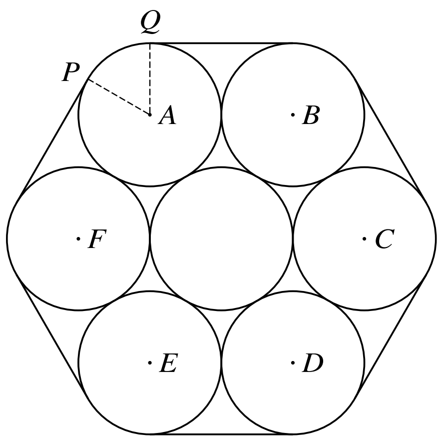
The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

(b) Find the exact perimeter of the plate. [3]

(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)$ cm^2. [5]

32. [9709/s21/12/q12]



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

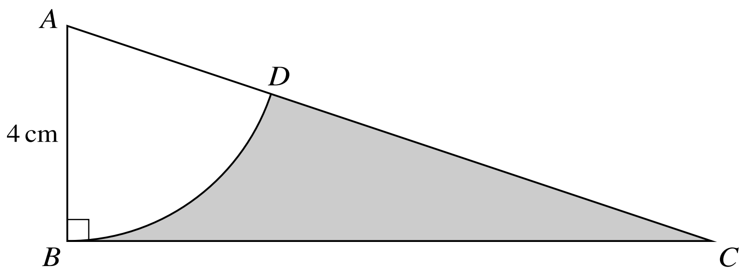
(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

(b) Find the length of the rope. [4]

(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

(d) Find the area of the complete region enclosed by the rope. [3]

33. [9709/s21/13/q5]

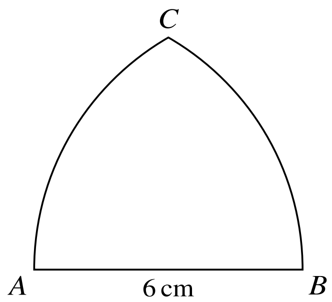


The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

(b) Find the perimeter of the shaded region. [4]

34. [9709/w21/11/q6]

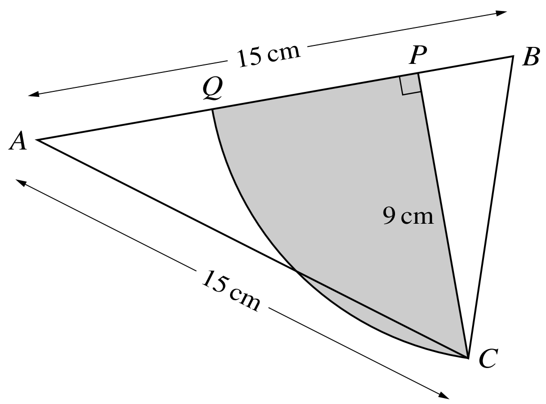


The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC. The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

(a) Find the perimeter of the plate, giving your answer in terms of π. [3]

(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$. [4]

35. [9709/w21/12/q7]

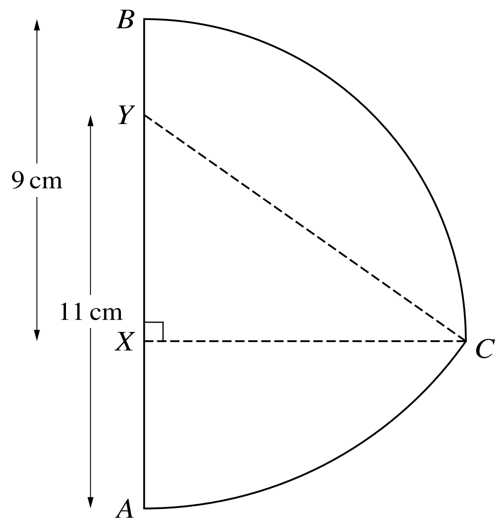


In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures. [2]

(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

36. [9709/w21/13/q5]

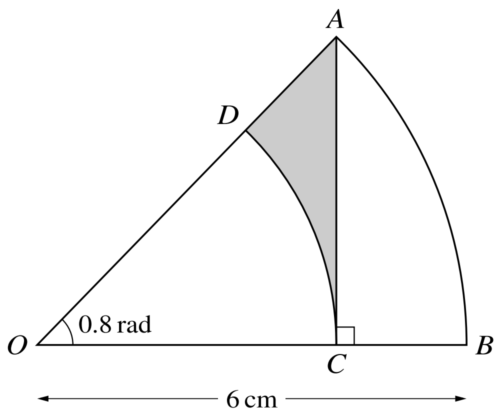


In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

(b) Find the perimeter of ABC. [6]

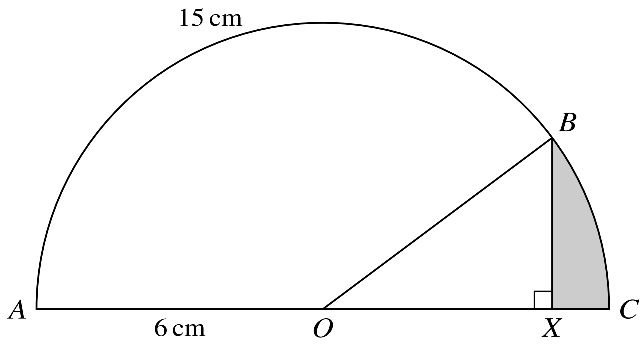
37. [9709/m20/12/q7]



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

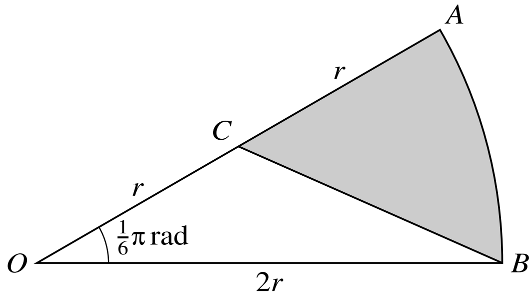
38. [9709/s20/11/q8]



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

39. [9709/s20/12/q7]



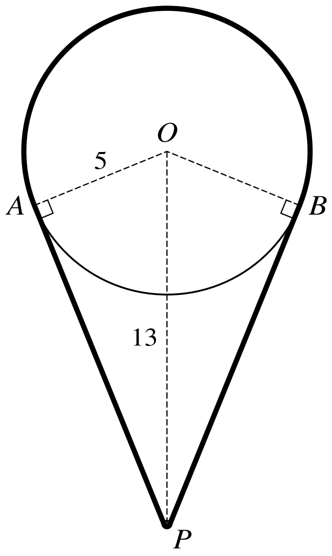
In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

(b) Find the exact perimeter of the shaded region. [2]

(c) Find the exact area of the shaded region. [3]

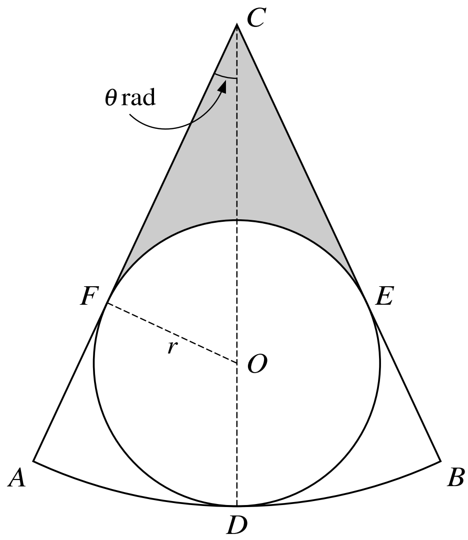
40. [9709/s20/13/q5]



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

41. [9709/w20/11/q10]



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

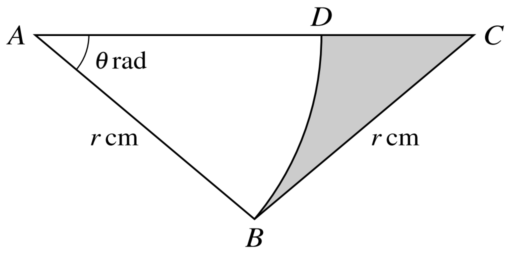
(a) Find CD in terms of r and $\sin\theta$. [3]

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

42. [9709/w20/12/q8]

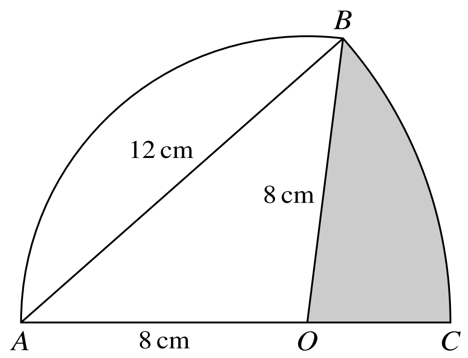


In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

43. [9709/w20/13/q9]



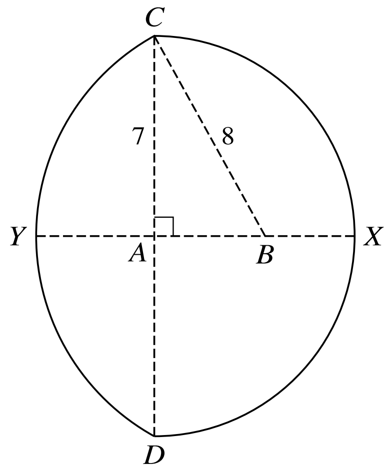
In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

(b) Find the area of the shaded region. [4]

(c) Find the perimeter of the shaded region. [3]

44. [9709/m19/12/q3]

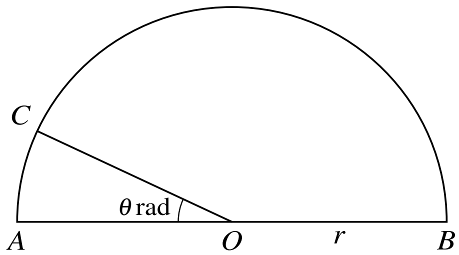


In the diagram, CXD is a semicircle of radius 7 cm with centre A and diameter CD. The straight line $YABX$ is perpendicular to CD, and the arc CYD is part of a circle with centre B and radius 8 cm. Find the total area of the region enclosed by the two arcs. [6]

45. [9709/s19/11/q3]

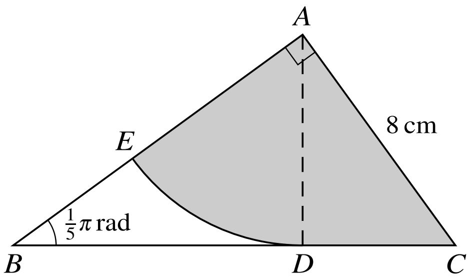
A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

46. [9709/s19/12/q5]



The diagram shows a semicircle with diameter AB, centre O and radius r. The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC. Find the value of θ correct to 2 significant figures. [5]

47. [9709/s19/13/q3]

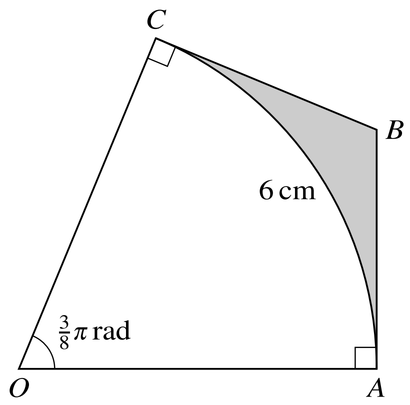


The diagram shows triangle ABC which is right-angled at A. Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D.

(i) Find the length of AD. [3]

(ii) Find the area of the shaded region. [3]

48. [9709/w19/11/q8]



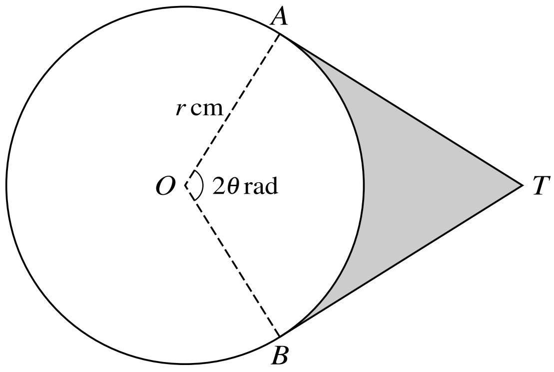
The diagram shows a sector OAC of a circle with centre O. Tangents AB and CB to the circle meet at B. The arc AC is of length 6 cm and angle $AOC = \frac{3}{8}\pi$ radians.

(i) Find the length of OA correct to 4 significant figures. [2]

(ii) Find the perimeter of the shaded region. [2]

(iii) Find the area of the shaded region. [4]

49. [9709/w19/12/q4]

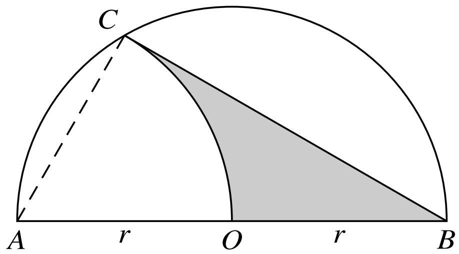


The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

50. [9709/w19/13/q4]

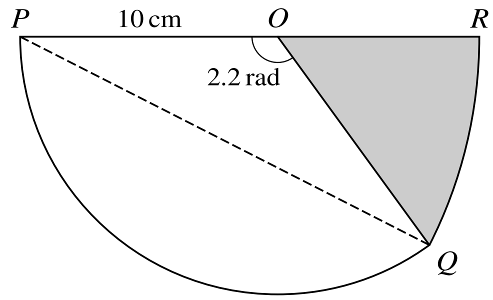


The diagram shows a semicircle ACB with centre O and radius r. Arc OC is part of a circle with centre A.

(i) Express angle CAO in radians in terms of π. [1]

(ii) Find the area of the shaded region in terms of r, π and $\sqrt{3}$, simplifying your answer. [4]

51. [9709/m18/12/q6]

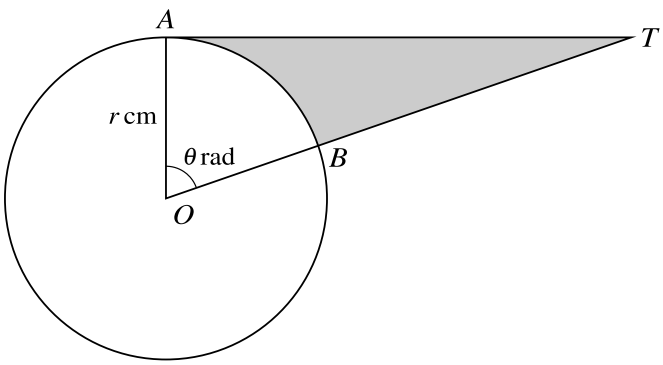


The diagram shows a sector POQ of a circle of radius 10 cm and centre O. Angle POQ is 2.2 radians. QR is an arc of a circle with centre P and POR is a straight line.

(i) Show that the length of PQ is 17.8 cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [4]

52. [9709/s18/11/q6]

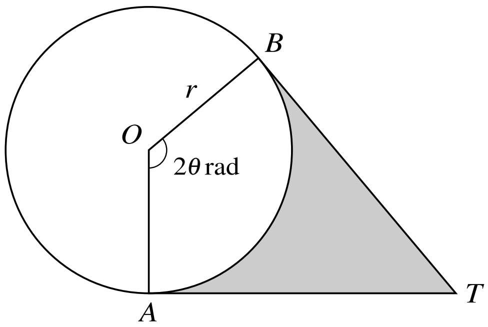


The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB = \theta$ radians and OBT is a straight line.

(i) Express the area of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. [4]

53. [9709/s18/12/q6]

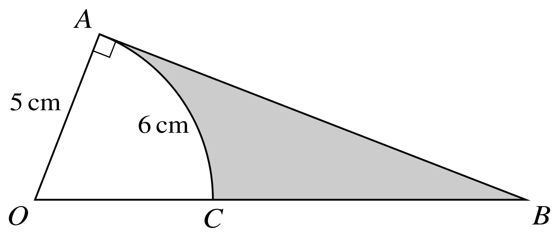


The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

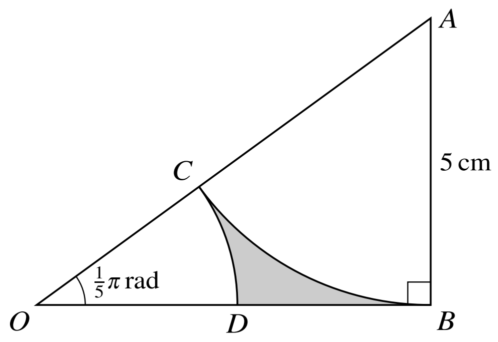
(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

54. [9709/s18/13/q5]



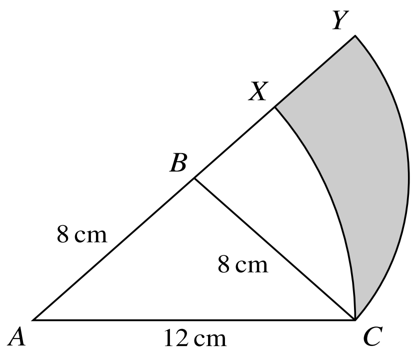
The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

55. [9709/w18/11/q9]



The diagram shows a triangle OAB in which angle ABO is a right angle, angle $AOB = \frac{1}{5}\pi$ radians and $AB = 5$ cm. The arc BC is part of a circle with centre A and meets OA at C. The arc CD is part of a circle with centre O and meets OB at D. Find the area of the shaded region. [8]

56. [9709/w18/12/q8]

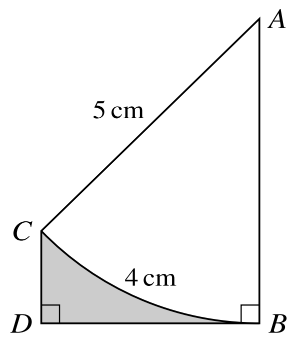


The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A, B, X and Y lie on a straight line.

(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

(ii) Find the perimeter of the shaded region. [4]

57. [9709/w18/13/q3]

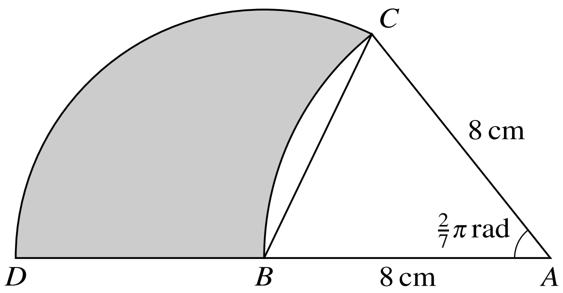


The diagram shows an arc BC of a circle with centre A and radius 5 cm. The length of the arc BC is 4 cm. The point D is such that the line BD is perpendicular to BA and DC is parallel to BA.

(i) Find angle BAC in radians. [1]

(ii) Find the area of the shaded region BDC. [5]

58. [9709/m17/12/q4]

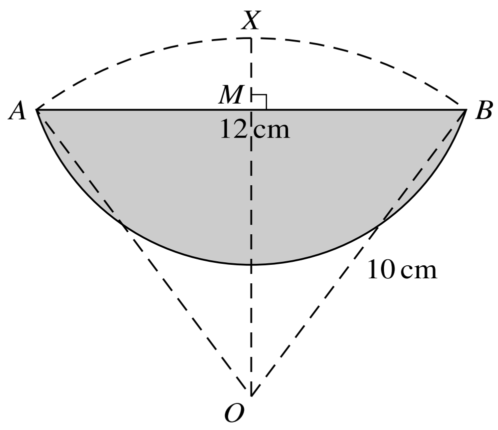


In the diagram, $AB = AC = 8$ cm and angle $CAB = \frac{2}{7}\pi$ radians. The circular arc BC has centre A, the circular arc CD has centre B and ABD is a straight line.

(i) Show that angle $CBD = \frac{9}{14}\pi$ radians. [1]

(ii) Find the perimeter of the shaded region. [5]

59. [9709/s17/11/q8]



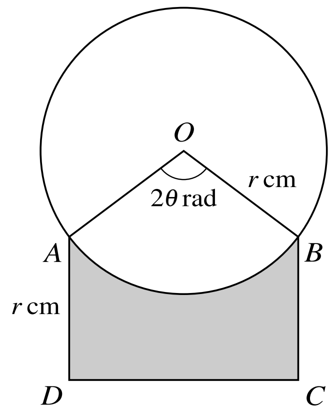
In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm. The length of the chord AB is 12 cm. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

(i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

60. [9709/s17/12/q4]

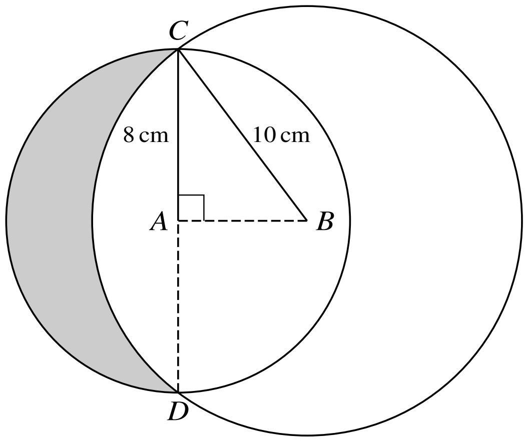


The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

61. [9709/s17/13/q7]

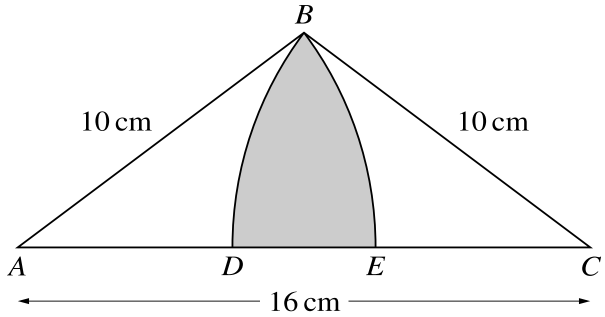


The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i) Find angle ABC in radians. [1]

(ii) Find the area of the shaded region. [6]

62. [9709/w17/11/q5]

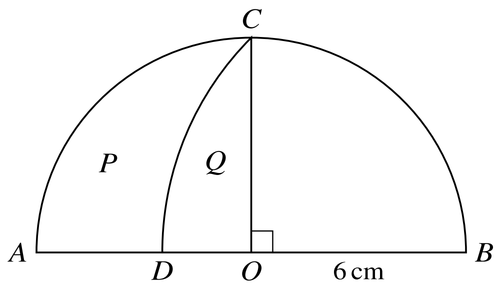


The diagram shows an isosceles triangle ABC in which $AC = 16$ cm and $AB = BC = 10$ cm. The circular arcs BE and BD have centres at A and C respectively, where D and E lie on AC.

(i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

(ii) Find the area of the shaded region. [5]

63. [9709/w17/12/q4]



The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB. The point D lies on AB, and DC is an arc of a circle with centre B.

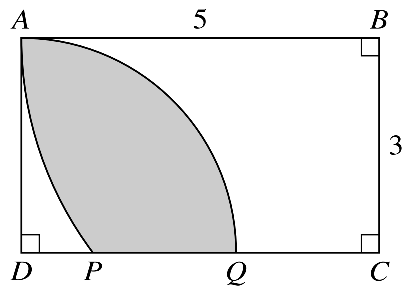
(i) Calculate the length of the arc DC. [3]

(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures. [4]

64. [9709/w17/13/q7]



The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B. Point Q lies on DC and AQ is an arc of a circle with centre D.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

(ii) Calculate the areas of the sectors BAP and DAQ. [3]

(iii) Calculate the area of the shaded region. [3]

65. [9709/m16/12/q9]

(a)

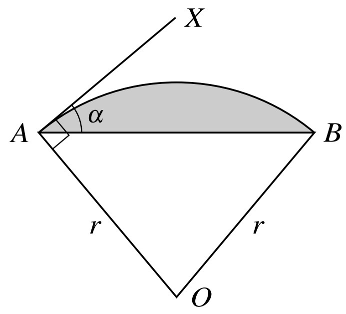


Fig. 1

In Fig. 1, OAB is a sector of a circle with centre O and radius r. AX is the tangent at A to the arc AB and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. [2]

(ii) Find the area of the shaded segment in terms of r and α. [2]

(b)

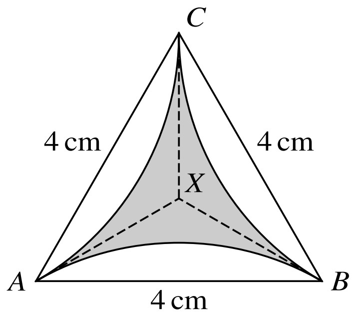
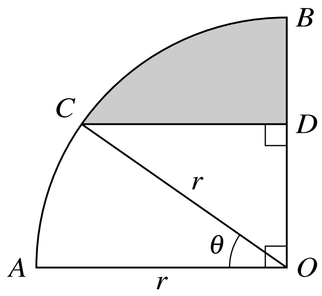


Fig. 2

In Fig. 2, ABC is an equilateral triangle of side 4 cm. The lines AX, BX and CX are tangents to the equal circular arcs AB, BC and CA. Use the results in part **(a)** to find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]

66. [9709/s16/11/q7]

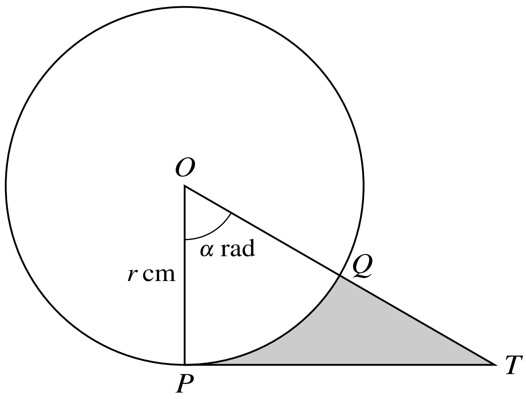


In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$, find the area of the shaded region. [3]

67. [9709/s16/12/q6]

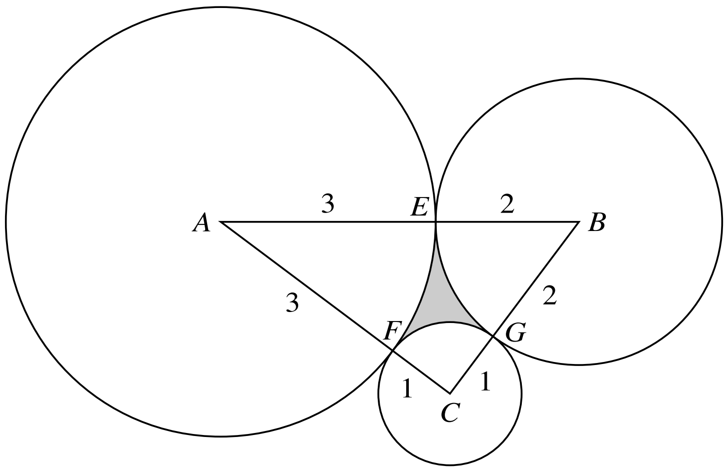


The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

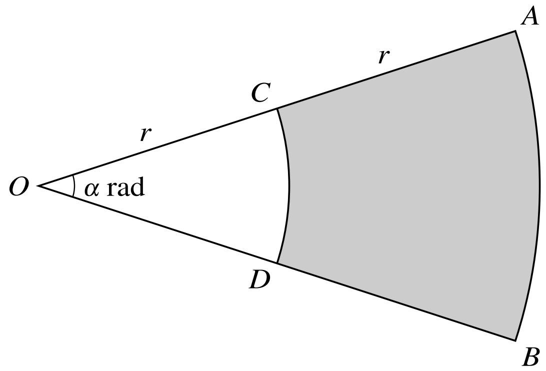
(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

68. [9709/s16/13/q6]



The diagram shows triangle ABC where $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG. [7]

69. [9709/w16/11/q3]

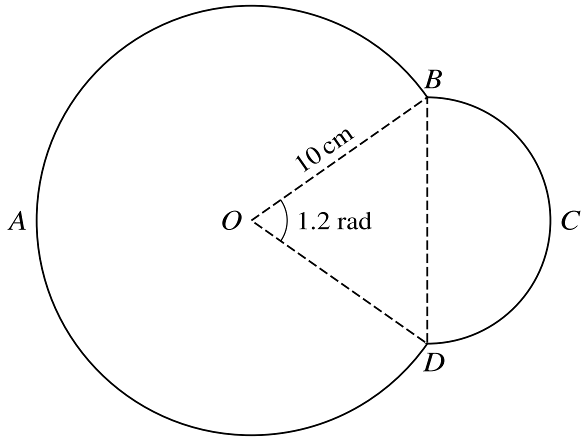


In the diagram OCA and ODB are radii of a circle with centre O and radius $2r$ cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

(i) Find the value of α. [2]

(ii) It is given that the area of the shaded region is $30\,\text{cm}^2$. Find the value of r. [3]

70. [9709/w16/12/q6]



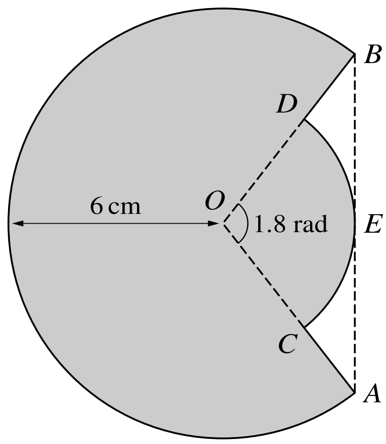
The diagram shows a metal plate *ABCD* made from two parts. The part *BCD* is a semicircle. The part *DAB* is a segment of a circle with centre *O* and radius 10 cm. Angle *BOD* is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

71. [9709/w16/13/q5]



The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle $COD = 1.8$ radians.

(i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]

(ii) Find the area of the shaded region. [4]

72. [9709/s15/11/q5]

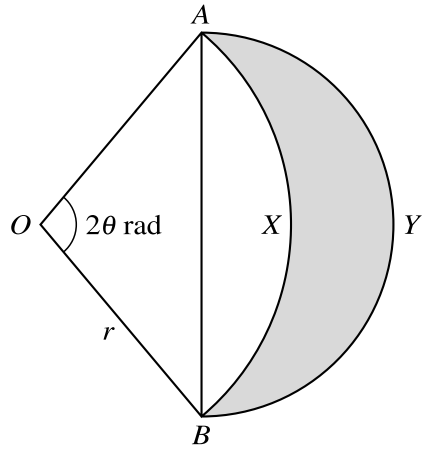
A piece of wire of length 24 cm is bent to form the perimeter of a sector of a circle of radius r cm.

(i) Show that the area of the sector, A cm^2, is given by $A = 12r - r^2$. [3]

(ii) Express A in the form $a - (r - b)^2$, where a and b are constants. [2]

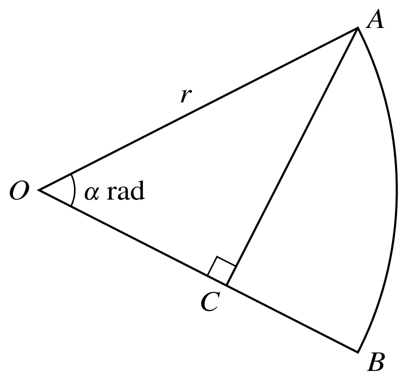
(iii) Given that r can vary, state the greatest value of A and find the corresponding angle of the sector. [2]

73. [9709/s15/12/q2]



In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]

74. [9709/s15/13/q11]



In the diagram, OAB is a sector of a circle with centre O and radius r. The point C on OB is such that angle ACO is a right angle. Angle AOB is α radians and is such that AC divides the sector into two regions of equal area.

(i) Show that $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$. [4]

It is given that the solution of the equation in part **(i)** is $\alpha = 0.9477$, correct to 4 decimal places.

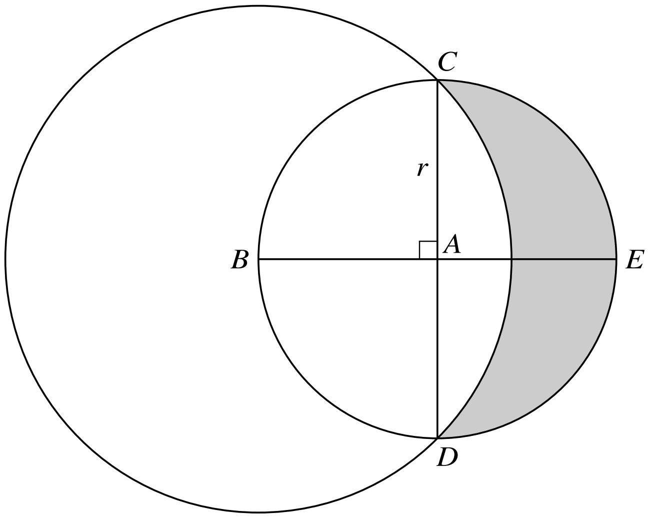
(ii) Find the ratio

perimeter of region OAC : perimeter of region ACB,

giving your answer in the form $k : 1$, where k is given correct to 1 decimal place. [5]

(iii) Find angle AOB in degrees. [1]

75. [9709/w15/11/q7]

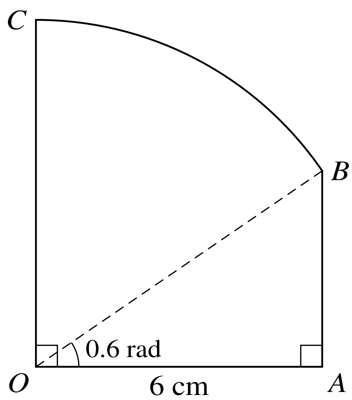


The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(ii) Find the area of the shaded region in terms of r. [6]

76. [9709/w15/12/q5]



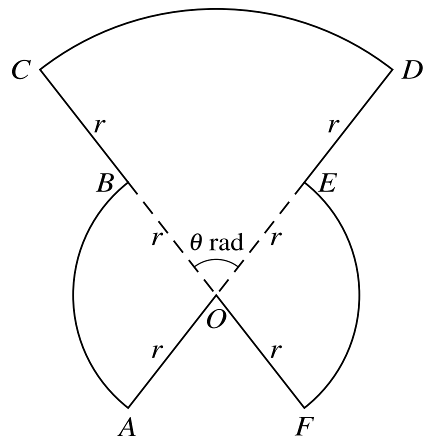
The diagram shows a metal plate $OABC$, consisting of a right-angled triangle OAB and a sector OBC of a circle with centre O. Angle $AOB = 0.6$ radians, $OA = 6$ cm and OA is perpendicular to OC.

(i) Show that the length of OB is 7.270 cm, correct to 3 decimal places. [1]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

77. [9709/w15/13/q4]



The diagram shows a metal plate $OABCDEF$ consisting of 3 sectors, each with centre O. The radius of sector COD is $2r$ and angle COD is θ radians. The radius of each of the sectors BOA and FOE is r, and $AOED$ and $CBOF$ are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$. [3]

(ii) Show that the perimeter of the metal plate is independent of θ. [4]

Chapter 4

Trigonometry

1. [9709/m25/12/q8]

A geometric progression is such that its second term is -120 and its sum to infinity is 160.

(a) Find the common ratio. [4]

(b) The first nine terms of the progression are now removed.

Find the sum to infinity of the remaining terms of the progression. [3]

2. [9709/s25/11/q1]

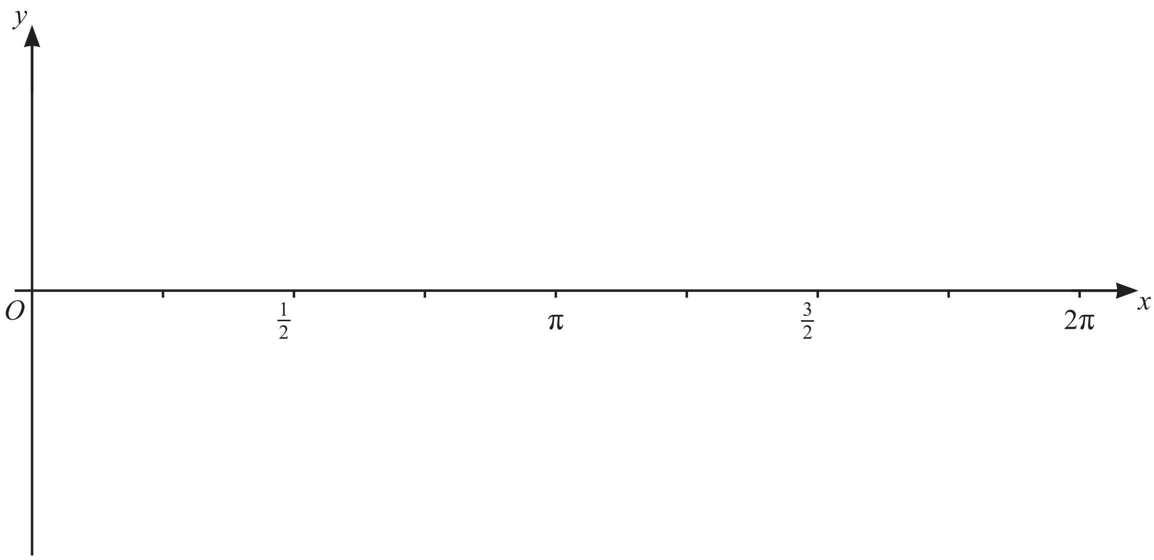
Solve the equation $6\sin\theta = 1 + \dfrac{2}{\sin\theta}$ for $-180° < \theta < 180°$. [4]

3. [9709/s25/12/q5]

The equation of a curve is $y = 4\cos 2x + 3$ for $0 \leqslant x \leqslant 2\pi$.

(a) State the greatest and least possible values of y. [2]

(b) Sketch the curve. [2]



(c) Hence determine the number of solutions of the equation $4\cos 2x + 3 = 2x - 1$ for $0 \leqslant x \leqslant 2\pi$. [1]

4. [9709/s25/12/q7]

(a) Prove the identity $\dfrac{\tan\theta+7}{\tan^2\theta-3} \equiv \dfrac{\sin\theta\cos\theta+7\cos^2\theta}{1-4\cos^2\theta}$. [3]

(b) Hence solve the equation $\dfrac{\sin\theta\cos\theta+7\cos^2\theta}{1-4\cos^2\theta} = \dfrac{5}{\tan\theta}$ for $0° \leqslant \theta \leqslant 180°$. [4]

5. [9709/s25/13/q2]

The first two terms of a geometric progression are

$$4\sin^2\theta, \quad 8\sin^3\theta,$$

where θ is an angle such that $0 < \theta < \frac{1}{6}\pi$.

Given that the sum to infinity of the progression is $\frac{1}{2}$, find the value of θ. Give your answer in the form $\sin^{-1} k$, where k is a rational number. [4]

6. [9709/s25/13/q5]

Solve the equation

$$4\sin\theta\tan\theta = 1 + 5\cos\theta$$

for $-180° < \theta < 180°$. [6]

7. [9709/s25/15/q3]

(a) Use completing the square to find the exact solutions of the equation $4x^2 - 4x - 1 = 0$. [2]

(b) Hence solve the equation $4\tan\theta = 4 + \dfrac{1}{\tan\theta}$ for $0° < \theta < 180°$. [3]

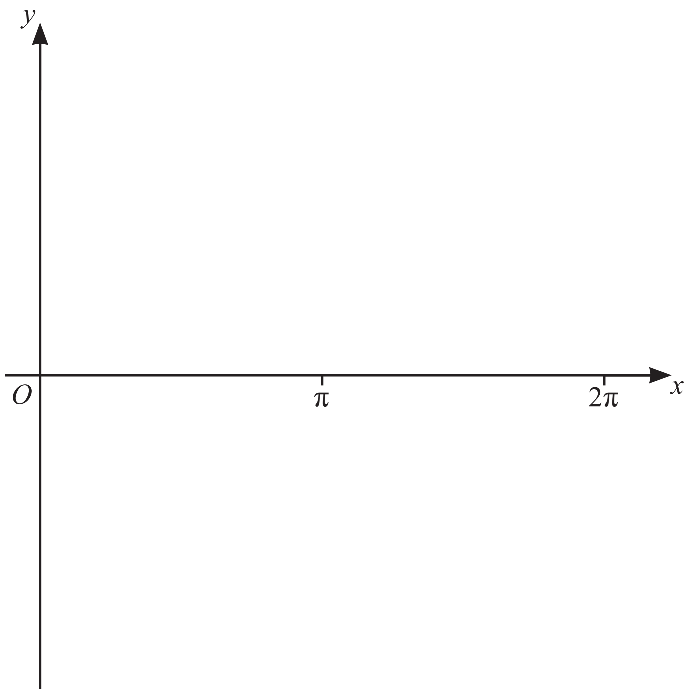
8. [9709/s25/15/q9]

Functions f and g are defined as follows.

$$f(x) = \cos x \text{ for } 0 \leqslant x \leqslant \pi$$
$$g(x) = 3\cos(x - \pi) + 2 \text{ for } \pi \leqslant x \leqslant 2\pi$$

(a) Describe fully the transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

(b) On the given axes, sketch the graphs of $y = f(x)$ and $y = g(x)$. [4]



(c) Find $g^{-1}f\left(\frac{1}{3}\pi\right)$. [4]

(d) Explain why the composite function fg cannot be formed. [1]

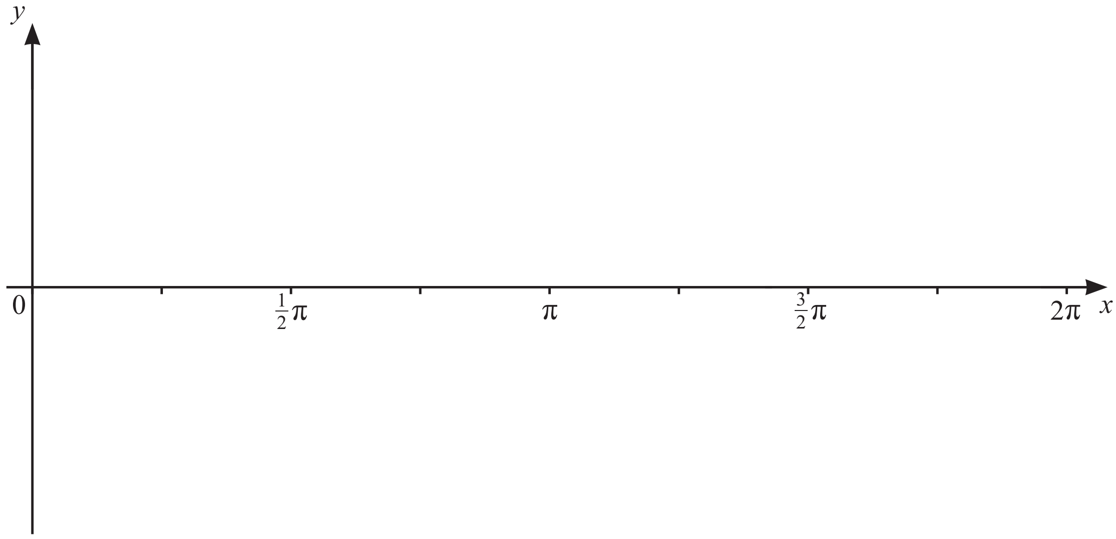
9. [9709/w25/11/q5]

(a) Show that $\tan^4\theta - 1 \equiv \dfrac{1 - 2\cos^2\theta}{\cos^4\theta}$. [3]

(b) Hence solve the equation $\cos^2\theta(\tan^4\theta - 1) = 7$ for $0° < \theta < 180°$. [4]

10. [9709/w25/12/q6]

(a) Sketch the graph of $y = 3\sin x + 2$ for $0 \leqslant x \leqslant 2\pi$. [2]



(b) Determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ of each of the following equations.

(i) $3\sin x + 2 = x$ [1]

(ii) $3\sin x + 2 = 5 - x$ [1]

(c) Solve the equation $3\sin x + 2 = 5\cos^2 x - 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

11. [9709/w25/13/q2]

(a) Solve the equation $\tan^{-1}(5x-3) = -\frac{1}{4}\pi$. [2]

....

....

....

....

....

....

....

....

(b) Solve the equation $5\cos^2\theta = 4\sin\theta + 4$ for $\frac{1}{2}\pi \leqslant \theta \leqslant 2\pi$. [4]

12. [9709/w25/15/q6]

(a) Show that the equation

$$6\sin\theta + \frac{1}{\tan\theta} = \frac{4}{\sin\theta}$$

can be written in the form

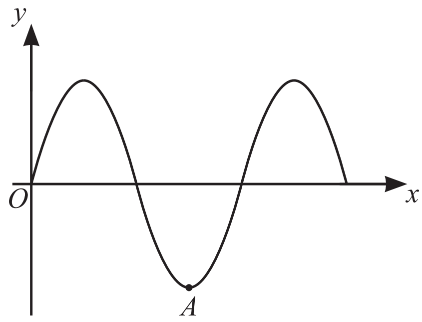
$$6\cos^2\theta - \cos\theta - 2 = 0.$$ [3]

(b) Hence, solve the equation

$$6\sin\theta + \frac{1}{\tan\theta} = \frac{4}{\sin\theta}$$

for $0° \leqslant \theta \leqslant 360°$. [4]

13. [9709/m24/12/q2]



The diagram shows part of the curve with equation $y = k\sin\frac{1}{2}x$, where k is a positive constant and x is measured in radians. The curve has a minimum point A.

(a) State the coordinates of A. [1]

(b) A sequence of transformations is applied to the curve in the following order.

Translation of 2 units in the negative y-direction

Reflection in the x-axis

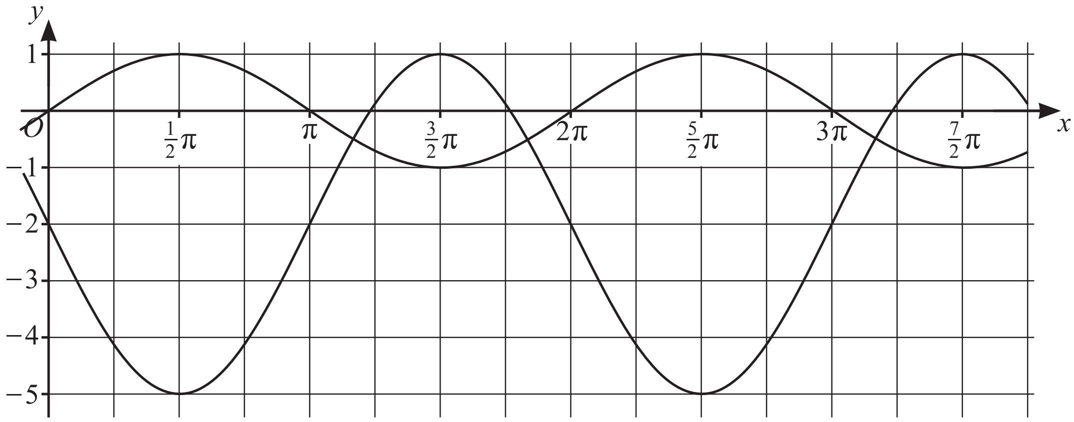
Find the equation of the new curve and determine the coordinates of the point on the new curve corresponding to A. [3]

14. [9709/m24/12/q4]

(a) Prove that $\frac{(\sin\theta+\cos\theta)^2-1}{\cos^2\theta} \equiv 2\tan\theta$. [3]

(b) Hence solve the equation $\frac{(\sin\theta+\cos\theta)^2-1}{\cos^2\theta} = 5\tan^3\theta$ for $-90^\circ < \theta < 90^\circ$. [3]

15. [9709/s24/11/q2]



The diagram shows two curves. One curve has equation $y = \sin x$ and the other curve has equation $y = \mathrm{f}(x)$.

(a) In order to transform the curve $y = \sin x$ to the curve $y = \mathrm{f}(x)$, the curve $y = \sin x$ is first reflected in the x-axis.

Describe fully a sequence of two further transformations which are required. [4]

(b) Find $\mathrm{f}(x)$ in terms of $\sin x$. [2]

16. [9709/s24/11/q5]

(a) Prove the identity $\dfrac{\sin^2 x - \cos x - 1}{1 + \cos x} \equiv -\cos x$. [3]

(b) Hence solve the equation $\dfrac{\sin^2 x - \cos x - 1}{2 + 2\cos x} = \dfrac{1}{4}$ for $0° \leqslant x \leqslant 360°$. [3]

17. [9709/s24/12/q3]

(a) Show that the equation $\frac{7\tan\theta}{\cos\theta}+12=0$ can be expressed as

$$12\sin^2\theta-7\sin\theta-12=0.$$ [3]

(b) Hence solve the equation $\frac{7\tan\theta}{\cos\theta}+12=0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

18. [9709/s24/12/q5]

The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

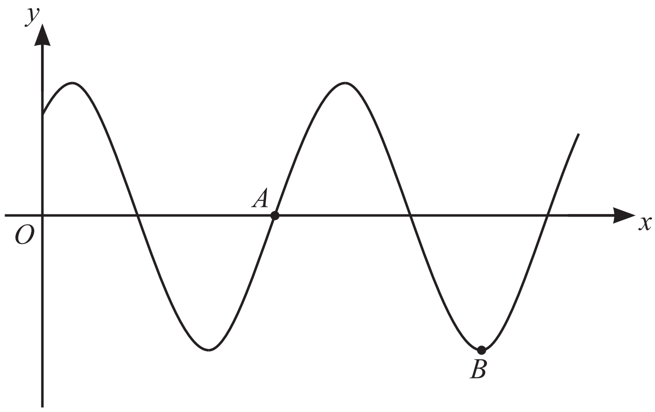
The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) **(i)** Find the sum to infinity of the progression in terms of θ. [2]

(ii) Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

19. [9709/s24/13/q2]

(a)



The diagram shows the curve $y = k\cos(x - \frac{1}{6}\pi)$ where k is a positive constant and x is measured in radians. The curve crosses the x-axis at point A and B is a minimum point.

Find the coordinates of A and B. [3]

(b) Find the exact value of t that satisfies the equation

$$3\sin^{-1}(3t) + 2\cos^{-1}\left(\tfrac{1}{2}\sqrt{2}\right) = \pi. \quad [2]$$

20. [9709/s24/13/q4]

(a) Show that the equation $\cos\theta(7\tan\theta - 5\cos\theta) = 1$ can be written in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are integers to be found. [3]

(b) Hence solve the equation $\cos 2x(7\tan 2x - 5\cos 2x) = 1$ for $0° < x < 180°$. [3]

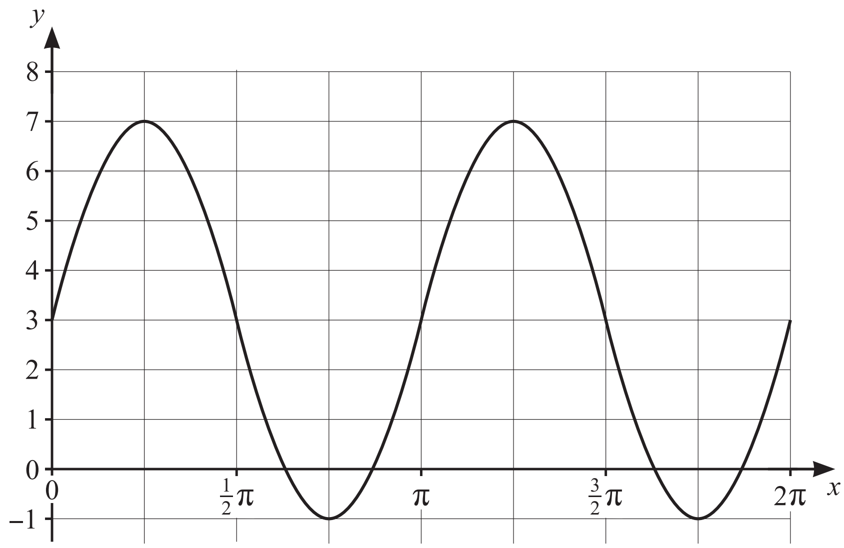
21. [9709/w24/11/q8]

(a) It is given that β is an angle between 90° and 180° such that $\sin\beta = a$.

Express $\tan^2\beta - 3\sin\beta\cos\beta$ in terms of a. [3]

(b) Solve the equation $\sin^2\theta + 2\cos^2\theta = 4\sin\theta + 3$ for $0° < \theta < 360°$. [5]

22. [9709/w24/12/q1]



The diagram shows the curve with equation $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$, where a, b and c are positive constants.

(a) State the values of a, b and c. [3]

(b) For these values of a, b and c, determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations:

(i) $a\sin(bx) + c = 7 - x$ [1]

(ii) $a\sin(bx) + c = 2\pi(x - 1)$. [1]

23. [9709/w24/13/q2]

Find the exact solution of the equation

$$\cos\frac{1}{6}\pi + \tan 2x + \frac{\sqrt{3}}{2} = 0 \text{ for } -\frac{1}{4}\pi < x < \frac{1}{4}\pi.$$ [2]

24. [9709/w24/13/q4]

Solve the equation $4\sin^4\theta + 12\sin^2\theta - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

25. [9709/m23/12/q7]

(a) By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

26. [9709/s23/11/q1]

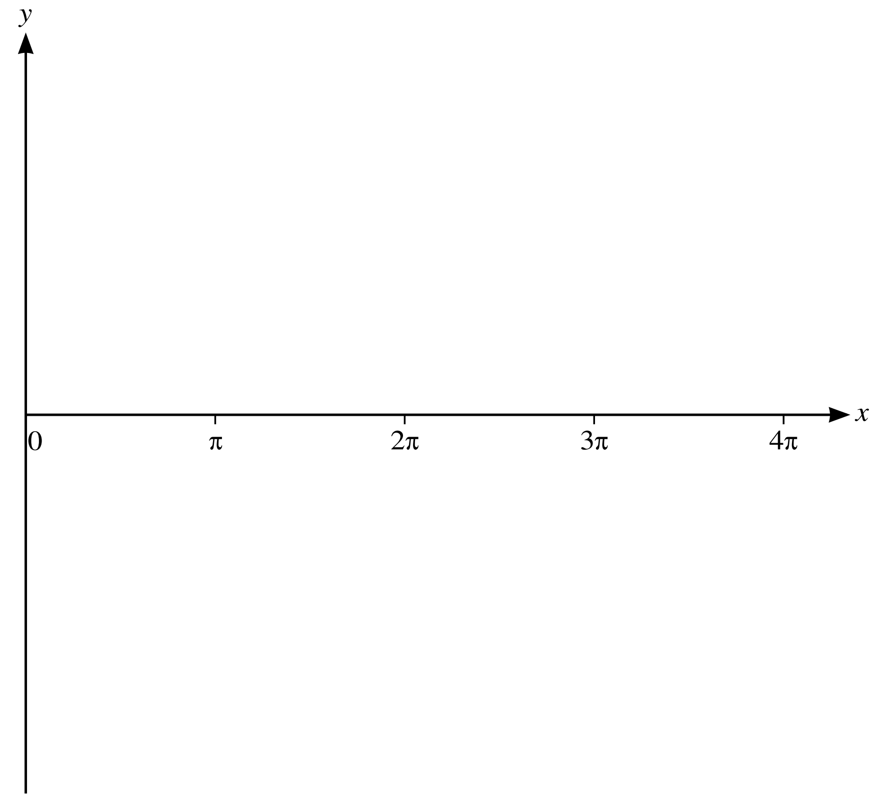
Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

27. [9709/s23/11/q7]

A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

(a) State greatest and least values of y. [2]

(b) Sketch the curve. [2]



(c) State the number of solutions of the equation

$$2 + 3\sin\frac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. [1]

28. [9709/s23/12/q7]

(a) **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

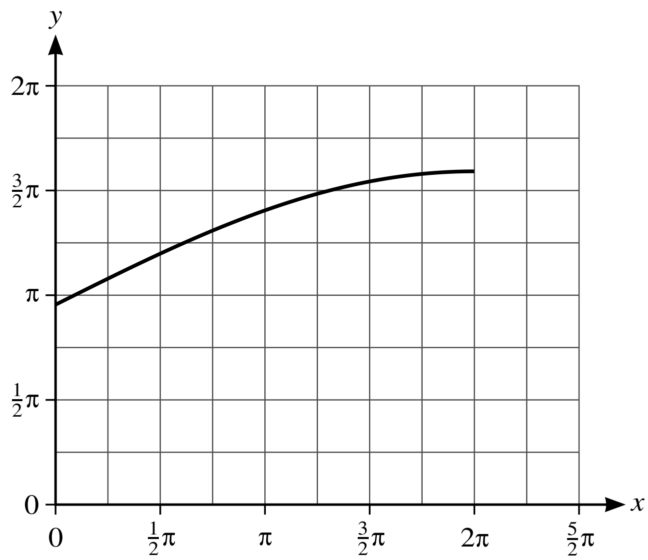
(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta + \sin\theta} + \dfrac{1 - \cos\theta}{\cos\theta - \sin\theta} \equiv \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$. [3]

(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta + \sin\theta} + \frac{1 - \cos\theta}{\cos\theta - \sin\theta} = 2(\cos\theta + \sin\theta - 1)$$

for $0 \leqslant \theta \leqslant \pi$. [3]

29. [9709/s23/12/q8]



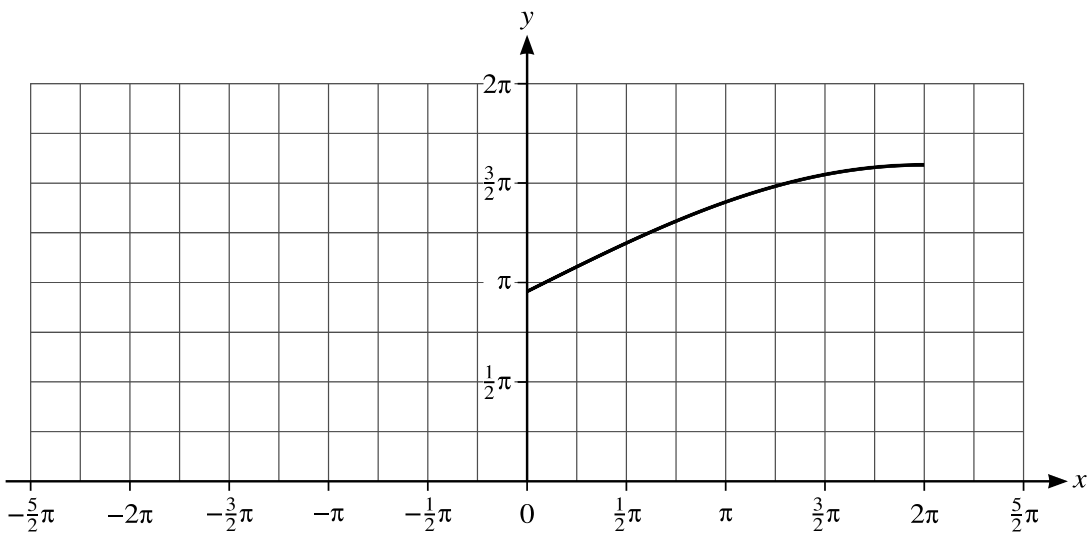
The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(c)



The diagram above shows part of the graph of the function $\mathrm{g}(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $\mathrm{g}(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = \mathrm{f}(x)$, making clear the order in which the transformations are applied. [6]

30. [9709/s23/13/q4]

(a) Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

31. [9709/w23/11/q5]

(a) Show that the equation

$$4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$$

may be expressed in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers to be found. [3]

(b) Hence solve the equation $4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$ for $0° \leqslant x \leqslant 360°$. [3]

32. [9709/w23/12/q2]

Find the exact solution of the equation

$$\tfrac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}\left(\tfrac{1}{2}\sqrt{3}\right).$$ [2]

33. [9709/w23/12/q5]

The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c} - 1}$, where b and c are integers to be found. [3]

34. [9709/w23/12/q7]

(a) Verify the identity $(2x-1)(4x^2+2x-1) \equiv 8x^3-4x+1$. [1]

(b) Prove the identity $\dfrac{\tan^2\theta+1}{\tan^2\theta-1} \equiv \dfrac{1}{1-2\cos^2\theta}$. [3]

(c) Using the results of **(a)** and **(b)**, solve the equation

$$\frac{\tan^2\theta+1}{\tan^2\theta-1} = 4\cos\theta,$$

for $0° \leqslant \theta \leqslant 180°$. [5]

35. [9709/w23/13/q3]

(a) Show that the equation

$$5\cos\theta - \sin\theta\tan\theta + 1 = 0$$

may be expressed in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $5\cos\theta - \sin\theta\tan\theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

36. [9709/m22/12/q7]

(a) Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

(b) Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

37. [9709/s22/11/q4]

(a) Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

(b) Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1 + \sin\theta} = \tan^2\theta(1 - \sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

38. [9709/s22/11/q8.b]

(a) The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

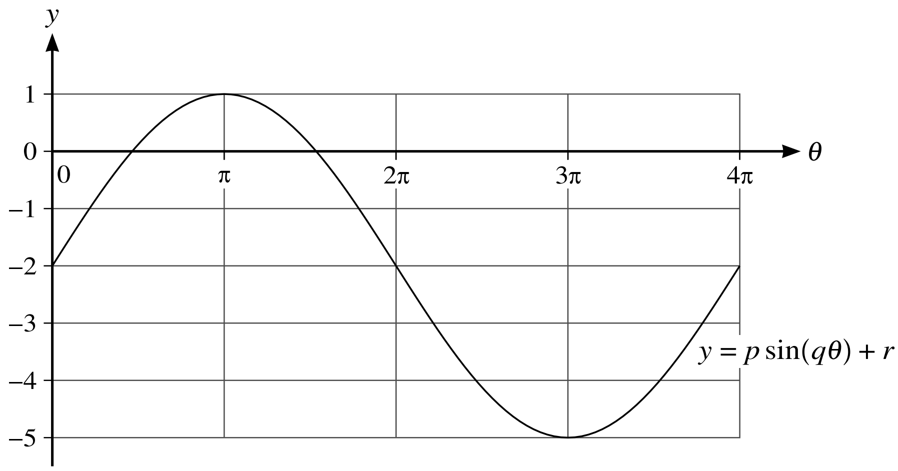
39. [9709/s22/12/q11]

The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

40. [9709/s22/13/q2]



The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.

(a) State the value of p. [1]

(b) State the value of q. [1]

(c) State the value of r. [1]

41. [9709/w22/11/q6]

(a) Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

42. [9709/w22/12/q7]

(a) Prove the identity $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$. [3]

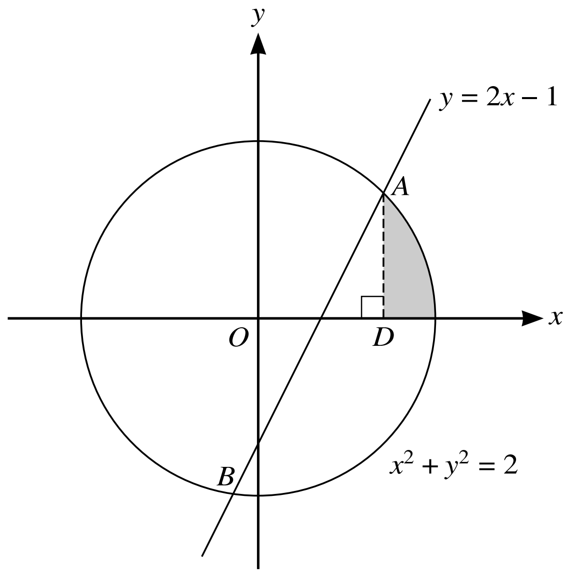
(b) Hence find the exact solutions of the equation $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}=2$ for $0\leqslant\theta\leqslant\pi$. [4]

43. [9709/w22/13/q6]

It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha}$. [5]

44. [9709/w22/13/q10.c]



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

(c) Find an exact expression for the perimeter of the shaded region. [2]

45. [9709/m21/12/q3]

Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

46. [9709/s21/11/q7]

(a) Prove the identity $\frac{1-2\sin^2\theta}{1-\sin^2\theta} \equiv 1-\tan^2\theta$. [2]

(b) Hence solve the equation $\frac{1-2\sin^2\theta}{1-\sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

47. [9709/s21/12/q10]

(a) Prove the identity $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

48. [9709/s21/13/q4]

(a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

(b) Hence express $\cos x$ in terms of k. [2]

(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

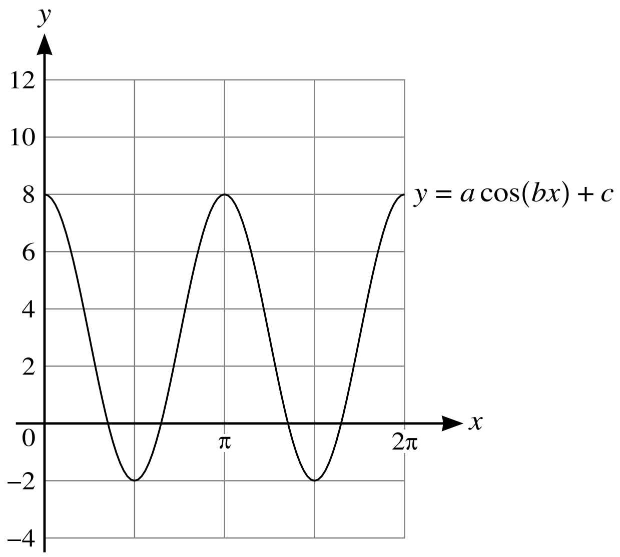
49. [9709/w21/11/q3]

Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

50. [9709/w21/11/q5]



The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ [1]

(ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

51. [9709/w21/12/q1]

Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

52. [9709/w21/13/q7]

(a) Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0. \qquad [4]$$

(b) Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

53. [9709/m20/12/q5]

Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

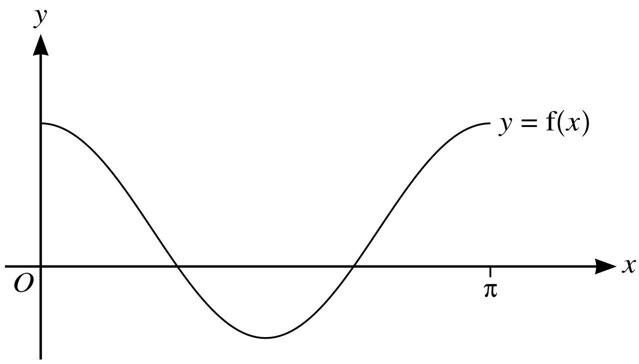
54. [9709/m20/12/q11]

(a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

55. [9709/s20/11/q4]



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = g(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

56. [9709/s20/11/q7]

(a) Prove the identity $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}\equiv\dfrac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{3}{\sin\theta}$, for $0\leqslant\theta\leqslant 2\pi$. [3]

57. [9709/s20/12/q2]

(a) Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

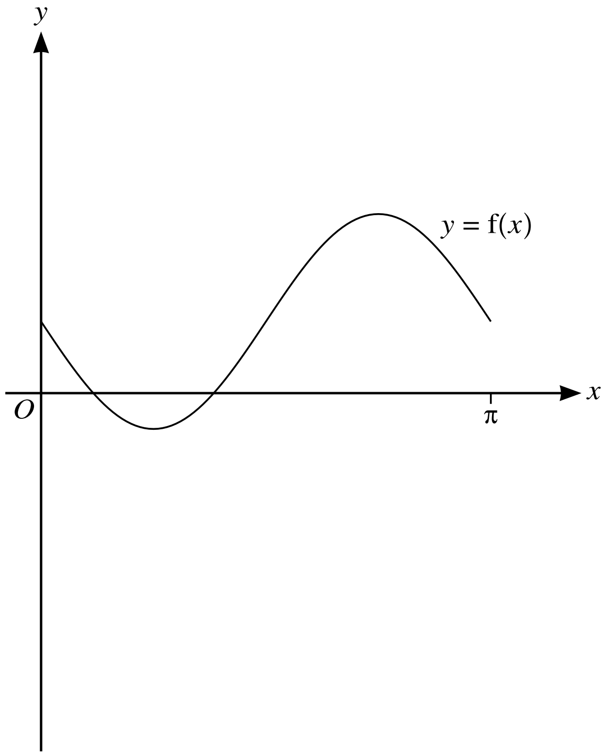
58. [9709/s20/12/q9]

Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

59. [9709/s20/13/q7]

(a) Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

60. [9709/s20/13/q8]

The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

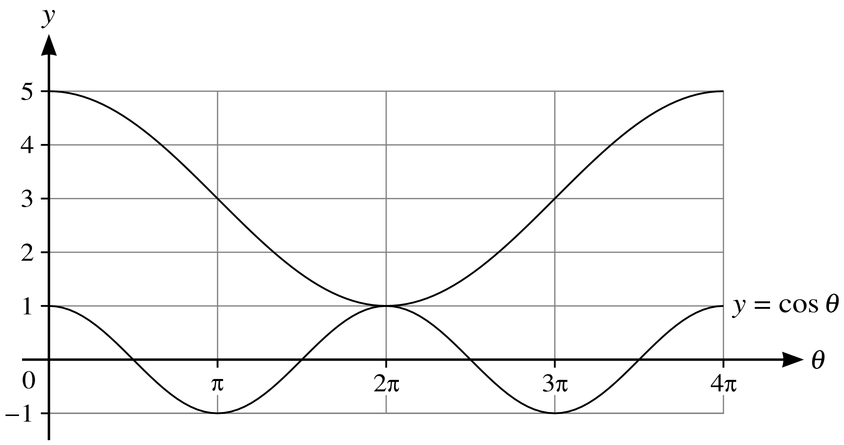
(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

61. [9709/w20/11/q4]



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

62. [9709/w20/11/q7]

(a) Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

63. [9709/w20/12/q6]

(a) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

64. [9709/w20/12/q11]

A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

(ii) $k = 1$ [1]

(iii) $k = 3$ [1]

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\mathrm{f}(x) = 3\cos 2x + 2,$$
$$\mathrm{g}(x) = \mathrm{f}(2x) + 4,$$
$$\mathrm{h}(x) = 2\mathrm{f}\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

(e) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{h}(x)$. [2]

65. [9709/w20/13/q3]

Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

66. [9709/w20/13/q7]

The first and second terms of an arithmetic progression are $\dfrac{1}{\cos^2\theta}$ and $-\dfrac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

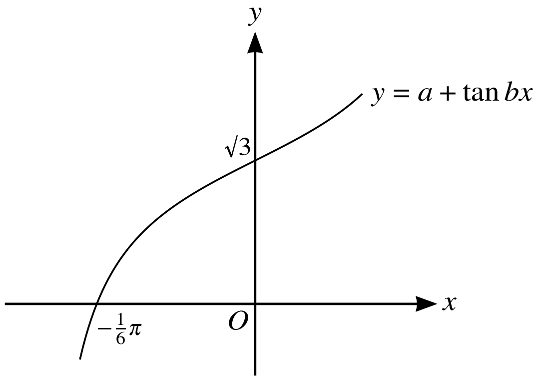
(a) Show that the common difference is $-\dfrac{1}{\cos^4\theta}$. [4]

(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

67. [9709/m19/12/q7]

(a) Solve the equation $3\sin^2 2\theta + 8\cos 2\theta = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

(b)



The diagram shows part of the graph of $y = a + \tan bx$, where x is measured in radians and a and b are constants. The curve intersects the x-axis at $\left(-\frac{1}{6}\pi,\ 0\right)$ and the y-axis at $(0,\ \sqrt{3})$. Find the values of a and b. [3]

68. [9709/s19/11/q6]

(i) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$. [3]

69. [9709/s19/11/q9]

The function f is defined by $f(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

(i) State the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

The function g is defined by $g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

(iv) For this value of p, find an expression for $g^{-1}(x)$. [2]

70. [9709/s19/12/q4]

Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x. [3]

(ii) In the case where $\tan x = 2$, express a in terms of b. [2]

71. [9709/s19/12/q6]

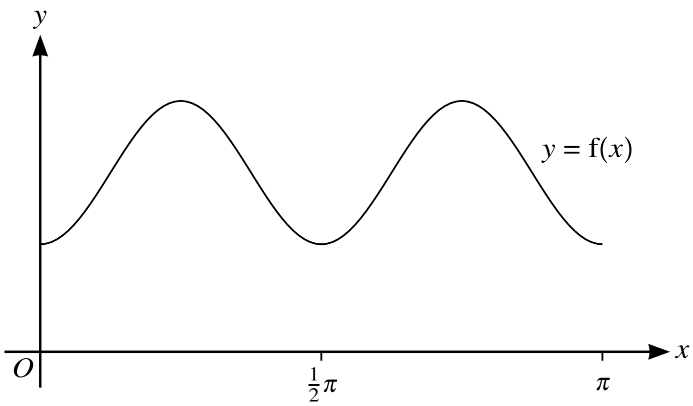
The equation of a curve is $y = 3\cos 2x$ and the equation of a line is $2y + \frac{3x}{\pi} = 5$.

(i) State the smallest and largest values of y for both the curve and the line for $0 \leqslant x \leqslant 2\pi$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 3\cos 2x$ and $2y + \frac{3x}{\pi} = 5$ for $0 \leqslant x \leqslant 2\pi$. [3]

(iii) State the number of solutions of the equation $6\cos 2x = 5 - \frac{3x}{\pi}$ for $0 \leqslant x \leqslant 2\pi$. [1]

72. [9709/s19/13/q9]



The function $f : x \mapsto p\sin^2 2x + q$ is defined for $0 \leqslant x \leqslant \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

(i) In terms of p and q, state the range of f. [2]

(ii) State the number of solutions of the following equations.

(a) $f(x) = p + q$ [1]

(b) $f(x) = q$ [1]

(c) $f(x) = \frac{1}{2}p + q$ [1]

(iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

73. [9709/w19/11/q5]

(i) Given that $4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$4\tan(2x - 20°) + 3\cos(2x - 20°) + \frac{1}{\cos(2x - 20°)} = 0$$

for $0° \leqslant x \leqslant 180°$. [4]

74. [9709/w19/12/q6]

(a) Given that $x > 0$, find the two smallest values of x, in radians, for which $3\tan(2x + 1) = 1$. Show all necessary working. [4]

(b) The function $\mathrm{f} : x \mapsto 3\cos^2 x - 2\sin^2 x$ is defined for $0 \leqslant x \leqslant \pi$.

(i) Express $\mathrm{f}(x)$ in the form $a\cos^2 x + b$, where a and b are constants. [1]

(ii) Find the range of f. [2]

75. [9709/w19/13/q7]

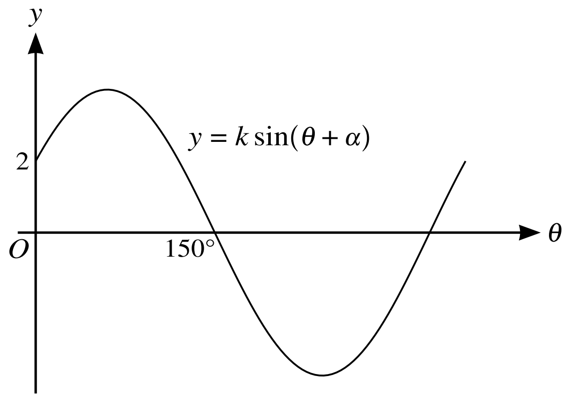
(i) Show that the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2\theta$. [2]

(ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

76. [9709/m18/12/q5]

(a) Express the equation $\frac{5 + 2\tan x}{3 + 2\tan x} = 1 + \tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leqslant x \leqslant \pi$. [4]

(b)



The diagram shows part of the graph of $y = k\sin(\theta + \alpha)$, where k and α are constants and $0° < \alpha < 180°$. Find the value of α and the value of k. [2]

77. [9709/s18/11/q4]

(i) Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

78. [9709/s18/12/q4]

The function f is such that $f(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $f\left(\frac{1}{3}\pi\right) = 5$ and $f(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

(ii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [3]

79. [9709/s18/12/q10]

(i) Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

80. [9709/s18/13/q7]

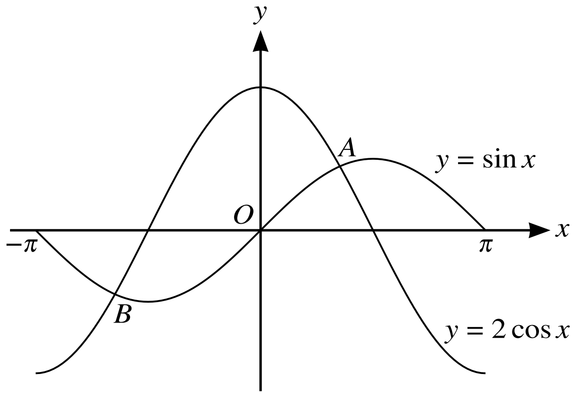
(a) **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

(ii) Find the y-coordinate of B. [2]

81. [9709/w18/11/q5]

(i) Show that the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

may be expressed as $9\cos^2\theta - 22\cos\theta + 4 = 0$. [3]

(ii) Hence solve the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

for $0° \leqslant \theta \leqslant 360°$. [3]

82. [9709/w18/12/q4]

Functions f and g are defined by

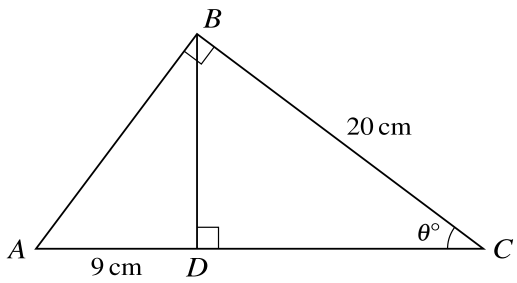
$$\mathrm{f} : x \mapsto 2 - 3\cos x \quad \text{for } 0 \leqslant x \leqslant 2\pi,$$

$$\mathrm{g} : x \mapsto \tfrac{1}{2}x \quad \text{for } 0 \leqslant x \leqslant 2\pi.$$

(i) Solve the equation $\mathrm{fg}(x) = 1$. [3]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [3]

83. [9709/w18/12/q6]



The diagram shows a triangle ABC in which $BC = 20$ cm and angle $ABC = 90°$. The perpendicular from B to AC meets AC at D and $AD = 9$ cm. Angle $BCA = \theta°$.

(i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC, show that $20\sin^2\theta = 9\cos\theta$. [4]

(ii) Hence, showing all necessary working, calculate θ. [3]

84. [9709/w18/13/q7]

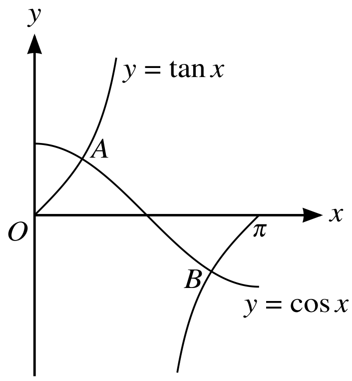
(i) Show that $\dfrac{\tan\theta + 1}{1 + \cos\theta} + \dfrac{\tan\theta - 1}{1 - \cos\theta} \equiv \dfrac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} = 0$$

for $0° < \theta < 90°$. [4]

85. [9709/m17/12/q5]



The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

(ii) Find by calculation the coordinates of B. [3]

86. [9709/s17/11/q3]

(i) Prove the identity $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} \equiv \dfrac{2}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

87. [9709/s17/11/q5]

The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

The line through P and Q meets the x-axis at $H(h, 0)$ and the y-axis at $K(0, k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

88. [9709/s17/12/q3]

(i) Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1 - \sin\theta}{1 + \sin\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

89. [9709/s17/12/q10]

The function f is defined by $\mathrm{f}(x) = 3\tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Solve the equation $\mathrm{f}(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$ and find the domain of f^{-1}. [5]

(iii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

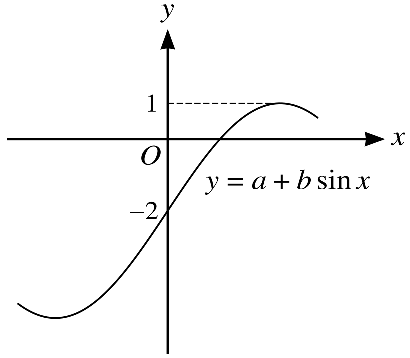
90. [9709/s17/13/q5]

(i) Show that the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ may be expressed as $\cos^2\theta = 2\sin^2\theta$. [3]

(ii) Hence solve the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ for $0° < \theta < 180°$. [3]

91. [9709/w17/11/q7]

(a)



The diagram shows part of the graph of $y = a + b\sin x$. Find the values of the constants a and b. [2]

(b) **(i)** Show that the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

may be expressed as $3\cos^2\theta - 2\cos\theta - 1 = 0$. [3]

(ii) Hence solve the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

for $-180° \leqslant \theta \leqslant 180°$. [4]

92. [9709/w17/12/q5]

(i) Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

93. [9709/w17/12/q6]

(a) The function f, defined by $f : x \mapsto a + b\sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{1}{6}\pi\right) = 4$ and $f\left(\frac{1}{2}\pi\right) = 3$.

(i) Find the values of the constants a and b. [3]

(ii) Evaluate $ff(0)$. [2]

(b) The function g is defined by $g : x \mapsto c + d\sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leqslant g(x) \leqslant 10$. Find the values of the constants c and d. [3]

94. [9709/m16/12/q4]

(a) Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

95. [9709/s16/11/q2]

Solve the equation $3\sin^2\theta = 4\cos\theta - 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

96. [9709/s16/12/q7]

(i) Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

97. [9709/s16/13/q8]

(i) Show that $3\sin x\tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x\tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x\tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

98. [9709/w16/11/q6]

(i) Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

99. [9709/w16/12/q2]

(i) Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]

100. [9709/w16/12/q10]

A function f is defined by $f : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Solve the equation $f(x) = 6$, giving answers in terms of π. [3]

The function g is defined by $g : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $g^{-1}(x)$. [3]

101. [9709/w16/13/q3]

Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

102. [9709/s15/11/q1]

Given that θ is an obtuse angle measured in radians and that $\sin\theta = k$, find, in terms of k, an expression for

(i) $\cos\theta$, [1]

(ii) $\tan\theta$, [2]

(iii) $\sin(\theta + \pi)$. [1]

103. [9709/s15/12/q5]

(i) Prove the identity $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \equiv \dfrac{\tan\theta - 1}{\tan\theta + 1}$. [1]

(ii) Hence solve the equation $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{6}$, for $0° \leqslant \theta \leqslant 180°$. [4]

104. [9709/s15/13/q4]

(i) Express the equation $3\sin\theta = \cos\theta$ in the form $\tan\theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

(ii) Solve the equation $3\sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

105. [9709/w15/11/q3]

Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

106. [9709/w15/11/q4]

(i) Show that the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$ [3]

(ii) Hence solve the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

107. [9709/w15/12/q4]

(i) Prove the identity $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 \equiv \dfrac{1 - \cos x}{1 + \cos x}$. [4]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$. [3]

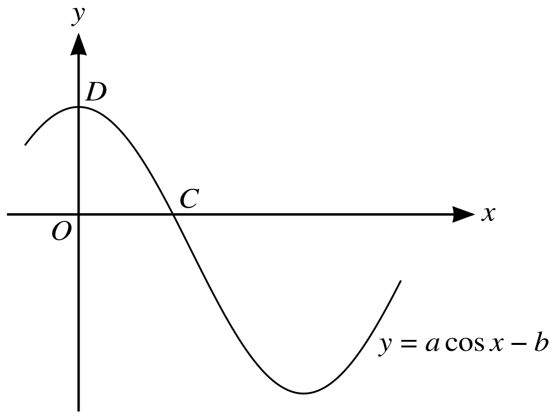
108. [9709/w15/13/q7]

(a) Show that the equation $\dfrac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ can be expressed as

$$3\cos^2\theta - 4\cos\theta - 4 = 0,$$

and hence solve the equation $\dfrac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ for $0° \leqslant \theta \leqslant 360°$. [6]

(b)



The diagram shows part of the graph of $y = a\cos x - b$, where a and b are constants. The graph crosses the x-axis at the point $C(\cos^{-1} c,\ 0)$ and the y-axis at the point $D(0,\ d)$. Find c and d in terms of a and b. [2]

Chapter 5

Binomial expansion

1. [9709/m25/12/q3]

(a) Find the complete expansion of $\left(2x - \frac{3}{x}\right)^4$. [4]

(b) Hence determine the coefficient of x^2 in the expansion of $\left(x^2 + 5\right)\left(2x - \frac{3}{x}\right)^4$. [2]

2. [9709/s25/11/q5]

(a) Find the first three terms, in ascending powers of x, in the expansion of each of the following expressions.

(i) $(2-px)^5$ [2]

....

(ii) $\left(1-\frac{1}{2}x\right)^4$ [2]

(b) Given that the coefficient of x^2 in the expansion of $(2-px)^5\left(1-\frac{1}{2}x\right)^4$ is 93, find the possible values of the constant p. [3]

3. [9709/s25/12/q3]

The coefficient of x^7 in the expansion of $\left(px^2 + \frac{4}{p}x\right)^5$ is 1280.

Find the value of the constant p. [4]

4. [9709/s25/13/q4]

(a) Find the first three terms in the expansion of $\left(2-\frac{3}{2}x\right)^5$ in ascending powers of x. [3]

(b) Use your answer to part **(a)**, with a suitable value of x, to find an approximation to 1.985^5. [3]

5. [9709/s25/15/q2]

In the expansion of $(3+ax)^5+(6-x)^4$, the coefficient of x^2 is six times the coefficient of x.

Find the possible values of the constant a. [5]

6. [9709/w25/11/q3]

In the expansion of

$$(px+3)^5 - \left(x^3 + \frac{p}{x}\right)^4,$$

the coefficient of x^4 is 216.

Find the value of the positive constant p. [5]

7. [9709/w25/12/q2]

Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^6$. [3]

8. [9709/w25/13/q1]

(a) Expand $\left(2-\frac{1}{2}x\right)^6$ in ascending powers of x up to and including the term in x^3. [3]

(b) Hence find the coefficient of x^3 in the expansion of $\left(3-x+2x^3\right)\left(2-\frac{1}{2}x\right)^6$. [2]

9. [9709/w25/15/q9]

In the expansion of $(p+qx)^4$, the coefficient of x is equal to the coefficient of x^2. The constants p and q are both positive.

(a) Find the ratio $p : q$. Give your answer in its simplest form. [3]

(b) It is given that the coefficient of x^3 is 486.

Find the values of p and q. [4]

10. [9709/m24/12/q6]

It is given that the coefficient of x^3 in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant a. [5]

11. [9709/s24/11/q3]

The coefficient of x^3 in the expansion of $(3+ax)^6$ is 160.

(a) Find the value of the constant a. [2]

(b) Hence find the coefficient of x^3 in the expansion of $(3+ax)^6(1-2x)$. [3]

12. [9709/s24/12/q1]

The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

13. [9709/s24/13/q1]

Find the coefficient of x^2 in the expansion of

$$(2-5x)(1+3x)^{10}.$$ [4]

14. [9709/w24/11/q1]

In the expansion of $\left(kx+\frac{2}{x}\right)^4$, where k is a positive constant, the term independent of x is equal to 150.

Find the value of k and hence determine the coefficient of x^2 in the expansion. [4]

15. [9709/w24/12/q4]

Find the term independent of x in the expansion of each of the following:

(a) $\left(x+\dfrac{3}{x^2}\right)^6$ [2]

(b) $(4x^3-5)\left(x+\dfrac{3}{x^2}\right)^6$. [4]

16. [9709/w24/13/q3]

(a) Find the coefficients of x^3 and x^4 in the expansion of $(3-ax)^5$, where a is a constant. Give your answers in terms of a. [3]

(b) Given that the coefficient of x^4 in the expansion of $(ax+7)(3-ax)^5$ is 240, find the positive value of a. [3]

17. [9709/m23/12/q6]

In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

18. [9709/s23/11/q2]

(a) Find the first three terms in the expansion, in ascending powers of x, of $(2 + 3x)^4$. [2]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

19. [9709/s23/12/q2]

The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

20. [9709/s23/13/q3]

(a) Give the complete expansion of $\left(x + \frac{2}{x}\right)^5$. [2]

(b) In the expansion of $(a + bx^2)\left(x + \frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

21. [9709/w23/11/q1]

(a) Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2. [2]

(b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

22. [9709/w23/12/q1]

The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a. [4]

23. [9709/w23/13/q4]

(a) Expand the following in ascending powers of x up to and including the term in x^2.

(i) $(1 + 2x)^5$. [1]

(ii) $(1 - ax)^6$, where a is a constant. [2]

In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of x^2 is -5.

(b) Find the possible values of a. [4]

24. [9709/m22/12/q3]

Find the term independent of x in each of the following expansions.

(a) $\left(3x + \dfrac{2}{x^2}\right)^6$ [3]

(b) $\left(3x + \dfrac{2}{x^2}\right)^6 (1 - x^3)$ [3]

25. [9709/s22/11/q3]

The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

(b) Given that $a + b = 216$, find the possible values of k. [3]

26. [9709/s22/12/q1]

The coefficient of x^4 in the expansion of $(3 + x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a. [4]

27. [9709/s22/13/q1]

The coefficient of x^3 in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p. [4]

28. [9709/w22/11/q4]

The coefficient of x^2 in the expansion of $\left(1+\frac{2}{p}x\right)^5+(1+px)^6$ is 70.

Find the possible values of the constant p. [6]

29. [9709/w22/13/q3]

(a) Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$. [2]

(b) Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$. [2]

30. [9709/m21/12/q1]

(a) Find the first three terms in the expansion, in ascending powers of x, of $(1+x)^5$. [1]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1-2x)^6$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1+x)^5(1-2x)^6$. [2]

31. [9709/s21/11/q3]

(a) Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x. [3]

(b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

32. [9709/s21/12/q4]

The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

33. [9709/s21/13/q7]

(a) Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

34. [9709/w21/11/q1]

(a) Expand $\left(1 - \frac{1}{2x}\right)^2$. [1]

(b) Find the first four terms in the expansion, in ascending powers of x, of $(1 + 2x)^6$. [2]

(c) Hence find the coefficient of x in the expansion of $\left(1 - \frac{1}{2x}\right)^2 (1 + 2x)^6$. [2]

35. [9709/w21/12/q8]

(a) It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

(b) It is given instead that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. [4]

36. [9709/w21/13/q2]

(a) Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$. [1]

(b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3, find the possible values of the constant a. [4]

37. [9709/m20/12/q6]

The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

38. [9709/s20/11/q2]

The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

39. [9709/s20/12/q1]

(a) Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

40. [9709/s20/13/q4]

(a) Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

41. [9709/w20/11/q5]

In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

42. [9709/w20/12/q1]

The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

43. [9709/w20/12/q11]

A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

(ii) $k = 1$ [1]

(iii) $k = 3$ [1]

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\begin{aligned} f(x) &= 3\cos 2x + 2, \\ g(x) &= f(2x) + 4, \\ h(x) &= 2f\left(x + \tfrac{1}{2}\pi\right). \end{aligned}$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

44. [9709/w20/13/q5]

In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

45. [9709/m19/12/q1]

The coefficient of x^3 in the expansion of $(1 - px)^5$ is -2160. Find the value of the constant p. [3]

46. [9709/s19/11/q1]

The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

47. [9709/s19/12/q1]

Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

48. [9709/s19/13/q2]

(i) In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

(ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

49. [9709/w19/11/q1]

Find the term independent of x in the expansion of $\left(2x + \frac{1}{4x^2}\right)^6$. [3]

50. [9709/w19/12/q1]

The coefficient of x^2 in the expansion of $(4 + ax)\left(1 + \frac{x}{2}\right)^6$ is 3. Find the value of the constant a. [4]

51. [9709/w19/13/q1]

(i) Expand $(1 + y)^6$ in ascending powers of y as far as the term in y^2. [1]

(ii) In the expansion of $\left(1 + (px - 2x^2)\right)^6$ the coefficient of x^2 is 48. Find the value of the positive constant p. [3]

52. [9709/m18/12/q2]

(i) Find the coefficients of x^2 and x^3 in the expansion of $(1 - 2x)^7$. [3]

(ii) Hence find the coefficient of x^3 in the expansion of $(2 + 5x)(1 - 2x)^7$. [2]

53. [9709/s18/13/q2]

Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

54. [9709/w18/12/q1]

Find the coefficient of $\frac{1}{x^2}$ in the expansion of $\left(3x + \frac{2}{3x^2}\right)^7$. [4]

55. [9709/w18/13/q1]

Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(x - \frac{2}{x}\right)^7$. [3]

56. [9709/m17/12/q2]

In the expansion of $\left(\frac{1}{ax} + 2ax^2\right)^5$, the coefficient of x is 5. Find the value of the constant a. [4]

57. [9709/s17/11/q1]

The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$. [4]

58. [9709/s17/12/q1]

(i) Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

(ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

59. [9709/s17/13/q1]

The coefficients of x and x^2 in the expansion of $(2 + ax)^7$ are equal. Find the value of the non-zero constant a. [3]

60. [9709/w17/12/q1]

Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

61. [9709/w17/13/q3]

(i) Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

(ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6.$$ [3]

62. [9709/m16/12/q1]

(i) Find the coefficients of x^4 and x^5 in the expansion of $(1-2x)^5$. [2]

(ii) It is given that, when $(1+px)(1-2x)^5$ is expanded, there is no term in x^5. Find the value of the constant p. [2]

63. [9709/s16/11/q1]

Find the term independent of x in the expansion of $\left(x - \frac{3}{2x}\right)^6$. [3]

64. [9709/s16/12/q4]

Find the term that is independent of x in the expansion of

(i) $\left(x - \frac{2}{x}\right)^6$, [2]

(ii) $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$. [4]

65. [9709/s16/13/q1]

Find the coefficient of x in the expansion of $\left(\frac{1}{x} + 3x^2\right)^5$. [3]

66. [9709/w16/11/q2]

Find the term independent of x in the expansion of $\left(2x + \frac{1}{2x^3}\right)^8$. [4]

67. [9709/w16/12/q4]

In the expansion of $(3 - 2x)\left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2. [6]

68. [9709/w16/13/q2]

The coefficient of x^3 in the expansion of $(1-3x)^6 + (1+ax)^5$ is 100. Find the value of the constant a. [4]

69. [9709/s15/11/q3]

(i) Find the first three terms, in ascending powers of x, in the expansion of

(a) $(1 - x)^6$, [2]

(b) $(1 + 2x)^6$. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $[(1 - x)(1 + 2x)]^6$. [3]

70. [9709/s15/12/q3]

(i) Find the coefficients of x^2 and x^3 in the expansion of $(2-x)^6$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(3x+1)(2-x)^6$. [2]

71. [9709/s15/13/q3]

(i) Write down the first 4 terms, in ascending powers of x, of the expansion of $(a-x)^5$. [2]

(ii) The coefficient of x^3 in the expansion of $(1-ax)(a-x)^5$ is -200. Find the possible values of the constant a. [4]

72. [9709/w15/11/q1]

In the expansion of $\left(1 - \frac{2x}{a}\right)(a + x)^5$, where a is a non-zero constant, show that the coefficient of x^2 is zero. [3]

73. [9709/w15/12/q2]

In the expansion of $(x + 2k)^7$, where k is a non-zero constant, the coefficients of x^4 and x^5 are equal. Find the value of k. [4]

74. [9709/w15/13/q2]

Find the coefficient of x in the expansion of $\left(\frac{x}{3} + \frac{9}{x^2}\right)^7$. [4]

Chapter 6

Series

1. [9709/m25/12/q5]

An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400. [6]

2. [9709/m25/12/q8]

A geometric progression is such that its second term is -120 and its sum to infinity is 160.

(a) Find the common ratio. [4]

(b) The first nine terms of the progression are now removed.

Find the sum to infinity of the remaining terms of the progression. [3]

3. [9709/s25/11/q3]

The third term of a geometric progression is 18 and the sum of the first three terms is 26. It is given that the common ratio is negative.

(a) Find the tenth term of the progression. Give your answer correct to 3 significant figures. [5]

(b) Find the exact value of the sum to infinity of the progression. [2]

4. [9709/s25/12/q10]

(a) The first, second and third terms of an arithmetic progression are $4k$, k^2 and $8k$ respectively, where k is a non-zero constant.

(i) Find the value of k. [2]

(ii) Find the sum of the first 20 terms of the progression. [3]

(b) The fourth and sixth terms of a geometric progression are 36 and 6 respectively. The common ratio of the progression is positive.

Find the sum to infinity of the progression. Give your answer in the form $\frac{a}{\sqrt{b}-c}$, where a, b and c are integers. [5]

5. [9709/s25/13/q2]

The first two terms of a geometric progression are

$$4\sin^2\theta,\quad 8\sin^3\theta,$$

where θ is an angle such that $0 < \theta < \frac{1}{6}\pi$.

Given that the sum to infinity of the progression is $\frac{1}{2}$, find the value of θ. Give your answer in the form $\sin^{-1} k$, where k is a rational number. [4]

6. [9709/s25/13/q6]

An arithmetic progression has first term a and common difference 2. The Nth term is 55 and the sum of the first $3N$ terms is 5760.

Find the values of N and a. [6]

7. [9709/s25/15/q6]

Each year, on her birthday, Ananya receives some money from each of her parents.

On Ananya's first birthday, her father gives her \$10. Every subsequent year, her father gives her \$5 more than he gave her the previous year.

On Ananya's first birthday, her mother also gives her \$10. Every subsequent year, her mother gives her 20% more than she gave her the previous year.

(a) Show that on Ananya's eleventh birthday she receives more from her mother than from her father. [3]

(b) Find the total amount of money Ananya receives up to and including her eighteenth birthday. [5]

8. [9709/w25/11/q2]

A geometric progression has first term a and common ratio $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$. It is given that the second term is 8 and the fifth term is $\frac{1}{8}$.

(a) Find the value of θ. Give your answer correct to 3 significant figures. [3]

(b) Find the exact value of the sum to infinity. [2]

9. [9709/w25/11/q9]

An arithmetic progression has first term 2 and common difference d. The sum of the first n terms is denoted by S_n.

(a) It is given that $(S_2 - 1)$, S_4, S_9 are the first three terms of a second arithmetic progression.

Find the value of d. [4]

(b) Hence find the difference between the values of the 15th terms of the two arithmetic progressions. [4]

10. [9709/w25/12/q8]

The first three terms of a geometric progression are a, b and c respectively, where a, b and c are positive constants. The first three terms of an arithmetic progression are a, b and $-3c$ respectively.

(a) Show that $a^2 - 10ac + 9c^2 = 0$. [3]

It is now given that $a = 9$ and c takes the smaller of its two possible values.

(b) **(i)** Find the sum to infinity of the geometric progression. [5]

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

11. [9709/w25/13/q4]

The first, second and third terms of a progression are 20, k and $k-5$ respectively.

(a) Given that the progression is arithmetic, find the 30th term. [2]

(b) Given instead that the progression is geometric, find the sum to infinity. [4]

12. [9709/w25/15/q2]

A geometric progression has first term $3+4\sqrt{2}$ and second term $5-\sqrt{2}$.

(a) Find the common ratio of the geometric progression. Give your answer in the form $\sqrt{2}+p$, where p is an integer to be found. [3]

(b) Find the sum to infinity of the geometric progression. [2]

13. [9709/m24/12/q8]

(a) An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

(b) A geometric progression a_1, a_2, a_3, ... is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S. The sum to infinity of the even-numbered terms (i.e. a_2, a_4, a_6, ...) is denoted by S_E.

Find the values of S and S_E. [4]

14. [9709/s24/11/q8]

(a) The first three terms of an arithmetic progression are $25, 4p-1$ and $13-p$, where p is a constant.

Find the value of the tenth term of the progression. [4]

(b) The first three terms of a geometric progression are $25, 4q-1$ and $13-q$, where q is a positive constant.

Find the sum to infinity of the progression. [4]

15. [9709/s24/13/q7]

The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference. [2]

(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100. [5]

16. [9709/s24/13/q10]

The geometric progression $a_1, a_2, a_3, \ldots$ has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r. [3]

(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

(c) Find the sum to infinity of the progression $a_2, a_5, a_8, \ldots$. [3]

17. [9709/w24/11/q10]

An arithmetic progression has first term 5 and common difference d, where $d > 0$. The second, fifth and eleventh terms of the arithmetic progression, in that order, are the first three terms of a geometric progression.

(a) Find the value of d. [3]

(b) The sum of the first 77 terms of the arithmetic progression is denoted by S_{77}. The sum of the first 10 terms of the geometric progression is denoted by G_{10}.

Find the value of $S_{77} - G_{10}$. [5]

18. [9709/w24/12/q2]

The first term of an arithmetic progression is -20 and the common difference is 5.

(a) Find the sum of the first 20 terms of the progression. [2]

It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

(b) Find the value of k. [3]

19. [9709/w24/13/q1]

An arithmetic progression has fourth term 15 and eighth term 25.

Find the 30th term of the progression. [3]

20. [9709/w24/13/q6]

The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

21. [9709/m23/12/q4]

The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

22. [9709/s23/11/q6]

The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

23. [9709/s23/12/q9]

The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

24. [9709/s23/13/q8]

A progression has first term a and second term $\dfrac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

25. [9709/w23/11/q7]

The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression. [7]

26. [9709/w23/12/q5]

The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\dfrac{b}{\sqrt{c} - 1}$, where b and c are integers to be found. [3]

27. [9709/w23/13/q5]

The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p. [3]

(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

28. [9709/m22/12/q4]

The first term of a geometric progression and the first term of an arithmetic progression are both equal to a.

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a. [6]

29. [9709/s22/11/q2]

The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15.

Find the sum of the first 50 terms of the progression. [5]

30. [9709/s22/12/q2]

The second and third terms of a geometric progression are 10 and 8 respectively.

Find the sum to infinity. [4]

31. [9709/s22/12/q4]

The first, second and third terms of an arithmetic progression are k, $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k. [2]

(b) Find the sum of the first 30 terms of the progression. [3]

32. [9709/s22/13/q3]

An arithmetic progression has first term 4 and common difference d. The sum of the first n terms of the progression is 5863.

(a) Show that $(n-1)d = \dfrac{11726}{n} - 8$. [1]

(b) Given that the nth term is 139, find the values of n and d, giving the value of d as a fraction. [4]

33. [9709/w22/11/q7]

A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

34. [9709/w22/12/q2]

The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

35. [9709/w22/12/q4]

A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term. [4]

36. [9709/w22/13/q9]

The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. [6]

37. [9709/m21/12/q9]

The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

38. [9709/s21/11/q2]

The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

39. [9709/s21/11/q5]

The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression. [4]

40. [9709/s21/12/q8]

The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b+3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

41. [9709/s21/13/q9]

(a) A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a+1)$ and common difference $(d+1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

42. [9709/w21/11/q4]

The first term of an arithmetic progression is a and the common difference is -4. The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a. [4]

The kth term of the arithmetic progression is zero.

(b) Find the value of k. [2]

43. [9709/w21/12/q5]

The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. [3]

(b) Hence find the exact sum of the first 25 terms of the progression. [3]

44. [9709/w21/12/q6]

The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

45. [9709/w21/13/q4]

The first term of an arithmetic progression is 84 and the common difference is -3.

(a) Find the smallest value of n for which the nth term is negative. [2]

It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

46. [9709/m20/12/q8]

A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

47. [9709/s20/11/q1]

The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

48. [9709/s20/11/q3]

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

49. [9709/s20/12/q4]

The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

50. [9709/s20/13/q8]

The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

51. [9709/w20/11/q8]

A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

52. [9709/w20/12/q2]

The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

53. [9709/w20/12/q4]

The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

54. [9709/m19/12/q6]

(i) The first and second terms of a geometric progression are p and $2p$ respectively, where p is a positive constant. The sum of the first n terms is greater than $1000p$. Show that $2^n > 1001$. [2]

(ii) In another case, p and $2p$ are the first and second terms respectively of an arithmetic progression. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find the values of n and p. [5]

55. [9709/s19/11/q8]

(a) The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

56. [9709/s19/12/q10]

(a) In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

57. [9709/s19/13/q5]

Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer A's weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

(i) Write down an expression for his total weight loss after x weeks. [1]

(ii) He reaches his 13 kg target during week n. Use your answer to part **(i)** to find the value of n. [2]

Boxer B's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression.

(iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

58. [9709/w19/11/q4]

A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run x km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

(i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

(ii) Find the total distance she runs over the 21 days. [2]

59. [9709/w19/12/q8]

(a) Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

(ii) Find the total distance she runs in the 21-day period. [2]

(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]

(ii) Find the fourth term of the progression. [2]

(iii) Find the sum to infinity of the progression. [2]

60. [9709/w19/13/q9]

The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]

(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

61. [9709/m18/12/q3]

On a certain day, the height of a young bamboo plant was found to be 40 cm. After exactly one day its height was found to be 41.2 cm. Two different models are used to predict its height exactly 60 days after it was first measured.

- Model A assumes that the daily amount of growth continues to be constant at the amount found for the first day.
- Model B assumes that the daily percentage rate of growth continues to be constant at the percentage rate of growth found for the first day.

(i) Using model A, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [2]

(ii) Using model B, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [3]

62. [9709/s18/11/q8]

(a) A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q. [2]

63. [9709/s18/12/q3]

A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

64. [9709/s18/13/q3]

The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

65. [9709/w18/11/q4]

The first term of a series is 6 and the second term is 2.

(i) For the case where the series is an arithmetic progression, find the sum of the first 80 terms. [3]

(ii) For the case where the series is a geometric progression, find the sum to infinity. [2]

66. [9709/w18/12/q5]

The first three terms of an arithmetic progression are 4, x and y respectively. The first three terms of a geometric progression are x, y and 18 respectively. It is given that both x and y are positive.

(i) Find the value of x and the value of y. [4]

(ii) Find the fourth term of each progression. [3]

67. [9709/w18/13/q5]

In an arithmetic progression the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420. Find n and a. [6]

68. [9709/s17/11/q4]

(a) An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of −28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated $2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

69. [9709/s17/12/q7]

(a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

70. [9709/s17/13/q2]

The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

(ii) Find the set of possible values that S can take. [2]

71. [9709/w17/11/q3]

(a) A geometric progression has first term $3a$ and common ratio r. A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r. [3]

(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. [3]

72. [9709/w17/12/q3]

(a) Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for $10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest $100. [2]

(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n+7)$. Find the 1st term and the common difference of the progression. [4]

73. [9709/w17/13/q1]

An arithmetic progression has first term -12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

74. [9709/m16/12/q3]

The 12th term of an arithmetic progression is 17 and the sum of the first 31 terms is 1023. Find the 31st term. [5]

75. [9709/s16/11/q9]

(a) The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x = \frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

76. [9709/s16/12/q9]

A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

77. [9709/s16/13/q4]

The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

78. [9709/w16/11/q5]

The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

79. [9709/w16/12/q8]

(a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

80. [9709/w16/13/q9]

(a) Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta + \sin^2\theta$, where $0 \leqslant \theta \leqslant \pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

81. [9709/s15/11/q7]

(a) The third and fourth terms of a geometric progression are $\frac{1}{3}$ and $\frac{2}{9}$ respectively. Find the sum to infinity of the progression. [4]

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector. [4]

82. [9709/s15/12/q8]

(a) The first, second and last terms in an arithmetic progression are 56, 53 and −22 respectively. Find the sum of all the terms in the progression. [4]

(b) The first, second and third terms of a geometric progression are $2k + 6$, $2k$ and $k + 2$ respectively, where k is a positive constant.

(i) Find the value of k. [3]

(ii) Find the sum to infinity of the progression. [2]

83. [9709/s15/13/q9]

(a) The first term of an arithmetic progression is -2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of θ for which the progression is convergent. [5]

84. [9709/w15/11/q8]

The first term of a progression is $4x$ and the second term is x^2.

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of x and the corresponding values of the third term. [4]

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term. [4]

85. [9709/w15/13/q6]

A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A : The height reached is reduced by 0.04 metres each time the ball bounces.

Model B : The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

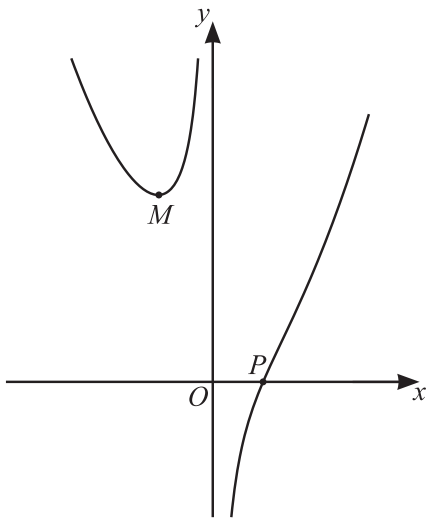
(b) using model B. [3]

(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

Chapter 7

Differentiation

1. [9709/m25/12/q2]



The diagram shows the curve with equation $y = 2x^2 - \frac{5}{x} + 3$. The curve crosses the x-axis at the point $P(1, 0)$ and M is a minimum point.

(a) Find the gradient of the curve at P. [2]

(b) Find the coordinates of M. Give each coordinate correct to 3 significant figures. [3]

2. [9709/m25/12/q9]

A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{5}{x^3}$. It is given that the curve has a stationary point at $\left(\frac{1}{2}, 9\right)$.

(a) Use the expression for $\frac{d^2y}{dx^2}$ to determine whether the stationary point is a maximum or a minimum point. [2]

(b) Find the equation of the curve. [7]

3. [9709/s25/11/q2]

The equation of a curve is such that $\frac{dy}{dx} = 4(2x-5)^3 - 9x^{\frac{1}{2}}$. The curve passes through the point $A\left(4, -\frac{11}{2}\right)$.

(a) Find the gradient of the normal to the curve at the point A. [2]

(b) Find the equation of the curve. [4]

4. [9709/s25/11/q7]

The equation of a curve is $y = 4x^2 + \frac{9}{x^2} - 8$.

(a) A point P is moving along the curve in such a way that its y-coordinate is decreasing at 5 units per second.

Find the rate at which the x-coordinate of point P is changing when $x = 2$. [4]

(b) Find the coordinates of the stationary points of the curve and determine their nature. [5]

5. **[9709/s25/12/q4]**

A point P is moving along the curve with equation $y = ax^{\frac{3}{2}} - 12x$ in such a way that the x-coordinate of P is increasing at a constant rate of 5 units per second.

(a) Find the rate at which the y-coordinate of P is changing when $x = 9$. Give your answer in terms of the constant a. [3]

....

(b) Given that the curve has a minimum point when $x = \frac{1}{4}$, find the value of a. [2]

6. [9709/s25/12/q9]

The equation of a curve is such that $\frac{d^2y}{dx^2} = -\frac{24}{x^3}$. It is given that the curve has a stationary point at $(-2, 19)$.

(a) Find an expression for $\frac{dy}{dx}$. [3]

(b) Find the x-coordinate of the other stationary point of the curve, and determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

....

....

....

....

....

....

....

....

....

(d) Find the equation of the normal to the curve at the point where $\frac{dy}{dx} = -\frac{9}{4}$ and x is positive. Express your answer in the form $px + qy + r = 0$, where p, q and r are integers. [4]

7. [9709/s25/13/q1]

A curve has equation $y = 2x + \frac{12}{x^2}$.

Find the equation of the tangent to the curve at the point $(-2, -1)$. Give your answer in the form $y = mx + c$. [4]

8. [9709/s25/13/q7]

A curve is such that $\frac{dy}{dx} = 3x^2 + 10x - 8$.

(a) Find the set of values of x for which y decreases as x increases. [3]

(b) It is given that the maximum point of the curve has y-coordinate 27.

Find the equation of the curve. [4]

9. [9709/s25/13/q10]

A curve C has equation $y = \dfrac{9}{2x-5} + 2x - 5$.

(a) Find the coordinates of the two stationary points. [4]

(b) Find $\dfrac{d^2y}{dx^2}$ and hence determine the nature of each stationary point. [3]

(c) The curve C is transformed to the curve C_1 using a translation of $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$ followed by reflection in the x-axis.

(i) State the coordinates of the maximum point of C_1. [1]

(ii) Find the equation of C_1 in the form $y = \dfrac{a}{bx+c} + dx + e$, where a, b, c, d and e are integers. [3]

10. [9709/s25/15/q8]

The equation of a curve is $y = x^3 + ax^2 + bx + 5$. The curve has a stationary point at (1, 9).

(a) Find the values of the constants a and b. [5]

(b) Find the coordinates of the other stationary point. [3]

(c) A point P is moving along part of the curve in such a way that the y-coordinate of P is increasing at a constant rate of 6 units per second.

Find the rate at which the x-coordinate of P is increasing when $x = 5$. [3]

11. [9709/w25/11/q11]

A curve passes through the point $P(4, 3)$ and is such that

$$\frac{dy}{dx} = \frac{8}{x^2} - \frac{10}{(2x-3)^2}.$$

(a) Find the equation of the normal to the curve at P. Give your answer in the form $y = mx + c$. [3]

(b) Find the rate of change of the gradient of the curve when $x = 4$. [3]

(c) Given that the curve also passes through the point $(-1, q)$, find the value of q. [5]

12. [9709/w25/12/q4]

The equation of a curve is such that $\frac{dy}{dx} = kx^3 + \frac{2}{x^2}$, where k is a constant. The curve passes through the point S (2, 20) and the gradient of the curve at S is $\frac{65}{2}$.

(a) Find the value of k. [1]

(b) The coordinates of a point T on the curve are (1, t).

Find the value of t. [5]

13. [9709/w25/12/q9]

The function f is defined by $f(x) = \dfrac{4}{(3x-6)^2} + \dfrac{1}{(3x-6)^3}$ for $x > 2$.

(a) Find an expression for $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [4]

(b) State whether f^{-1} exists. Give a reason for your answer. [1]

The function g is defined by $g(x) = 4x - 3$ for $x > a$.

(c) Find the range of g in terms of the constant a. [1]

(d) Find the set of values of a for which the composite function fg exists. [2]

14. [9709/w25/13/q3]

The equation of a curve is $y = f(x)$, where $f(x) = \frac{1}{2}x^{\frac{2}{3}}(x-2)^2$. The following points lie on the curve. Non-exact values of the y-coordinates are given correct to 6 decimal places.

$$A(8, 72),\ B(8.001, k),\ C(8.01, 72.300388),\ D(8.1, 75.038882)$$

(a) Find the value of k. Give your answer correct to 6 decimal places. [1]

The table below shows the gradients of the chords AB and AC, given correct to 4 decimal places.

Chord	AB	AC	AD
Gradient of chord	30.0039	30.0388	

(b) Find the gradient of the chord AD. Give your answer correct to 4 decimal places. [1]

(c) State what the values in the table suggest about the value of $f'(8)$. [1]

15. [9709/w25/13/q11]

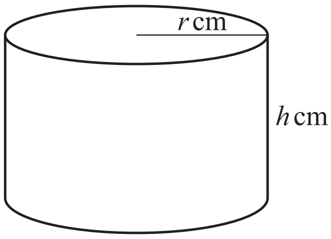
The equation of a curve is $y = \frac{8}{3x-8} - \frac{6}{x-1}$.

(a) Find the coordinates of the point at which the tangent to the curve at the point (3, 5) intersects the line $y = -8x$. [6]

(b) **(i)** Find the x-coordinates of each of the stationary points of the curve. [3]

(ii) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of each of the stationary points. [4]

16. [9709/w25/15/q7]



A manufacturer wishes to design an open cylindrical tank, as shown in the diagram. The tank will have a base but no top. The outside of the tank will have a fixed surface area of $600\pi\,\text{cm}^2$. The radius r cm and height h cm of the tank can vary.

(a) Show that the volume, $V\,\text{cm}^3$, of the tank is given by

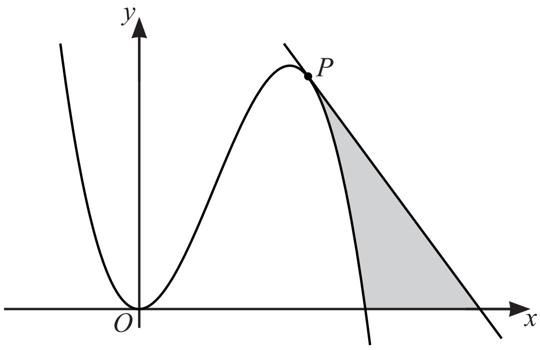
$$V = \frac{\pi r(600 - r^2)}{2}.$$

[3]

(b) Find the exact value of r which corresponds to the maximum value of V. [3]

(c) Hence, find the maximum value of V. [2]

17. [9709/w25/15/q11]



The diagram shows the curve with equation $y = 4x^2 - x^3$ and the tangent to the curve at the point P. The point P has x-coordinate 3.

(a) Find the equation of the tangent to the curve at the point P. Give your answer in the form $y = mx + c$. [5]

....

(b) The shaded region is bounded by the curve, the x-axis and the tangent to the curve at P.

Find the exact area of the shaded region. [6]

The graph of $y = 4x^2 - x^3$ is transformed by a stretch of scale factor $\frac{1}{3}$ in the x-direction. The point Q is the image of P under this transformation. The transformed shaded region is bounded by the transformed curve, the x-axis and the tangent to the transformed curve at Q.

(c) (i) Find the equation of the transformed curve in the form $y = mx^2 + nx^3$, where m and n are integers to be found. [1]

(c) (ii) State the coordinates of Q and the area of the transformed shaded region. [2]

18. [9709/m24/12/q5]

A curve has the equation $y = \frac{3}{2x^2 - 5}$.

Find the equation of the normal to the curve at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

19. [9709/s24/11/q4]

The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (2x-1)\sqrt{3x-2}-2$. The following points lie on the curve. Non-exact values have been given correct to 5 decimal places.

$A(2,4)$, $B(2.0001,k)$, $C(2.001,4.00625)$, $D(2.01,4.06261)$, $E(2.1,4.63566)$, $F(3,11.22876)$

(a) Find the value of k. Give your answer correct to 5 decimal places. [1]

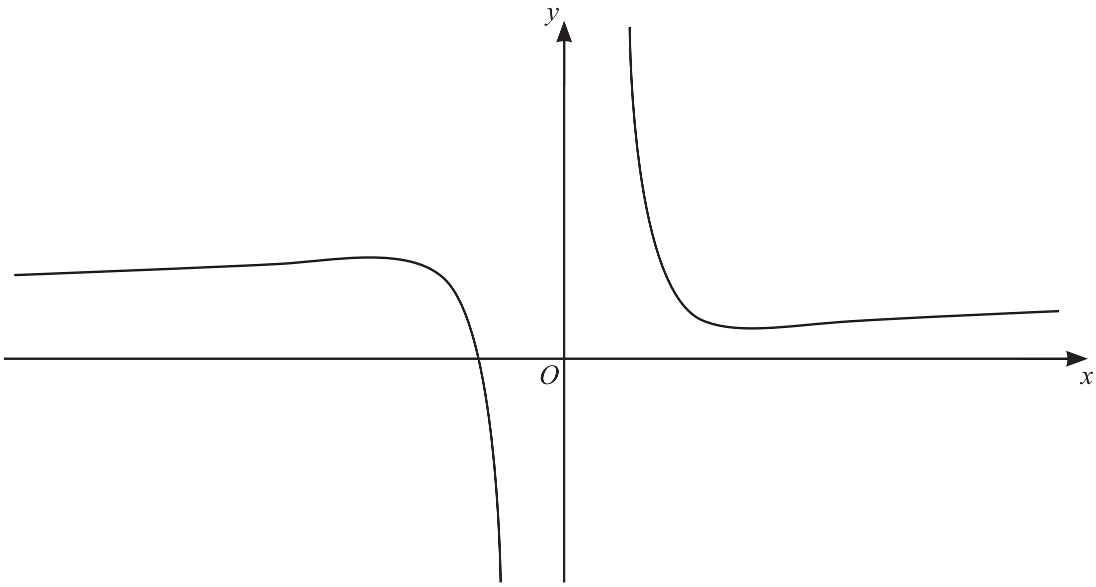
The table shows the gradients of the chords AB, AC, AD and AF.

Chord	AB	AC	AD	AE	AF
Gradient of chord	6.2501	6.2511	6.2608		7.2288

(b) Find the gradient of the chord AE. Give your answer correct to 4 decimal places. [1]

(c) Deduce the value of $\mathrm{f}'(2)$ using the values in the table. [1]

20. [9709/s24/11/q11]



A function is defined by $f(x) = \frac{4}{x^3} - \frac{3}{x} + 2$ for $x \neq 0$. The graph of $y = f(x)$ is shown in the diagram.

(a) Find the set of values of x for which $f(x)$ is decreasing. [5]

(b) A triangle is bounded by the y-axis, the normal to the curve at the point where $x = 1$ and the tangent to the curve at the point where $x = -1$.

Find the area of the triangle. Give your answer correct to 3 significant figures. [8]

21. [9709/s24/12/q10]

The equation of a curve is $y = (5-2x)^{\frac{3}{2}} + 5$ for $x < \frac{5}{2}$.

(a) A point P is moving along the curve in such a way that the y-coordinate of point P is decreasing at 5 units per second.

Find the rate at which the x-coordinate of point P is increasing when $y = 32$. [4]

(b) Point A on the curve has y-coordinate 32. Point B on the curve is such that the gradient of the curve at B is -3.

Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

22. [9709/s24/13/q5]

The equation of a curve is $y = 2x^2 - \frac{1}{2x} + 3$.

(a) Find the coordinates of the stationary point. [3]

(b) Determine the nature of the stationary point. [2]

(c) For positive values of x, determine whether the curve shows a function that is increasing, decreasing or neither. Give a reason for your answer. [2]

23. [9709/w24/11/q2]

The curve $y = x^2 - \frac{a}{x}$ has a stationary point at $(-3, b)$.

Find the values of the constants a and b. [4]

24. [9709/w24/11/q5]

The equation of a curve is such that $\frac{dy}{dx} = 4x - 3\sqrt{x} + 1$.

(a) Find the x-coordinate of the point on the curve at which the gradient is $\frac{11}{2}$. [3]

(b) Given that the curve passes through the point (4, 11), find the equation of the curve. [4]

25. [9709/w24/11/q9]

The equation of a curve is $y = 4 + 5x + 6x^2 - 3x^3$.

(a) Find the set of values of x for which y decreases as x increases. [4]

(b) It is given that $y = 9x + k$ is a tangent to the curve.

Find the value of the constant k. [4]

26. [9709/w24/12/q10]

A function f with domain $x > 0$ is such that $f'(x) = 8(2x-3)^{\frac{1}{3}} - 10x^{\frac{2}{3}}$. It is given that the curve with equation $y = f(x)$ passes through the point (1, 0).

(a) Find the equation of the normal to the curve at the point (1, 0). [3]

(b) Find $f(x)$. [4]

It is given that the equation $f'(x) = 0$ can be expressed in the form

$$125x^2 - 128x + 192 = 0.$$

(c) Determine, making your reasoning clear, whether f is an increasing function, a decreasing function or neither. [3]

27. [9709/w24/13/q11]

The equation of a curve is $y = kx^{\frac{1}{2}} - 4x^2 + 2$, where k is a constant.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of k. [2]

(b) It is given that $k = 2$.

Find the coordinates of the stationary point and determine its nature. [4]

(c) Points A and B on the curve have x-coordinates 0.25 and 1 respectively. For a different value of k, the tangents to the curve at the points A and B meet at a point with x-coordinate 0.6.

Find this value of k. [6]

28. [9709/m23/12/q3]

A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the x-coordinate of the point on the curve at which the x- and y-coordinates are increasing at the same rate. [4]

29. [9709/m23/12/q10]

At the point (4, −1) on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

(b) Find the equation of the curve. [4]

(c) Find the coordinates of the stationary point. [3]

(d) Determine the nature of the stationary point. [2]

30. [9709/s23/11/q5]

The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

31. [9709/s23/11/q9]

Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25+h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

32. [9709/s23/11/q11]

The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

(b) Determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

(d) Find the coordinates of any other stationary points on the curve. [2]

33. [9709/s23/12/q11]

The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

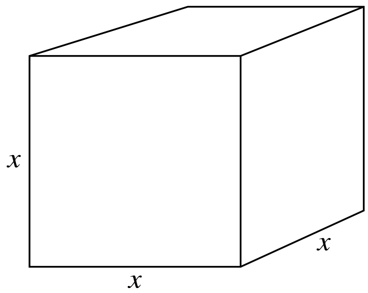
where k is a positive constant.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the x-coordinate of the stationary point in terms of k. [2]

(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

34. [9709/w23/11/q3]



The diagram shows a cubical closed container made of a thin elastic material which is filled with water and frozen. During the freezing process the length, x cm, of each edge of the container increases at the constant rate of 0.01 cm per minute. The volume of the container at time t minutes is V cm^3.

Find the rate of increase of V when $x = 20$. [3]

35. [9709/w23/12/q3]

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P\,(2,\ 8)$.

(a) Find the equation of the normal to the curve at P. [2]

(b) Find the equation of the curve. [4]

36. [9709/w23/12/q10]

The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (4x-3)^{\frac{5}{3}} - \frac{20}{3}x$.

(a) Find the x-coordinates of the stationary points of the curve and determine their nature. [6]

(b) State the set of values for which the function f is increasing. [1]

37. [9709/w23/13/q9]

A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

(a) Find the equation of the normal to the curve at the point $A\,(4,\ 3)$, giving your answer in the form $y = mx + c$. [3]

A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at A the rate of increase of the x-coordinate is $3\,\text{cm}\,\text{s}^{-1}$.

(b) Find the rate of increase of the y-coordinate at A. [2]

At A the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the y-coordinate is constant at $5\,\text{cm}\,\text{s}^{-1}$.

(c) As the point moves down the normal, find the rate of change of its x-coordinate. [3]

38. [9709/m22/12/q11]

It is given that a curve has equation $y = k(3x - k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

The function f has a stationary value at $x = a$ and is defined by

$$f(x) = 4(3x - 4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

(c) The function g is defined by $g(x) = -(3x + 1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

39. [9709/s22/11/q10]

The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

(b) Find the equation of the curve. [5]

(c) Show that the curve has no other stationary points. [3]

(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x-coordinate of A at the point where $x = 1$. [3]

40. [9709/s22/12/q9]

The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point of the curve and determine its nature. [4]

41. [9709/s22/13/q10.b]

The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} f(x)\, dx$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

42. [9709/w22/11/q3]

A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P, which has x-coordinate 9.

Find the y-coordinate of P. [5]

43. [9709/w22/11/q8.a]

The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

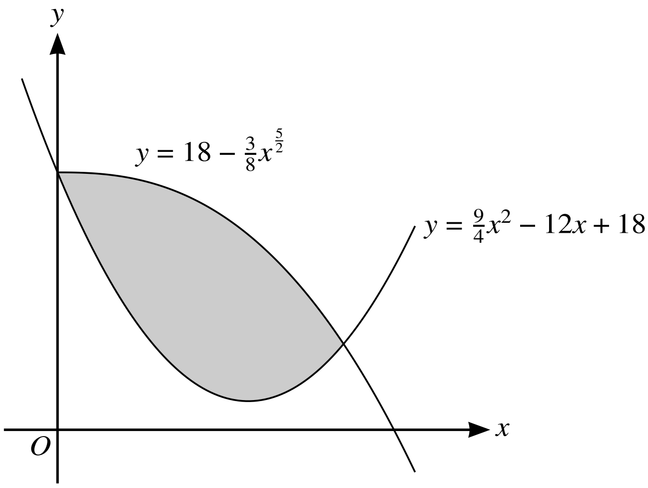
(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx - d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

44. [9709/w22/12/q11.c]

(a) Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]



The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region. [5]

(c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

45. [9709/w22/13/q4]

A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is $4\,\text{m}$, giving your answer in cm per minute. [5]

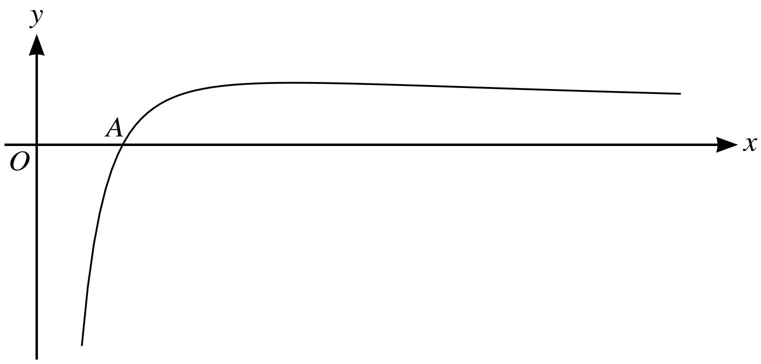
46. [9709/m21/12/q6.a]

A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6}{(3x-2)^3}$ and $A\,(1,\ -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

47. [9709/m21/12/q11.bc]



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

(b) Find the equation of the tangent to the curve at A. [4]

(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

48. [9709/s21/11/q11]

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

49. [9709/s21/12/q3]

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2,\ k)$ $B\,(2.9,\ 2.8025)$ $C\,(2.99,\ 2.9800)$ $D\,(2.999,\ 2.9980)$ $E\,(3,\ 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

50. [9709/s21/12/q11]

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

(b) Find the equation of the curve. [4]

(c) Find $\frac{d^2y}{dx^2}$. [2]

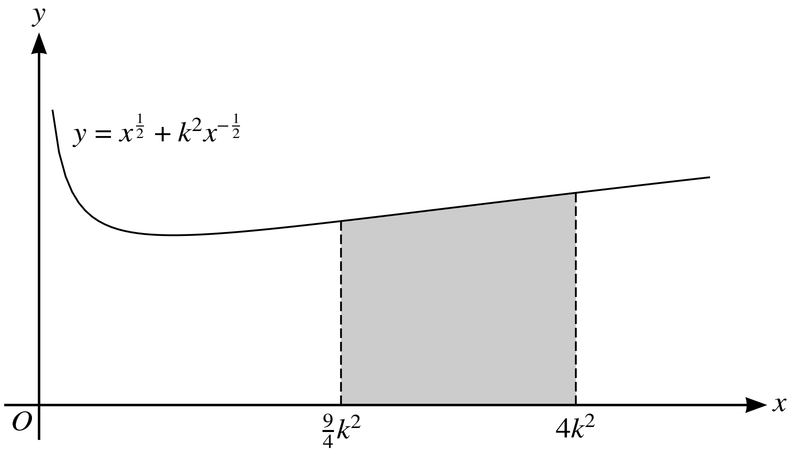
(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

51. [9709/s21/13/q2]

The function f is defined by $f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

52. [9709/s21/13/q11]



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

53. [9709/w21/11/q9]

A curve has equation $y = \mathrm{f}(x)$, and it is given that $\mathrm{f}'(x) = 2x^2 - 7 - \dfrac{4}{x^2}$.

(a) Given that $\mathrm{f}(1) = -\frac{1}{3}$, find $\mathrm{f}(x)$. [4]

(b) Find the coordinates of the stationary points on the curve. [5]

(c) Find $\mathrm{f}''(x)$. [1]

(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

54. [9709/w21/12/q9]

The volume $V\,\mathrm{m}^3$ of a large circular mound of iron ore of radius $r\,\mathrm{m}$ is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of $1.5\,\mathrm{m}^3$ per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

55. [9709/w21/12/q10]

The function f is defined by $f(x) = x^2 + \dfrac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

(c) Given that this is the only stationary point of the curve, find the range of f. [2]

56. [9709/w21/13/q3.b]

(a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

57. [9709/w21/13/q10]

A curve has equation $y = \mathrm{f}(x)$ and it is given that

$$\mathrm{f}'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

(a) Find $\mathrm{f}''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

It is now given that $k = -3$ and the minimum point is at $(2,\ 3\tfrac{1}{2})$.

(b) Find $\mathrm{f}(x)$. [4]

(c) Find the coordinates of the other stationary point and determine its nature. [4]

58. [9709/m20/12/q1]

The function f is defined by $f(x) = \dfrac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

59. [9709/m20/12/q4]

A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

60. [9709/m20/12/q10]

The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

(b) Determine the nature of the stationary point. [3]

(c) Find the equation of the curve. [4]

61. [9709/s20/11/q9]

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each stationary point. [2]

62. [9709/s20/12/q3]

A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of $600\,\text{cm}^3$ per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

(b) Find the rate of increase of the radius after 30 seconds. [3]

63. [9709/s20/12/q10]

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each of the stationary points. [2]

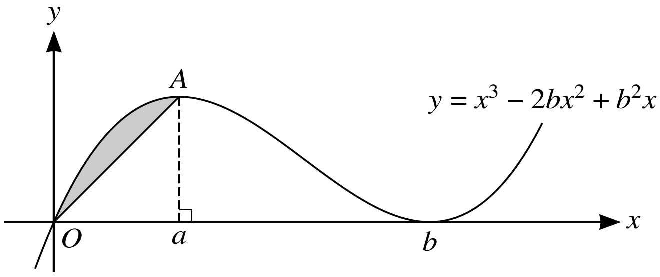
64. [9709/s20/13/q6]

A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

65. [9709/s20/13/q11.a]



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

66. [9709/w20/11/q3]

Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is $10\,\text{cm}$. [3]

67. [9709/w20/11/q6]

The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

68. [9709/w20/12/q7.a]

The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

(b) Find the equation of the curve. [4]

69. [9709/w20/13/q8]

The equation of a curve is $y = 2x + 1 + \frac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [3]

(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

70. [9709/w20/13/q10.a]

A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}\right) dx = \frac{13}{12}$.

Find the value of k. [5]

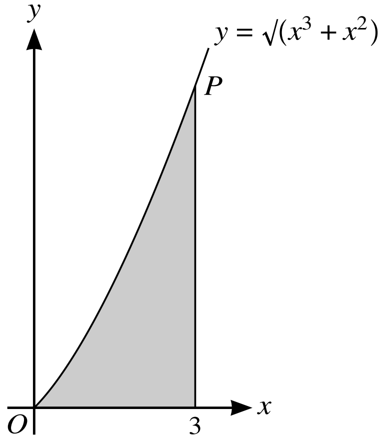
71. [9709/m19/12/q4]

A curve has equation $y = (2x - 1)^{-1} + 2x$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

(ii) Find the x-coordinates of the stationary points and, showing all necessary working, determine the nature of each stationary point. [4]

72. [9709/m19/12/q9.b]

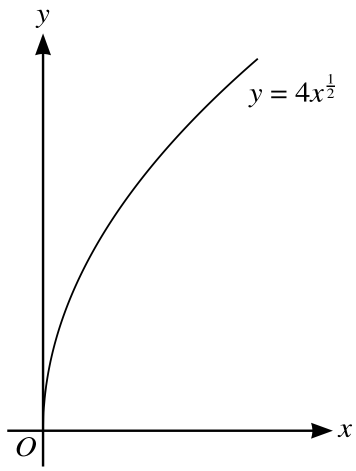


The diagram shows part of the curve with equation $y = \sqrt{(x^3 + x^2)}$. The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

(ii) P is the point on the curve with x-coordinate 3. Find the y-coordinate of the point where the normal to the curve at P crosses the y-axis. [6]

73. [9709/m19/12/q10]



The diagram shows the curve with equation $y = 4x^{\frac{1}{2}}$.

(i) The straight line with equation $y = x + 3$ intersects the curve at points A and B. Find the length of AB. [6]

(ii) The tangent to the curve at a point T is parallel to AB. Find the coordinates of T. [3]

(iii) Find the coordinates of the point of intersection of the normal to the curve at T with the line AB. [3]

74. [9709/s19/12/q3]

A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P(2, 9)$ lies on the curve.

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

(ii) Find the equation of the curve. [3]

75. [9709/s19/12/q9]

The curve C_1 has equation $y = x^2 - 4x + 7$. The curve C_2 has equation $y^2 = 4x + k$, where k is a constant. The tangent to C_1 at the point where $x = 3$ is also the tangent to C_2 at the point P. Find the value of k and the coordinates of P. [8]

76. [9709/s19/13/q8]

A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k. [8]

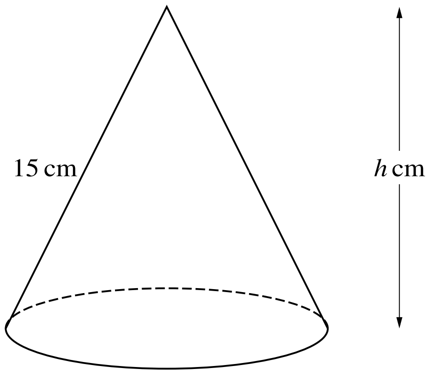
77. [9709/w19/11/q2]

An increasing function, f, is defined for $x > n$, where n is an integer. It is given that $f'(x) = x^2 - 6x + 8$. Find the least possible value of n. [3]

78. [9709/w19/11/q3]

The line $y = ax + b$ is a tangent to the curve $y = 2x^3 - 5x^2 - 3x + c$ at the point $(2, 6)$. Find the values of the constants a, b and c. [5]

79. [9709/w19/12/q5]



The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

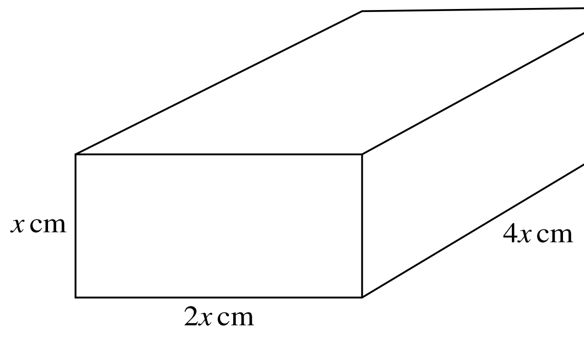
[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

80. [9709/w19/13/q3]

The equation of a curve is $y = x^3 + x^2 - 8x + 7$. The curve has no stationary points in the interval $a < x < b$. Find the least possible value of a and the greatest possible value of b. [4]

81. [9709/w19/13/q5]



The dimensions of a cuboid are x cm, $2x$ cm and $4x$ cm, as shown in the diagram.

(i) Show that the surface area S cm^2 and the volume V cm^3 are connected by the relation

$$S = 7V^{\frac{2}{3}}.$$ [3]

(ii) When the volume of the cuboid is 1000 cm^3 the surface area is increasing at 2 cm^2 s^{-1}. Find the rate of increase of the volume at this instant. [4]

82. [9709/m18/12/q8]

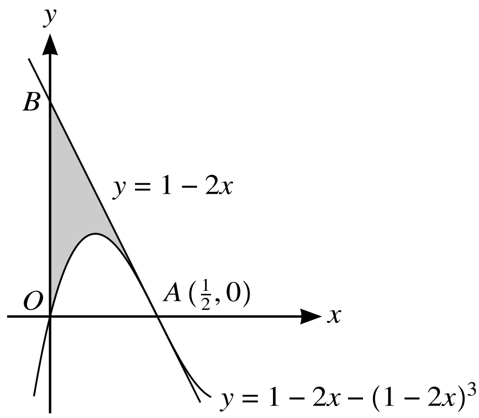
A curve has equation $y = \frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 8x$.

(i) Find the x-coordinates of the stationary points. [5]

(ii) Find $\frac{d^2y}{dx^2}$. [1]

(iii) Find, showing all necessary working, the nature of each stationary point. [2]

83. [9709/m18/12/q11.a]



The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1 - 2x)^3 \, dx$. [2]

(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

84. [9709/s18/11/q2]

A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

85. [9709/s18/11/q3]

A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$. The point (1, 1) lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

86. [9709/s18/11/q10]

The curve with equation $y = x^3 - 2x^2 + 5x$ passes through the origin.

(i) Show that the curve has no stationary points. [3]

(ii) Denoting the gradient of the curve by m, find the stationary value of m and determine its nature. [5]

(iii) Showing all necessary working, find the area of the region enclosed by the curve, the x-axis and the line $x = 6$. [4]

87. [9709/s18/13/q4]

A curve with equation $y = \mathrm{f}(x)$ passes through the point $A\,(3,\ 1)$ and crosses the y-axis at B. It is given that $\mathrm{f}'(x) = (3x - 1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

88. [9709/s18/13/q8]

(i) The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

(ii) The function f is defined by $\mathrm{f}(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

89. [9709/w18/11/q10]

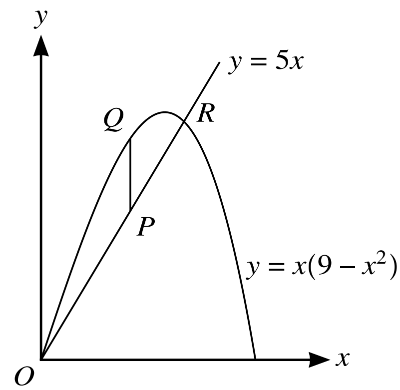
A curve has equation $y = \frac{1}{2}(4x - 3)^{-1}$. The point A on the curve has coordinates $\left(1, \frac{1}{2}\right)$.

(i) (a) Find and simplify the equation of the normal through A. [5]

(b) Find the x-coordinate of the point where this normal meets the curve again. [3]

(ii) A point is moving along the curve in such a way that as it passes through A its x-coordinate is decreasing at the rate of 0.3 units per second. Find the rate of change of its y-coordinate at A. [2]

90. [9709/w18/12/q3]



The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R. Point P lies on the line $y = 5x$ between O and R and the x-coordinate of P is t. Point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the length of PQ in terms of t, simplifying your answer. [2]

(ii) Given that t can vary, find the maximum value of the length of PQ. [3]

91. [9709/w18/13/q2]

The function f is defined by $f(x) = x^3 + 2x^2 - 4x + 7$ for $x \geqslant -2$. Determine, showing all necessary working, whether f is an increasing function, a decreasing function or neither. [4]

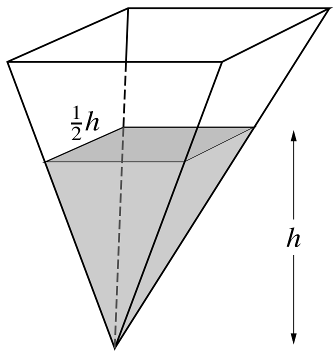
92. [9709/w18/13/q8]

A curve passes through (0, 11) and has an equation for which $\frac{dy}{dx} = ax^2 + bx - 4$, where a and b are constants.

(i) Find the equation of the curve in terms of a and b. [3]

(ii) It is now given that the curve has a stationary point at (2, 3). Find the values of a and b. [5]

93. [9709/m17/12/q3]



The diagram shows a water container in the form of an inverted pyramid, which is such that when the height of the water level is h cm the surface of the water is a square of side $\frac{1}{2}h$ cm.

(i) Express the volume of water in the container in terms of h. [1]

[The volume of a pyramid having a base area A and vertical height h is $\frac{1}{3}Ah$.]

Water is steadily dripping into the container at a constant rate of 20 cm^3 per minute.

(ii) Find the rate, in cm per minute, at which the water level is rising when the height of the water level is 10 cm. [4]

94. [9709/m17/12/q7]

The function f is defined for $x \geqslant 0$ by $\mathrm{f}(x) = (4x + 1)^{\frac{3}{2}}$.

(i) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$. [3]

The first, second and third terms of a geometric progression are respectively $\mathrm{f}(2)$, $\mathrm{f}'(2)$ and $k\mathrm{f}''(2)$.

(ii) Find the value of the constant k. [5]

95. [9709/m17/12/q9]

The point A (2, 2) lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. [3]

The normal to the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. [4]

The tangents at A and B intersect each other at C.

(iii) Find the coordinates of C. [4]

96. [9709/s17/11/q6]

The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is $2000\,\text{cm}^3$.

(i) Express h in terms of x and show that the total surface area of the prism, $A\,\text{cm}^2$, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

97. [9709/s17/12/q5]

A curve has equation $y = 3 + \frac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

98. [9709/s17/12/q9]

The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

99. [9709/s17/13/q6]

The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k. [6]

100. [9709/w17/11/q1]

A curve has equation $y = 2x^{\frac{3}{2}} - 3x - 4x^{\frac{1}{2}} + 4$. Find the equation of the tangent to the curve at the point (4, 0). [4]

101. [9709/w17/11/q2]

A function f is defined by $\mathrm{f} : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a. [4]

102. [9709/w17/11/q4]

Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm^3, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

(i) Show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$. [1]

(ii) Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]

(iii) Find the maximum volume of a cone that can be made by these machines. [1]

103. [9709/w17/12/q7]

Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates $(4, 7)$ and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

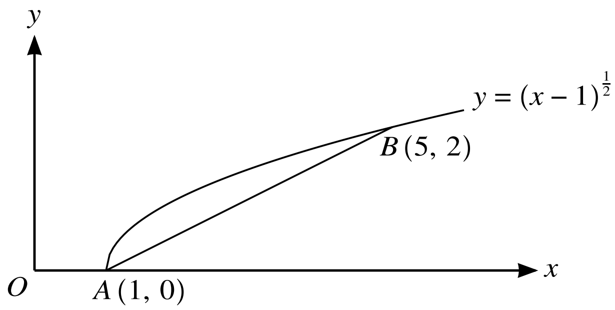
(i) In the case where L passes through the mid-point of AB, find the value of m. [4]

(ii) Find the set of values of m for which L does not meet the curve. [4]

104. [9709/w17/13/q4]

The function f is such that $f(x) = (2x - 1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

105. [9709/w17/13/q11]



The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A\,(1, 0)$ and $B\,(5, 2)$ lying on the curve.

(i) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [2]

(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB. [5]

(iii) Find the perpendicular distance between the line AB and the tangent parallel to AB. Give your answer correct to 2 decimal places. [3]

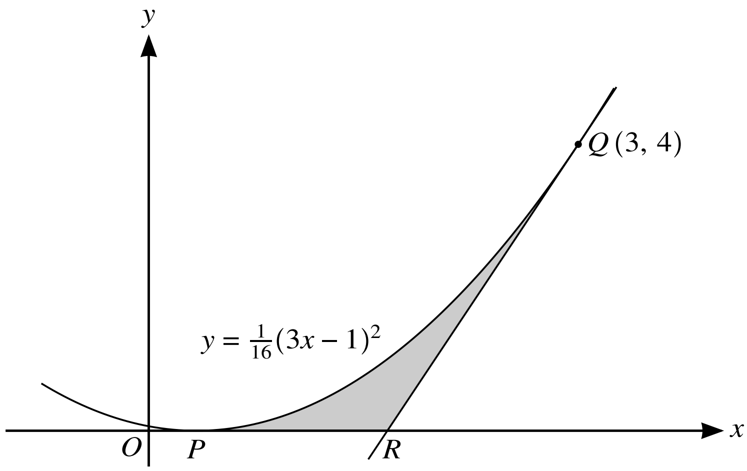
106. [9709/m16/12/q6]

A vacuum flask (for keeping drinks hot) is modelled as a closed cylinder in which the internal radius is r cm and the internal height is h cm. The volume of the flask is $1000\,\text{cm}^3$. A flask is most efficient when the total internal surface area, $A\,\text{cm}^2$, is a minimum.

(i) Show that $A = 2\pi r^2 + \dfrac{2000}{r}$. [3]

(ii) Given that r can vary, find the value of r, correct to 1 decimal place, for which A has a stationary value and verify that the flask is most efficient when r takes this value. [5]

107. [9709/m16/12/q10.12]



The diagram shows part of the curve $y = \frac{1}{16}(3x-1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

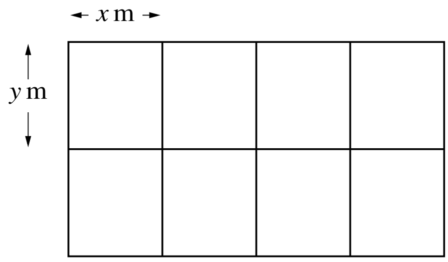
(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

108. [9709/s16/11/q5]



A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures x m by y m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

(i) Show that the total area of land used for the sheep pens, A m^2, is given by

$$A = 384x - 9.6x^2. \qquad [3]$$

(ii) Given that x and y can vary, find the dimensions of each sheep pen for which the value of A is a maximum. (There is no need to verify that the value of A is a maximum.) [3]

109. [9709/s16/11/q8]

A curve has equation $y = 3x - \frac{4}{x}$ and passes through the points $A\,(1,\ -1)$ and $B\,(4,\ 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

110. [9709/s16/13/q5]

A curve has equation $y = 8x + (2x-1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

111. [9709/s16/13/q7]

The point $P(x, y)$ is moving along the curve $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$ in such a way that the rate of change of y is constant. Find the values of x at the points at which the rate of change of x is equal to half the rate of change of y. [7]

112. [9709/w16/11/q11]

The point $P(3,\ 5)$ lies on the curve $y = \dfrac{1}{x-1} - \dfrac{9}{x-5}$.

(i) Find the x-coordinate of the point where the normal to the curve at P intersects the x-axis. [5]

(ii) Find the x-coordinate of each of the stationary points on the curve and determine the nature of each stationary point, justifying your answers. [6]

113. [9709/w16/12/q7]

The equation of a curve is $y = 2 + \dfrac{3}{2x - 1}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

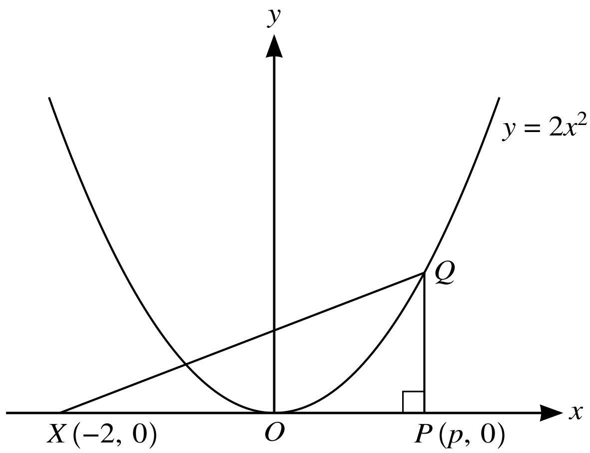
(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

114. [9709/w16/13/q4]

The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. [4]

115. [9709/s15/11/q2]



The diagram shows the curve $y = 2x^2$ and the points $X(-2, 0)$ and $P(p, 0)$. The point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the area, A, of triangle XPQ in terms of p. [2]

The point P moves along the x-axis at a constant rate of 0.02 units per second and Q moves along the curve so that PQ remains parallel to the y-axis.

(ii) Find the rate at which A is increasing when $p = 2$. [3]

116. [9709/s15/11/q5.3]

A piece of wire of length 24 cm is bent to form the perimeter of a sector of a circle of radius r cm.

(i) Show that the area of the sector, A cm^2, is given by $A = 12r - r^2$. [3]

(ii) Express A in the form $a - (r - b)^2$, where a and b are constants. [2]

(iii) Given that r can vary, state the greatest value of A and find the corresponding angle of the sector. [2]

117. [9709/s15/11/q9]

The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the coordinates of the other stationary point in terms of p. [4]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

118. [9709/s15/12/q4]

Variables u, x and y are such that $u = 2x(y - x)$ and $x + 3y = 12$. Express u in terms of x and hence find the stationary value of u. [5]

119. [9709/s15/12/q6]

A tourist attraction in a city centre is a big vertical wheel on which passengers can ride. The wheel turns in such a way that the height, h m, of a passenger above the ground is given by the formula $h = 60(1 - \cos kt)$. In this formula, k is a constant, t is the time in minutes that has elapsed since the passenger started the ride at ground level and kt is measured in radians.

(i) Find the greatest height of the passenger above the ground. [1]

One complete revolution of the wheel takes 30 minutes.

(ii) Show that $k = \frac{1}{15}\pi$. [2]

(iii) Find the time for which the passenger is above a height of 90 m. [3]

120. [9709/s15/13/q8]

The function f is defined by $f(x) = \frac{1}{x+1} + \frac{1}{(x+1)^2}$ for $x > -1$.

(i) Find $f'(x)$. [3]

(ii) State, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

The function g is defined by $g(x) = \frac{1}{x+1} + \frac{1}{(x+1)^2}$ for $x < -1$.

(iii) Find the coordinates of the stationary point on the curve $y = g(x)$. [4]

121. [9709/w15/11/q5]

A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

122. [9709/w15/12/q3]

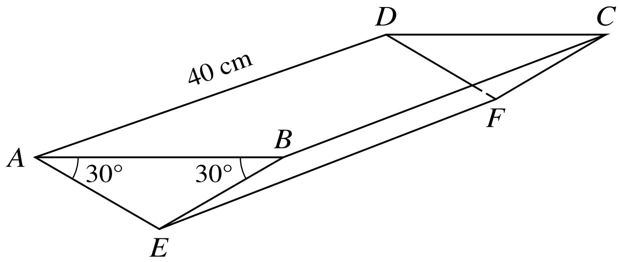


Fig. 1

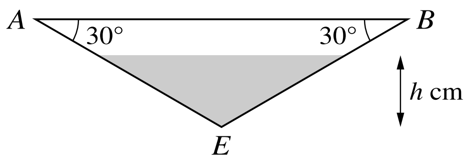


Fig. 2

Fig. 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are identical isosceles triangles. Angle ABE = angle $BAE = 30°$. The length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal. Water is poured into the tank at a constant rate of $200\,\text{cm}^3\,\text{s}^{-1}$. The depth of water, t seconds after filling starts, is h cm (see Fig. 2).

(i) Show that, when the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by $V = (40\sqrt{3})h^2$. [3]

(ii) Find the rate at which h is increasing when $h = 5$. [3]

123. [9709/w15/12/q9.23]

The curve $y = f(x)$ has a stationary point at $(2,\ 10)$ and it is given that $f''(x) = \dfrac{12}{x^3}$.

(i) Find $f(x)$. [6]

(ii) Find the coordinates of the other stationary point. [2]

(iii) Find the nature of each of the stationary points. [2]

124. [9709/w15/13/q3]

(i) Express $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f, where $\mathrm{f}(x) = x^3 - 3x^2 + 7x - 8$, is defined for $x \in \mathbb{R}$. Find $\mathrm{f}'(x)$ and state, with a reason, whether f is an increasing function, a decreasing function or neither. [3]

125. [9709/w15/13/q9]

A curve passes through the point $A\,(4,\ 6)$ and is such that $\frac{dy}{dx} = 1 + 2x^{-\frac{1}{2}}$. A point P is moving along the curve in such a way that the x-coordinate of P is increasing at a constant rate of 3 units per minute.

(i) Find the rate at which the y-coordinate of P is increasing when P is at A. [3]

(ii) Find the equation of the curve. [3]

(iii) The tangent to the curve at A crosses the x-axis at B and the normal to the curve at A crosses the x-axis at C. Find the area of triangle ABC. [5]

Chapter 8

Integration

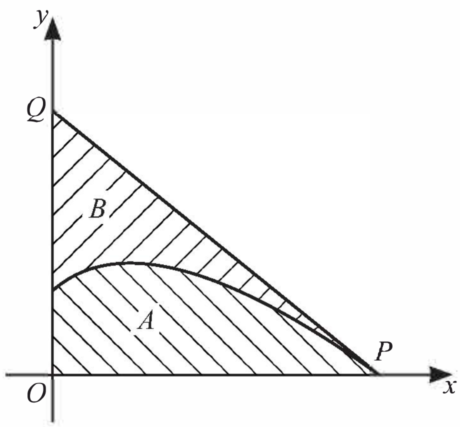
1. [9709/m25/12/q9]

A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{5}{x^3}$. It is given that the curve has a stationary point at $\left(\frac{1}{2}, 9\right)$.

(a) Use the expression for $\frac{d^2y}{dx^2}$ to determine whether the stationary point is a maximum or a minimum point. [2]

(b) Find the equation of the curve. [7]

2. [9709/m25/12/q10]



The diagram shows the curve with equation

$$y = 4(3x+4)^{\frac{1}{2}} - 2x - 6$$

for values of x such that $0 \leqslant x \leqslant 7$. The tangent to the curve at the point $P\ (7, 0)$ meets the y-axis at the point Q. Region A is bounded by the curve and the two axes. Region B is bounded by the curve, the line segment PQ and the y-axis.

(a) Find the area of region A. [4]

(b) Find the area of region B. [5]

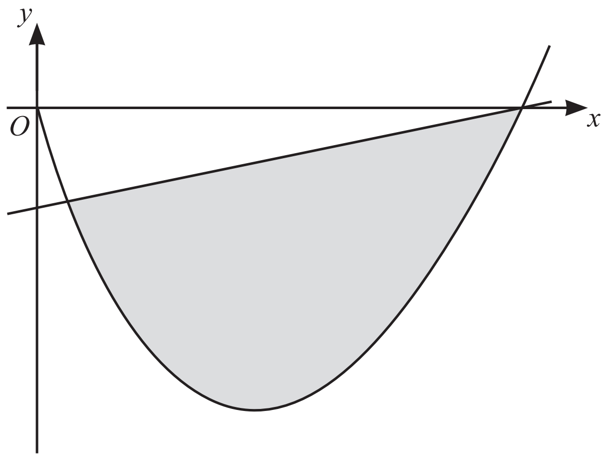
3. [9709/s25/11/q2]

The equation of a curve is such that $\frac{dy}{dx} = 4(2x-5)^3 - 9x^{\frac{1}{2}}$. The curve passes through the point $A\left(4, -\frac{11}{2}\right)$.

(a) Find the gradient of the normal to the curve at the point A. [2]

(b) Find the equation of the curve. [4]

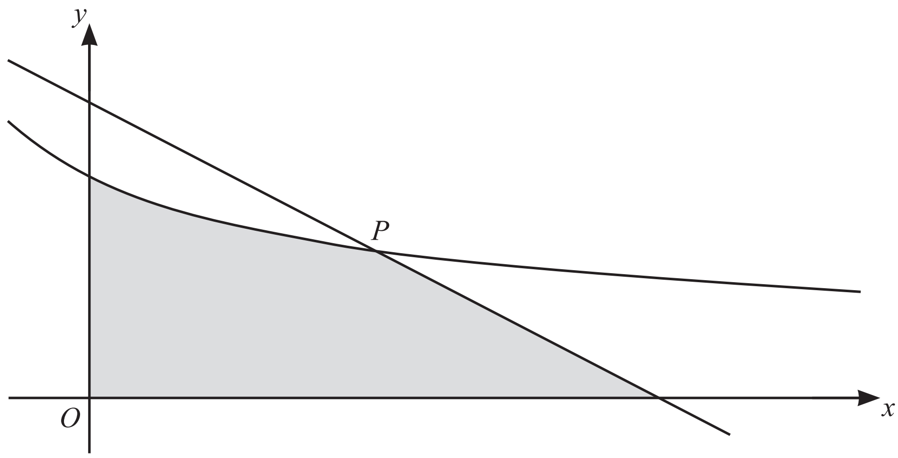
4. [9709/s25/11/q4]



The diagram shows the curve with equation $y = 5x^{\frac{3}{2}} - 20x$ and the line with equation $y = x - 16$. The x-coordinates of the points of intersection of the curve and line are 1 and 16.

Find the area of the shaded region between the curve and the line. [5]

5. [9709/s25/12/q6]



The diagram shows the curve with equation $y = \frac{9}{(5x+4)^{\frac{1}{2}}}$ and the line $y = 6 - 3x$. The line and the curve intersect at the point P which has y-coordinate 3.

Find the area of the shaded region. [6]

6. [9709/s25/12/q9]

The equation of a curve is such that $\frac{d^2y}{dx^2} = -\frac{24}{x^3}$. It is given that the curve has a stationary point at $(-2, 19)$.

(a) Find an expression for $\frac{dy}{dx}$. [3]

(b) Find the x-coordinate of the other stationary point of the curve, and determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

....

....

....

....

....

....

....

....

....

(d) Find the equation of the normal to the curve at the point where $\frac{dy}{dx} = -\frac{9}{4}$ and x is positive. Express your answer in the form $px + qy + r = 0$, where p, q and r are integers. [4]

7. [9709/s25/13/q3]

Given that $\int_1^3 \left(\frac{a}{(4x-3)^2} + 2\right) dx = 12$, find the value of the constant a. [4]

8. [9709/s25/13/q7]

A curve is such that $\frac{dy}{dx} = 3x^2 + 10x - 8$.

(a) Find the set of values of x for which y decreases as x increases. [3]

(b) It is given that the maximum point of the curve has y-coordinate 27.

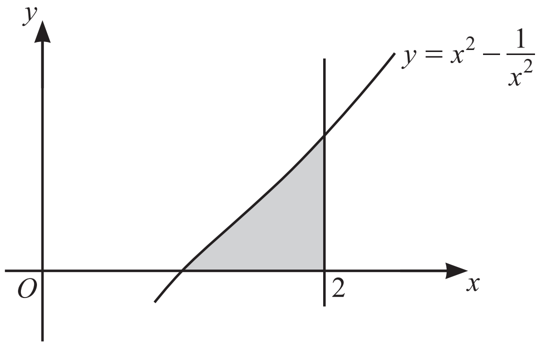
Find the equation of the curve. [4]

9. [9709/s25/15/q1]

The equation of a curve is such that $\frac{dy}{dx} = 12(2x-5)^2 + 8x$. It is given that the curve passes through the point (2, 4).

Find an equation of the curve. [4]

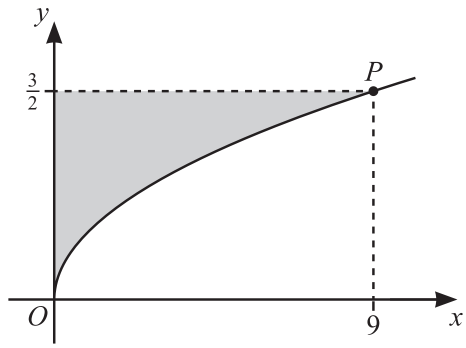
10. [9709/s25/15/q4]



The diagram shows part of the curve $y = x^2 - \frac{1}{x^2}$. The shaded region is bounded by the curve, the line $x = 2$ and the x-axis.

Find the volume formed when the shaded region is rotated through 360° about the x-axis, giving your answer correct to 2 decimal places. [5]

11. [9709/w25/11/q8]



The diagram shows the curve with equation $y = \frac{1}{2}\sqrt{x}$ and the point P with coordinates $\left(9, \frac{3}{2}\right)$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{3}{2}$.

(a) Find the area of the shaded region. [3]

(b) The shaded region is rotated through 360° about the ***y*-axis**.

Find the exact volume of the solid produced. [4]

12. [9709/w25/12/q4]

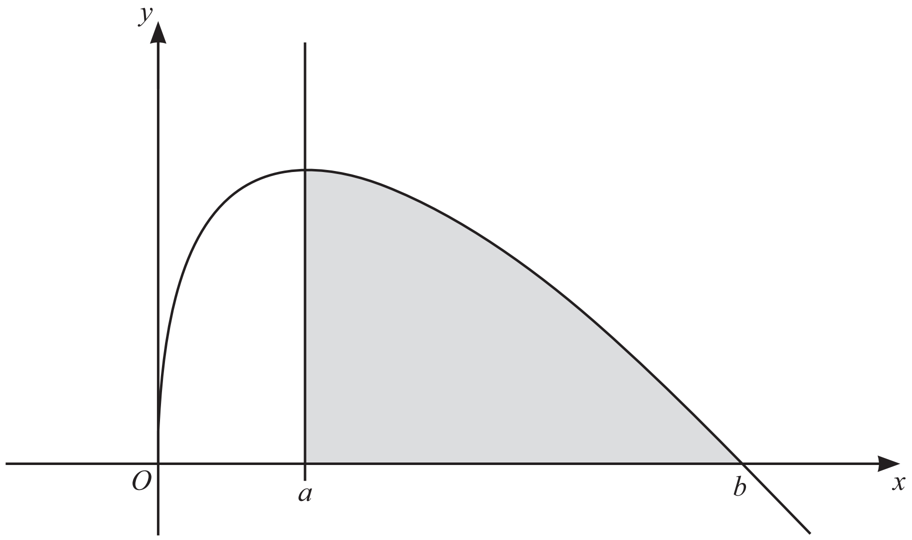
The equation of a curve is such that $\frac{dy}{dx} = kx^3 + \frac{2}{x^2}$, where k is a constant. The curve passes through the point S (2, 20) and the gradient of the curve at S is $\frac{65}{2}$.

(a) Find the value of k. [1]

(b) The coordinates of a point T on the curve are (1, t).

Find the value of t. [5]

13. [9709/w25/12/q5]

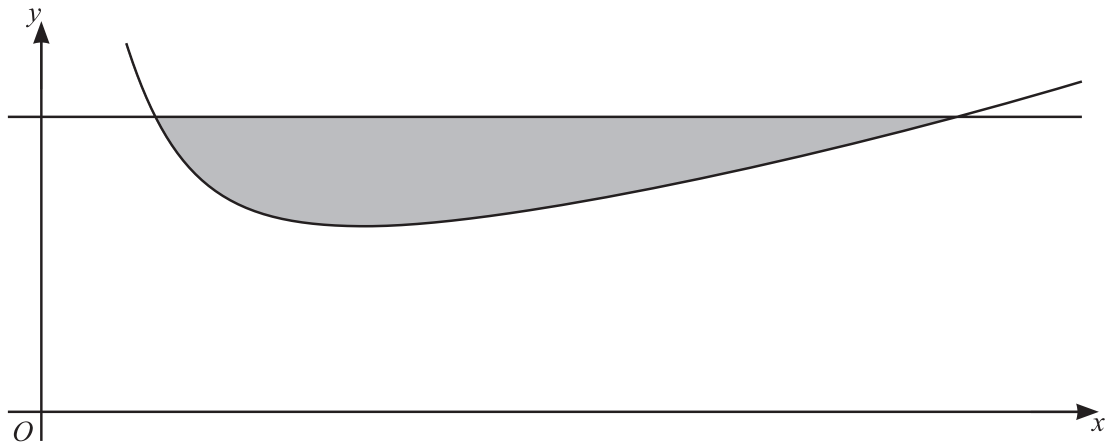


The equation of a curve is $y = 4x^{\frac{1}{2}} - x$. The curve has a maximum point when $x = a$ and crosses the x-axis at the point with coordinates $(b, 0)$, where $b > 0$. The shaded region is bounded by the curve, the line $x = a$ and the x-axis (see diagram).

(a) Find the value of a. [3]

(b) Find the exact area of the shaded region. [5]

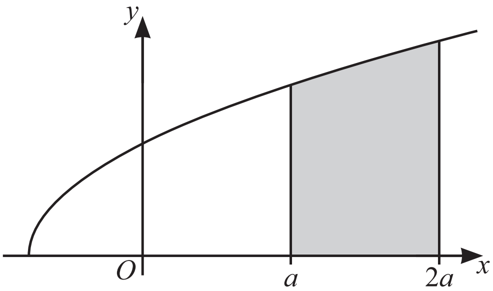
14. [9709/w25/13/q9]



The diagram shows part of the curve with equation $y = \frac{1}{2}x + \frac{4}{x}$ and the line $y = 4.5$.

Find the exact volume of the solid formed when the shaded region is rotated through 360° about the x-axis. [8]

15. [9709/w25/15/q8]



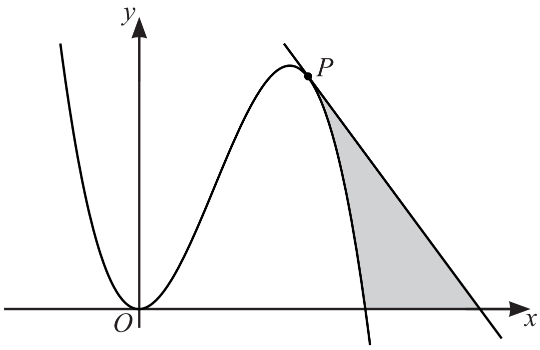
The diagram shows the curve with equation $y = \sqrt{6x+5}$. The shaded region is bounded by the curve, the x-axis and the lines $x = a$ and $x = 2a$, where a is a positive constant.

The shaded region is rotated through 360° about the x-axis to form a solid. The solid has volume, V, such that $V \geqslant 46\pi$.

(a) Show that $9a^2 + 5a - 46 \geqslant 0$. [4]

(b) Find the range of possible values of a. [3]

16. [9709/w25/15/q11]



The diagram shows the curve with equation $y = 4x^2 - x^3$ and the tangent to the curve at the point P. The point P has x-coordinate 3.

(a) Find the equation of the tangent to the curve at the point P. Give your answer in the form $y = mx + c$. [5]

....

(b) The shaded region is bounded by the curve, the x-axis and the tangent to the curve at P.

Find the exact area of the shaded region. [6]

The graph of $y = 4x^2 - x^3$ is transformed by a stretch of scale factor $\frac{1}{3}$ in the x-direction. The point Q is the image of P under this transformation. The transformed shaded region is bounded by the transformed curve, the x-axis and the tangent to the transformed curve at Q.

(c) **(i)** Find the equation of the transformed curve in the form $y = mx^2 + nx^3$, where m and n are integers to be found. [1]

(c) **(ii)** State the coordinates of Q and the area of the transformed shaded region. [2]

17. [9709/m24/12/q1]

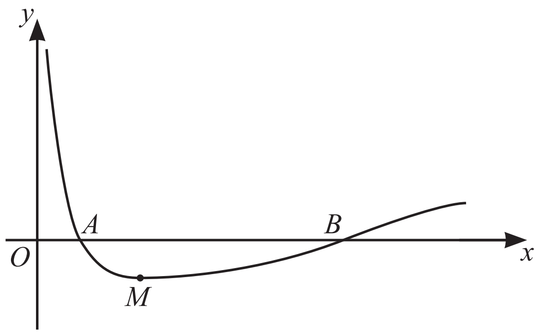
Find the exact value of $\int_3^{\infty} \frac{2}{x^2}\,dx$. [3]

18. [9709/m24/12/q3]

A curve is such that $\frac{dy}{dx} = 3(4x+5)^{\frac{1}{2}}$. It is given that the points $(1, 9)$ and $(5, a)$ lie on the curve.

Find the value of a. [5]

19. [9709/m24/12/q11]

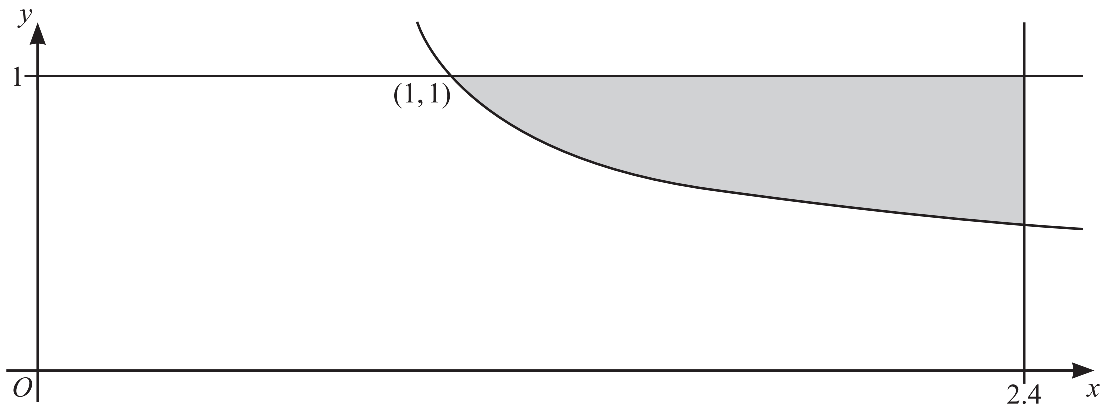


The diagram shows the curve with equation $y = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$ for $x > 0$. The curve crosses the x-axis at points A and B and has a minimum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the area of the region bounded by the curve and the line segment AB. [7]

20. [9709/s24/11/q9]



The diagram shows part of the curve with equation $y = \dfrac{1}{(5x-4)^{\frac{1}{3}}}$ and the lines $x = 2.4$ and $y = 1$. The curve intersects the line $y = 1$ at the point $(1, 1)$.

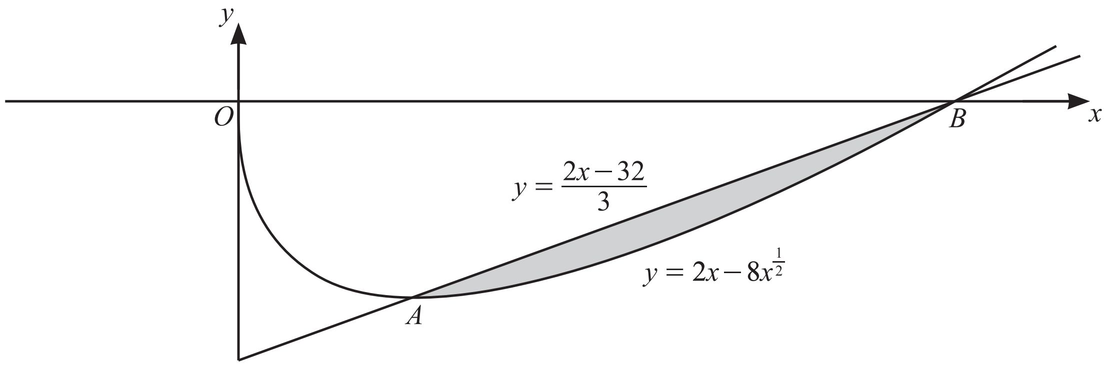
Find the exact volume of the solid generated when the shaded region is rotated through 360° about the x-axis. [6]

21. [9709/s24/12/q6]

The curve with equation $y = 2x - 8x^{\frac{1}{2}}$ has a minimum point at A and intersects the positive x-axis at B.

(a) Find the coordinates of A and B. [4]

(b)



The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line AB. It is given that the equation of AB is $y = \dfrac{2x - 32}{3}$.

Find the area of the shaded region between the curve and the line. [5]

22. [9709/s24/12/q9]

A function f is such that $f'(x) = 6(2x-3)^2 - 6x$ for $x \in \mathbb{R}$.

(a) Determine the set of values of x for which $f(x)$ is decreasing. [4]

(b) Given that $f(1) = -1$, find $f(x)$. [4]

23. [9709/s24/13/q6]

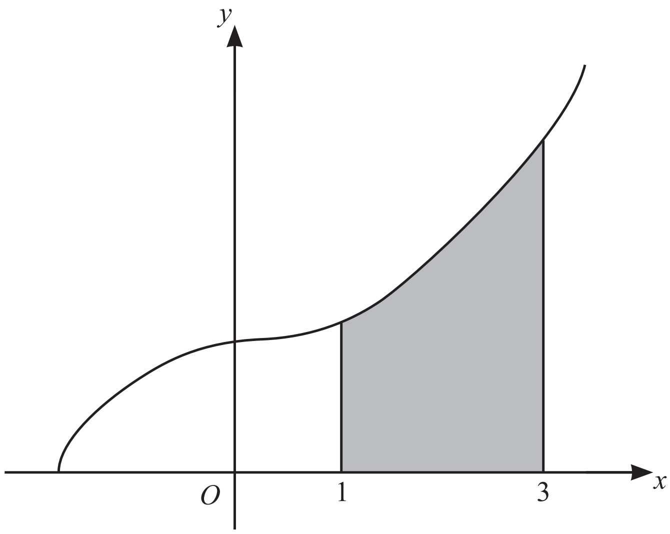
A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{dy}{dx} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

24. [9709/s24/13/q9]



The diagram shows the curve with equation $y = \sqrt{2x^3 + 10}$.

(a) Find the equation of the tangent to the curve at the point where $x = 3$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [5]

(b) The region shaded in the diagram is enclosed by the curve and the straight lines $x = 1$, $x = 3$ and $y = 0$.

Find the volume of the solid obtained when the shaded region is rotated through 360° about the x-axis. [3]

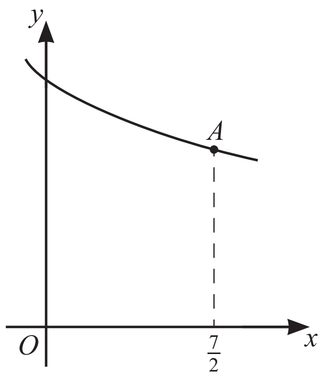
25. [9709/w24/11/q5]

The equation of a curve is such that $\frac{dy}{dx} = 4x - 3\sqrt{x} + 1$.

(a) Find the x-coordinate of the point on the curve at which the gradient is $\frac{11}{2}$. [3]

(b) Given that the curve passes through the point (4, 11), find the equation of the curve. [4]

26. [9709/w24/11/q7]



The diagram shows part of the curve with equation $y = \dfrac{12}{\sqrt[3]{2x+1}}$. The point A on the curve has coordinates $\left(\frac{7}{2}, 6\right)$.

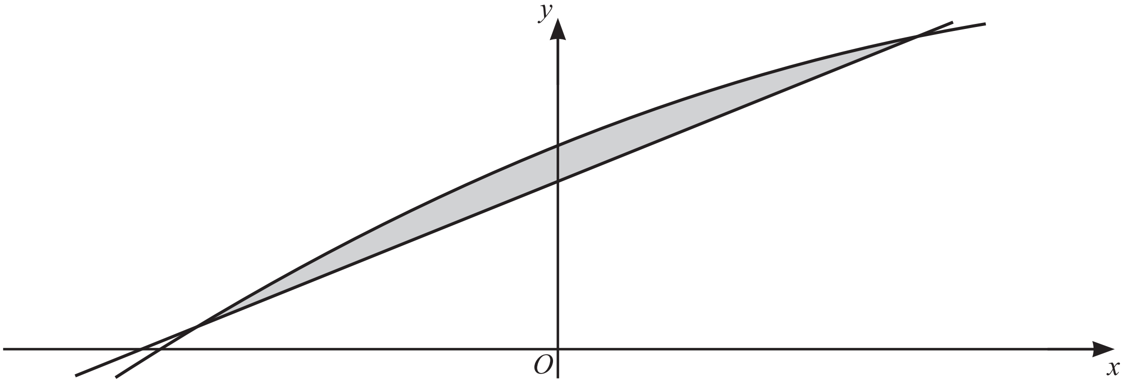
(a) Find the equation of the tangent to the curve at A. Give your answer in the form $y = mx + c$. [4]

(b) Find the area of the region bounded by the curve and the lines $x = 0$, $x = \frac{7}{2}$ and $y = 0$. [4]

27. [9709/w24/12/q7]

(a) By expressing $-2x^2+8x+11$ in the form $-a(x-b)^2+c$, where a, b and c are positive integers, find the coordinates of the vertex of the graph with equation $y=-2x^2+8x+11$. [3]

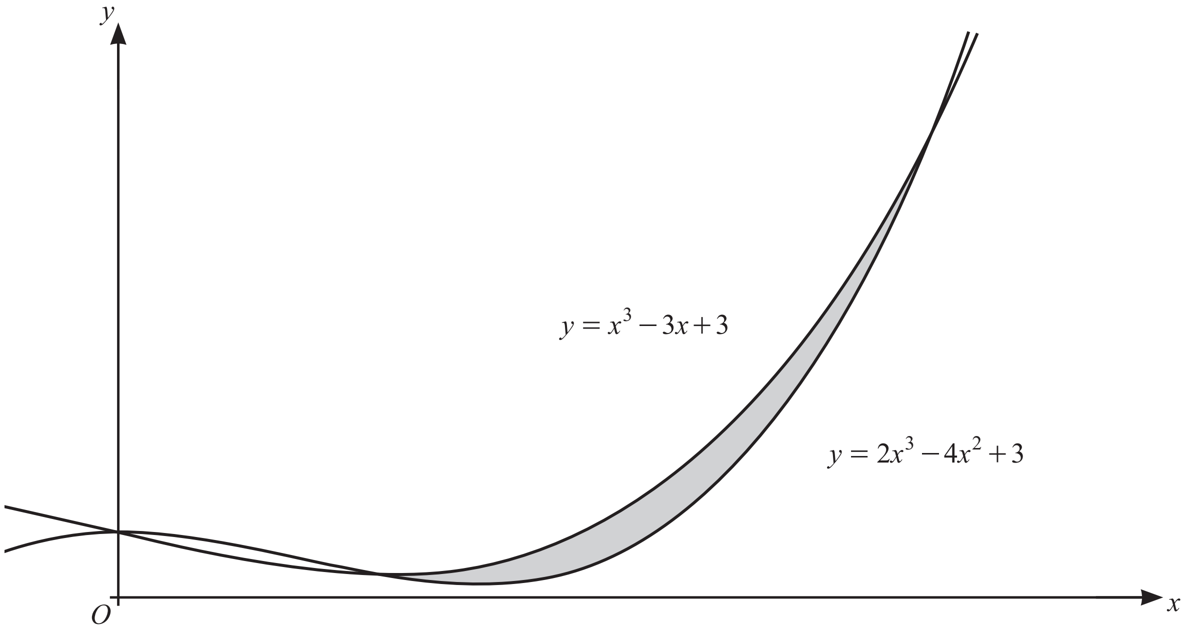
(b)



The diagram shows part of the curve with equation $y=-2x^2+8x+11$ and the line with equation $y=8x+9$.

Find the area of the shaded region. [5]

28. [9709/w24/13/q9]



The diagram shows the curves with equations $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3$.

(a) Find the x-coordinates of the points of intersection of the curves. [3]

(b) Find the area of the shaded region. [4]

29. [9709/m23/12/q10]

At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

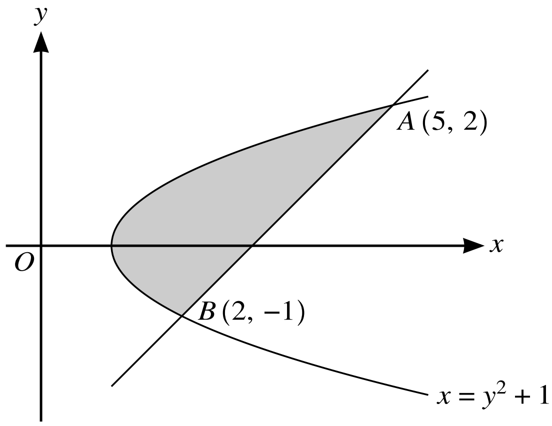
(a) Show that $k = -2$. [1]

(b) Find the equation of the curve. [4]

(c) Find the coordinates of the stationary point. [3]

(d) Determine the nature of the stationary point. [2]

30. [9709/m23/12/q11]

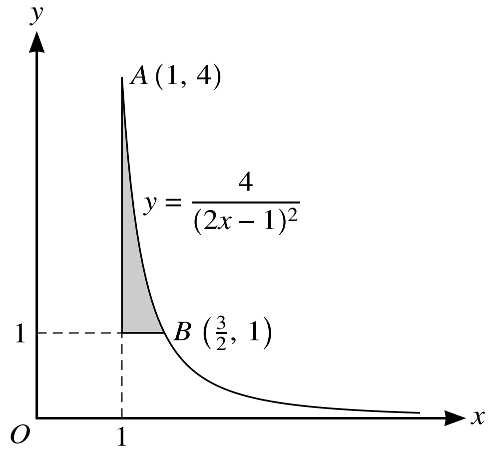


The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5,\ 2)$ and $B\,(2,\ -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the ***y*-axis**. [9]

31. [9709/s23/11/q10]



The diagram shows part of the curve with equation $y = \dfrac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1, 4)$ and $B, \left(\frac{3}{2}, 1\right)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

32. [9709/s23/11/q11]

The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

(b) Determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

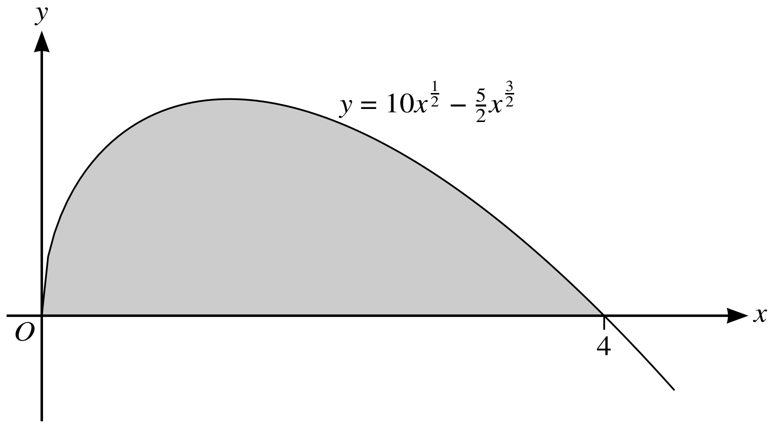
(d) Find the coordinates of any other stationary points on the curve. [2]

33. [9709/s23/12/q1]

The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

34. [9709/s23/12/q5]



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$.

Find the area of the shaded region. [4]

35. [9709/s23/13/q9]

A curve which passes through (0, 3) has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 1 - \dfrac{2}{(x-1)^3}$.

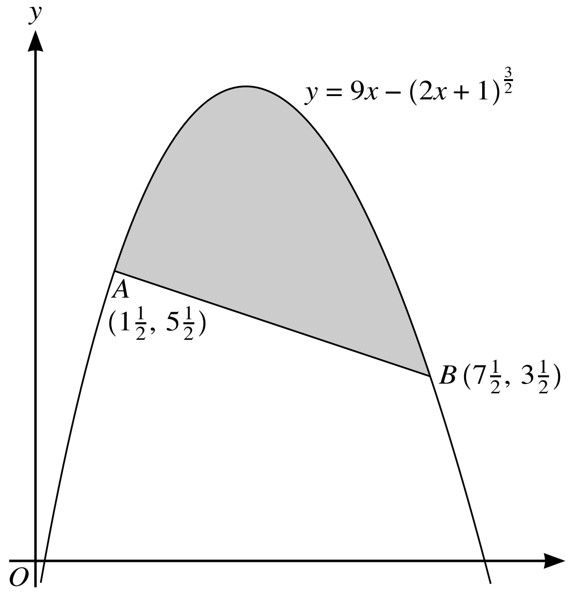
(a) Find the equation of the curve. [4]

The tangent to the curve at(0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation($2x + 1)(x - 1)^2 - 1 = 0$. [4]

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

36. [9709/s23/13/q10]



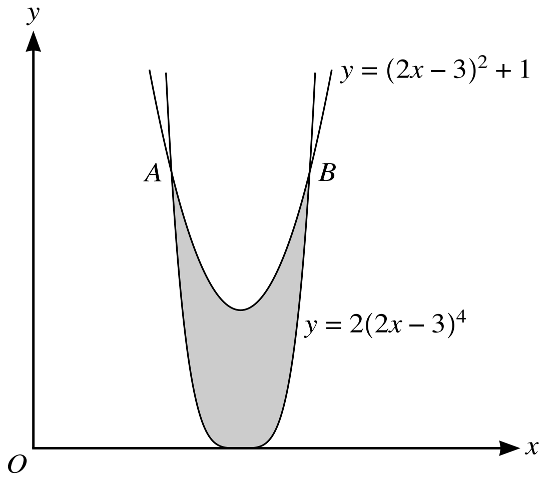
The diagram shows the points $A\left(1\frac{1}{2}, 5\frac{1}{2}\right)$ and $B\left(7\frac{1}{2}, 3\frac{1}{2}\right)$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

(b) Verify that the line AB is the normal to the curve at A. [3]

(c) Find the area of the shaded region. [5]

37. [9709/w23/11/q8]



The diagram shows the curves with equations $y = 2(2x-3)^4$ and $y = (2x-3)^2 + 1$ meeting at points A and B.

(a) By using the substitution $u = 2x - 3$ find, by calculation, the coordinates of A and B. [4]

(b) Find the exact area of the shaded region. [5]

38. [9709/w23/11/q10]

A curve has a stationary point at $(2, -10)$ and is such that $\frac{d^2y}{dx^2} = 6x$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Find the equation of the curve. [3]

(c) Find the coordinates of the other stationary point and determine its nature. [3]

(d) Find the equation of the tangent to the curve at the point where the curve crosses the y-axis. [2]

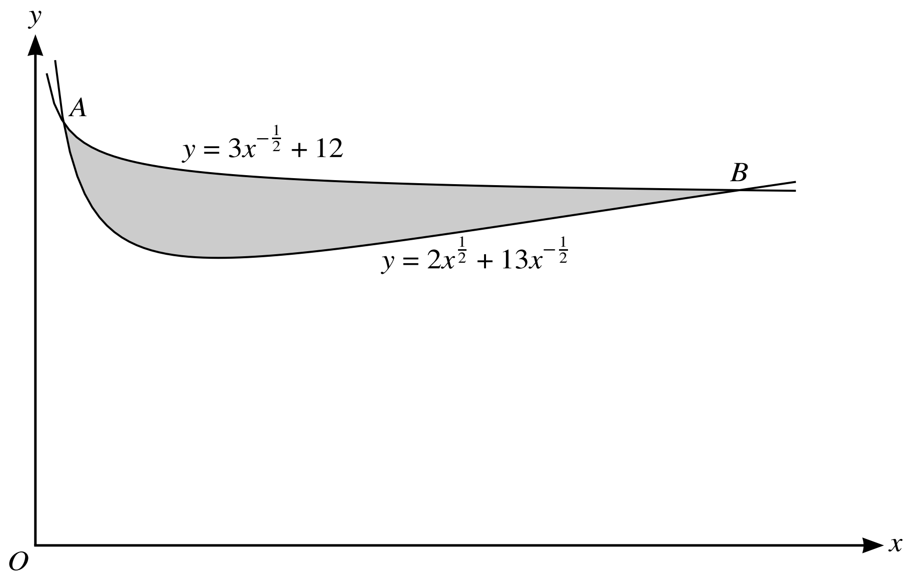
39. [9709/w23/12/q3]

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P. [2]

(b) Find the equation of the curve. [4]

40. [9709/w23/12/q9]



The diagram shows curves with equations $y = 2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}$ and $y = 3x^{-\frac{1}{2}} + 12$. The curves intersect at points A and B.

(a) Find the coordinates of A and B. [4]

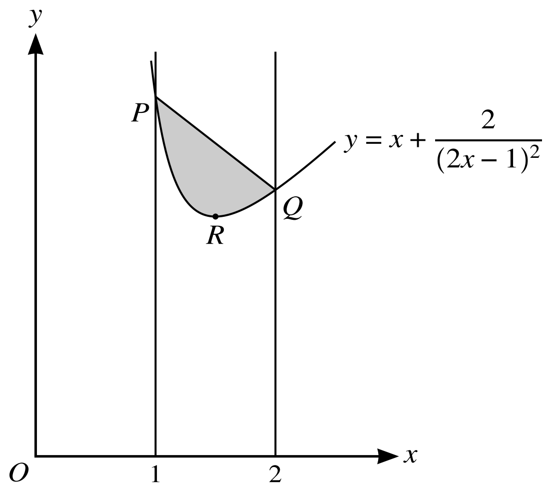
(b) Hence find the area of the shaded region. [5]

41. [9709/w23/13/q1]

A curve is such that its gradient at a point (x, y) is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve. [4]

42. [9709/w23/13/q11]



The diagram shows part of the curve with equation $y = x + \dfrac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at P and Q respectively and R is the stationary point on the curve.

(a) Verify that the x-coordinate of R is $\frac{3}{2}$ and find the y-coordinate of R. [4]

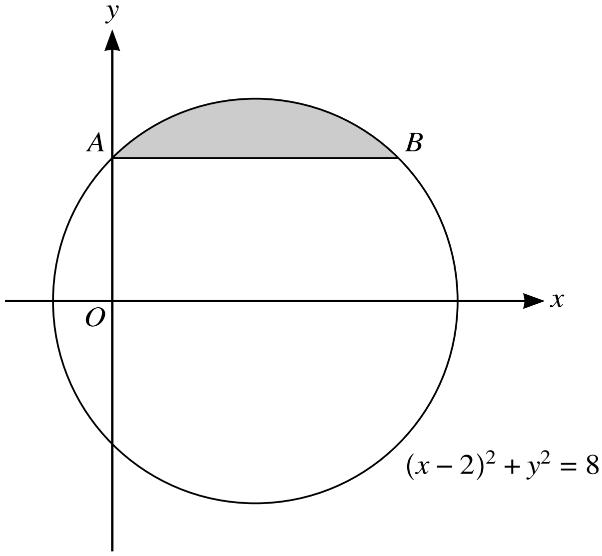
(b) Find the exact value of the area of the shaded region. [6]

43. [9709/m22/12/q1]

A curve with equation $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $\mathrm{f}(8) = 5$.

Find $\mathrm{f}(x)$. [4]

44. [9709/m22/12/q8]

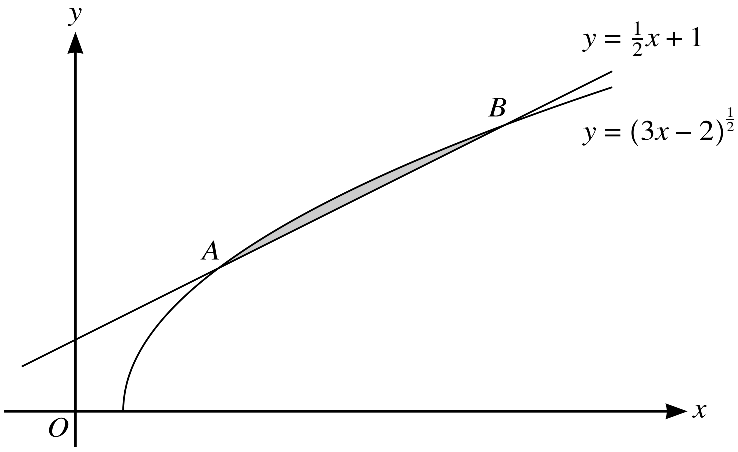


The diagram shows the circle with equation $(x-2)^2 + y^2 = 8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

45. [9709/s22/11/q7]



The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

(b) Hence find the area of the region enclosed between the curve and the line. [5]

46. [9709/s22/11/q10.b]

The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

(b) Find the equation of the curve. [5]

(c) Show that the curve has no other stationary points. [3]

(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

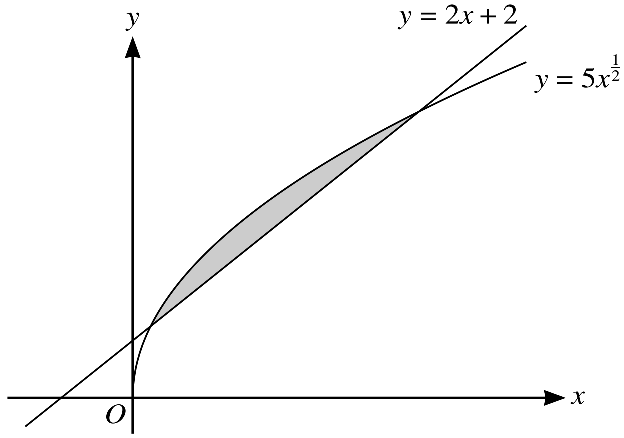
Find the rate of increase of the x-coordinate of A at the point where $x = 1$. [3]

47. [9709/s22/12/q3]

The equation of a curve is such that $\frac{dy}{dx} = 3(4x-7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $\left(4, \frac{5}{2}\right)$.

Find the equation of the curve. [4]

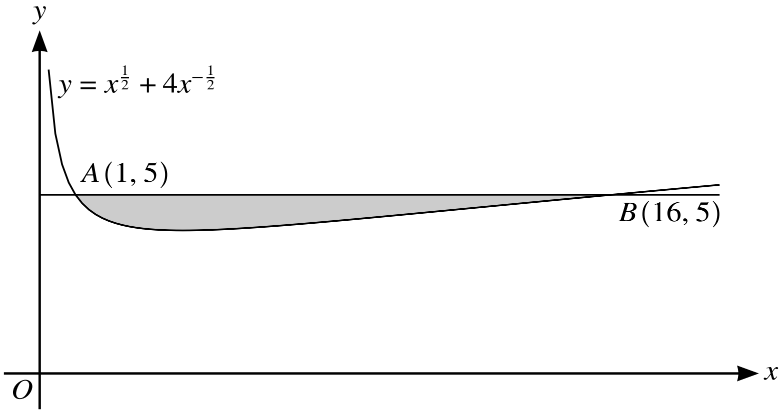
48. [9709/s22/12/q6]



The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

49. [9709/s22/13/q8]



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A(1, 5)$ and $B(16, 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

(b) Calculate the area of the shaded region. [4]

50. [9709/s22/13/q10.a]

The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} f(x)\,dx$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

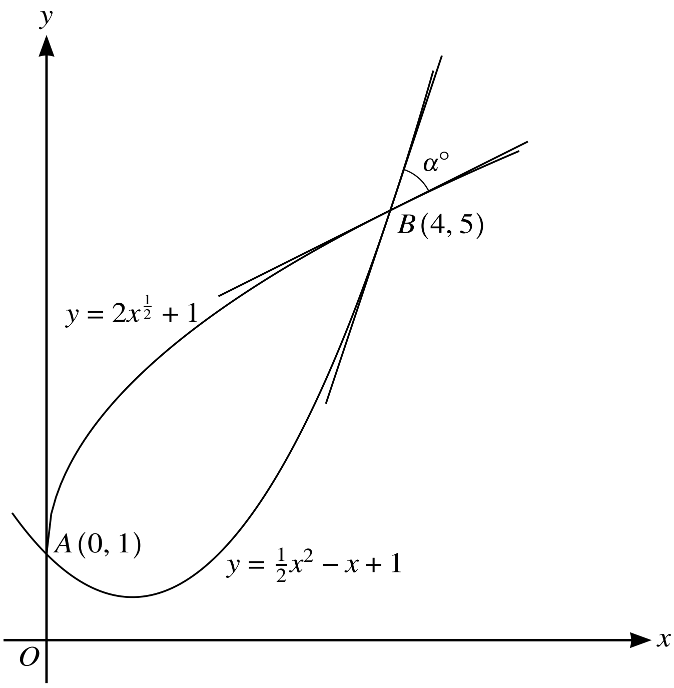
51. [9709/w22/11/q2]

The equation of a curve is such that $\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P\,(6, 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

(b) Find the equation of the curve. [4]

52. [9709/w22/11/q10]



Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

(b) Find α. [5]

53. [9709/w22/12/q8]

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

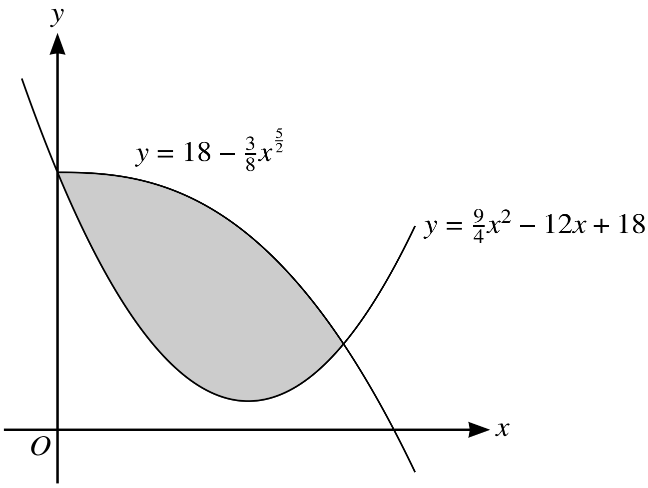
(a) Find the equation of the curve. [4]

(b) Find the x-coordinate of the stationary point. [2]

(c) State the set of values of x for which y increases as x increases. [1]

54. [9709/w22/12/q11.b]

(a) Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]



The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region. [5]

(c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

55. [9709/w22/13/q7]

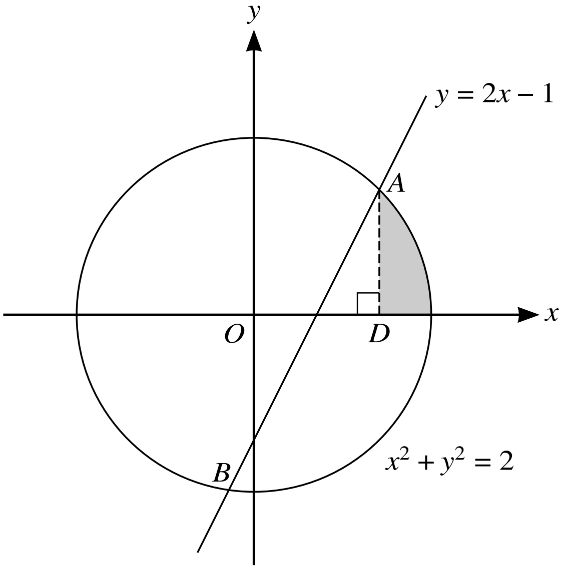
The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = \dfrac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. [4]

(b) Find $\mathrm{f}(x)$ given that the curve passes through the point $(-1, 5)$. [3]

56. [9709/w22/13/q10.b]



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

(c) Find an exact expression for the perimeter of the shaded region. [2]

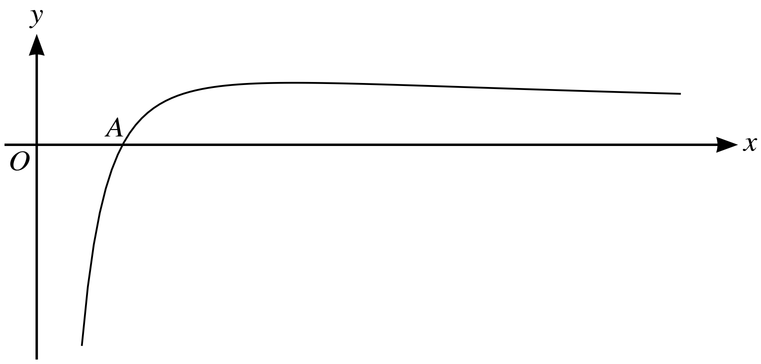
57. [9709/m21/12/q6.b]

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and A (1, −3) lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

58. [9709/m21/12/q11.d]



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

(b) Find the equation of the tangent to the curve at A. [4]

(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

59. [9709/s21/11/q1]

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $\left(\frac{1}{2}, 4\right)$.

Find the equation of the curve. [4]

60. [9709/s21/11/q11.d]

The equation of a curve is $y = 2\sqrt{3x + 4} - x$.

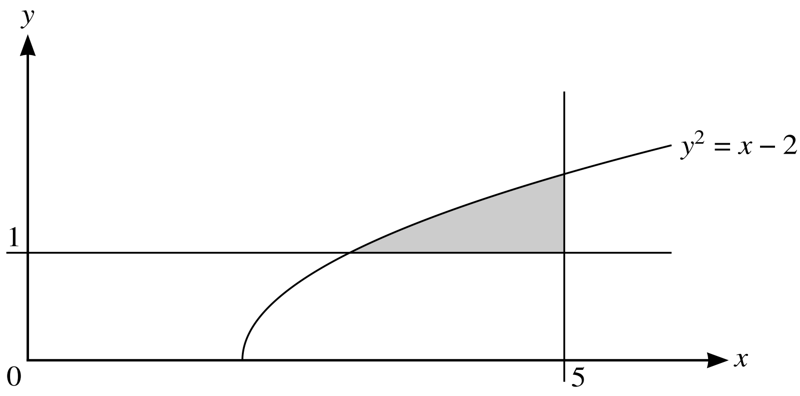
(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

61. [9709/s21/12/q9]



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

62. [9709/s21/12/q11.b]

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

(b) Find the equation of the curve. [4]

(c) Find $\frac{d^2y}{dx^2}$. [2]

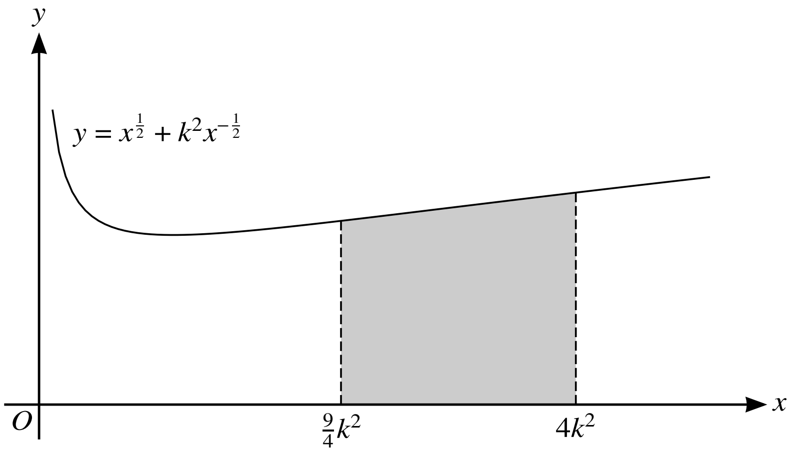
(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

63. [9709/s21/13/q1]

A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

64. [9709/s21/13/q11.c]



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

65. [9709/w21/11/q9.a]

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

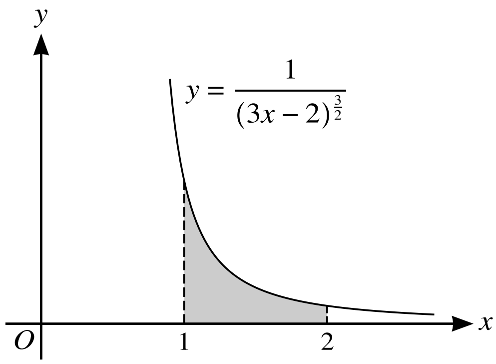
(b) Find the coordinates of the stationary points on the curve. [5]

(c) Find $f''(x)$. [1]

(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

66. [9709/w21/11/q10]

(a) Find $\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]



The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

The normal to the curve at the point $(1, 1)$ crosses the y-axis at the point A.

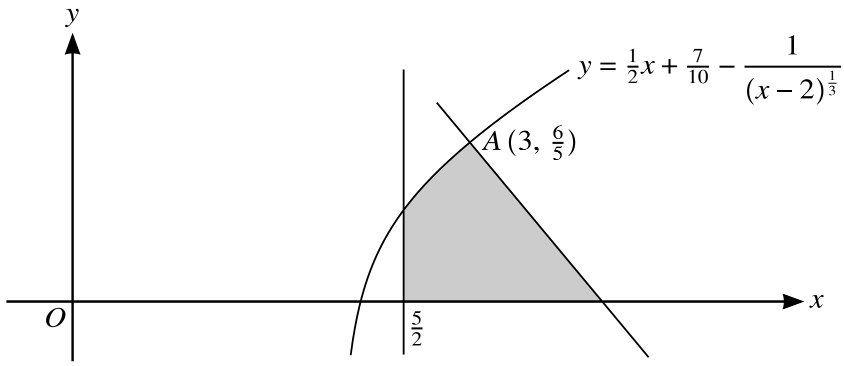
(c) Find the y-coordinate of A. [4]

67. [9709/w21/12/q4]

A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $\left(2,\ 5\frac{2}{3}\right)$.

Find the equation of the curve. [4]

68. [9709/w21/12/q11]

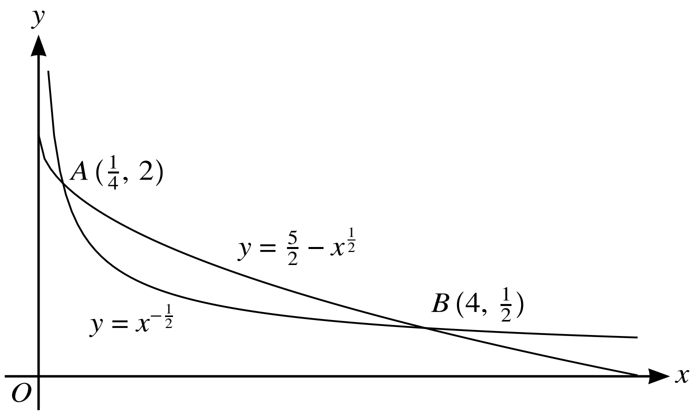


The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \dfrac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\left(3, \frac{6}{5}\right)$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

69. [9709/w21/13/q8]



The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4},\ 2\right)$ and $B\left(4,\ \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1,\ 1)$ intersects the y-axis at the point $(0,\ p)$.

Find the value of p. [4]

70. [9709/w21/13/q10.b]

A curve has equation $y = \mathrm{f}(x)$ and it is given that

$$\mathrm{f}'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

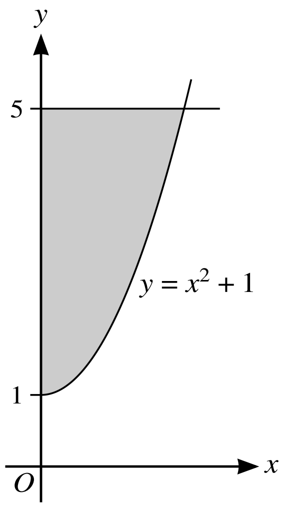
(a) Find $\mathrm{f}''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

It is now given that $k = -3$ and the minimum point is at $(2,\ 3\tfrac{1}{2})$.

(b) Find $\mathrm{f}(x)$. [4]

(c) Find the coordinates of the other stationary point and determine its nature. [4]

71. [9709/m20/12/q3]



The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

72. [9709/m20/12/q10.c]

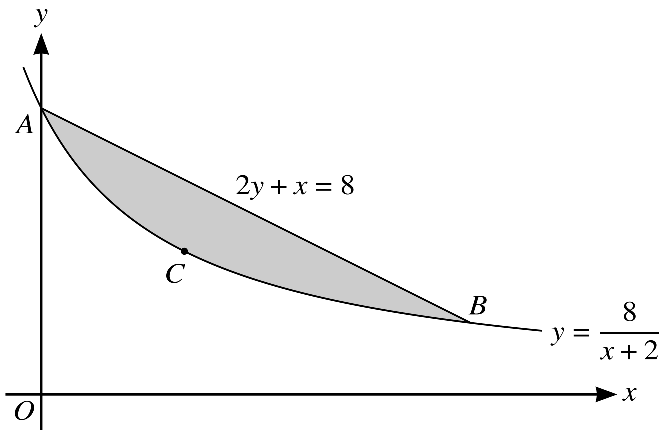
The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

(b) Determine the nature of the stationary point. [3]

(c) Find the equation of the curve. [4]

73. [9709/s20/11/q11]

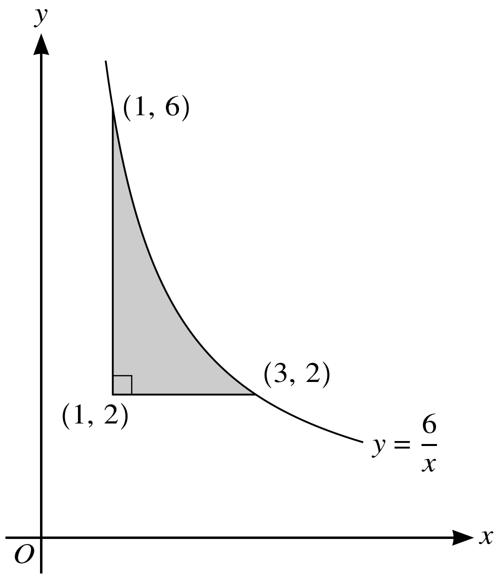


The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

74. [9709/s20/12/q8]



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

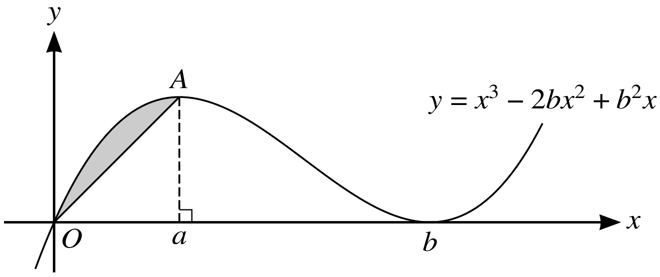
(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

75. [9709/s20/13/q2]

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point (4, 7) lies on the curve.

Find the equation of the curve. [4]

76. [9709/s20/13/q11.b]



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

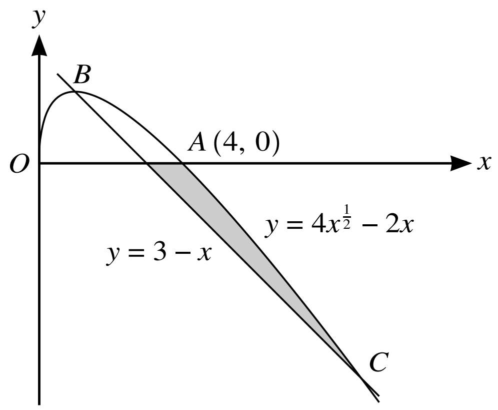
(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

77. [9709/w20/11/q2]

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

78. [9709/w20/11/q12]



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4,\ 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

(b) Show that B is a stationary point on the curve. [2]

(c) Find the area of the shaded region. [6]

79. [9709/w20/12/q7.b]

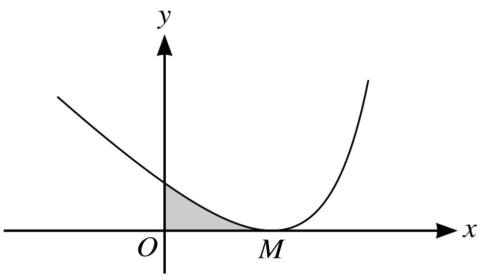
The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

(b) Find the equation of the curve. [4]

80. [9709/w20/12/q10]



The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

(b) Find, by calculation, the x-coordinate of M. [2]

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

81. [9709/w20/13/q2]

The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^\infty f(x)\,dx$. [3]

(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

82. [9709/w20/13/q10.b]

A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

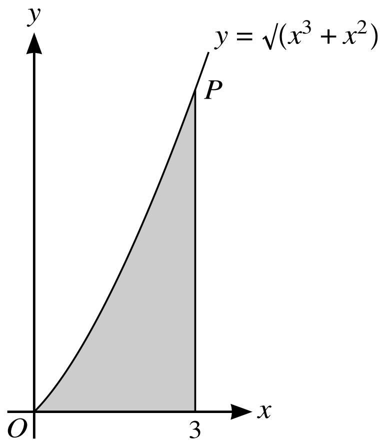
(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}\right) \mathrm{d}x = \frac{13}{12}$.

Find the value of k. [5]

83. [9709/m19/12/q2]

A curve with equation $y = \mathrm{f}(x)$ passes through the points (0, 2) and (3, −1). It is given that $\mathrm{f}'(x) = kx^2 - 2x$, where k is a constant. Find the value of k. [5]

84. [9709/m19/12/q9.a]



The diagram shows part of the curve with equation $y = \sqrt{(x^3 + x^2)}$. The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

(ii) P is the point on the curve with x-coordinate 3. Find the y-coordinate of the point where the normal to the curve at P crosses the y-axis. [6]

85. [9709/s19/11/q10]

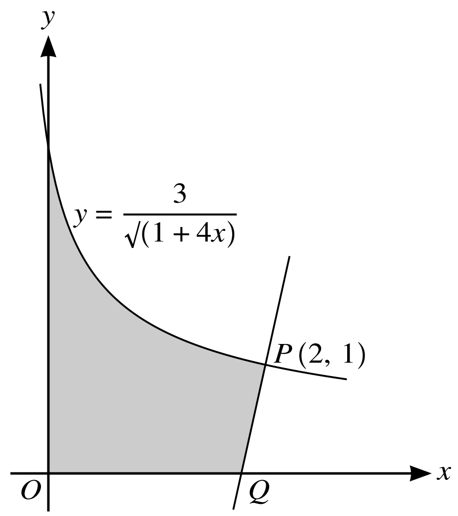
A curve for which $\frac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

(ii) Find the x-coordinate of the other stationary point on the curve. [1]

(iii) Determine the nature of each of the stationary points. [2]

86. [9709/s19/11/q11]

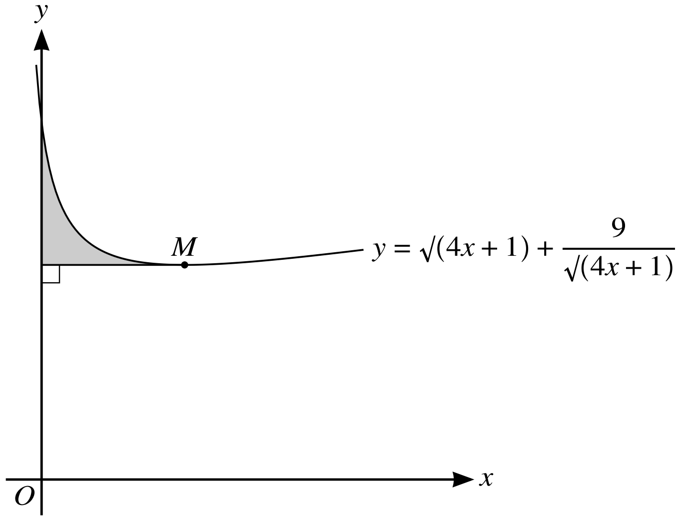


The diagram shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$ and a point $P\,(2,\ 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

87. [9709/s19/12/q11]



The diagram shows part of the curve $y = \sqrt{(4x+1)} + \dfrac{9}{\sqrt{(4x+1)}}$ and the minimum point M.

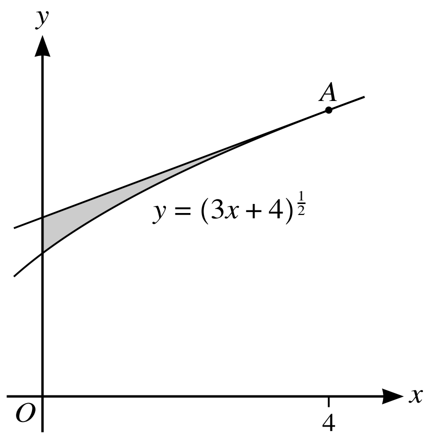
(i) Find expressions for $\dfrac{dy}{dx}$ and $\int y\,dx$. [6]

(ii) Find the coordinates of M. [3]

The shaded region is bounded by the curve, the y-axis and the line through M parallel to the x-axis.

(iii) Find, showing all necessary working, the area of the shaded region. [3]

88. [9709/s19/13/q10]



The diagram shows part of the curve with equation $y = (3x + 4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

(iii) A point is moving along the curve. At the point P the y-coordinate is increasing at half the rate at which the x-coordinate is increasing. Find the x-coordinate of P. [3]

89. [9709/w19/11/q9]

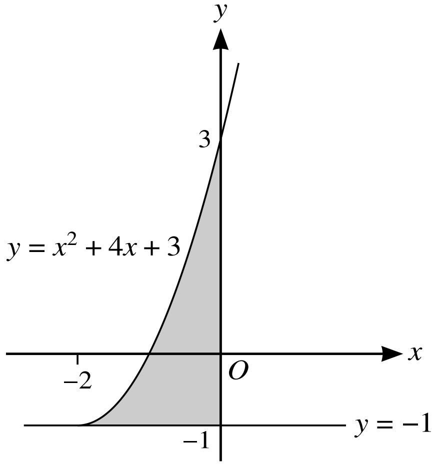
A curve for which $\frac{dy}{dx} = (5x - 1)^{\frac{1}{2}} - 2$ passes through the point (2, 3).

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

90. [9709/w19/11/q11]



The diagram shows a shaded region bounded by the y-axis, the line $y = -1$ and the part of the curve $y = x^2 + 4x + 3$ for which $x \geqslant -2$.

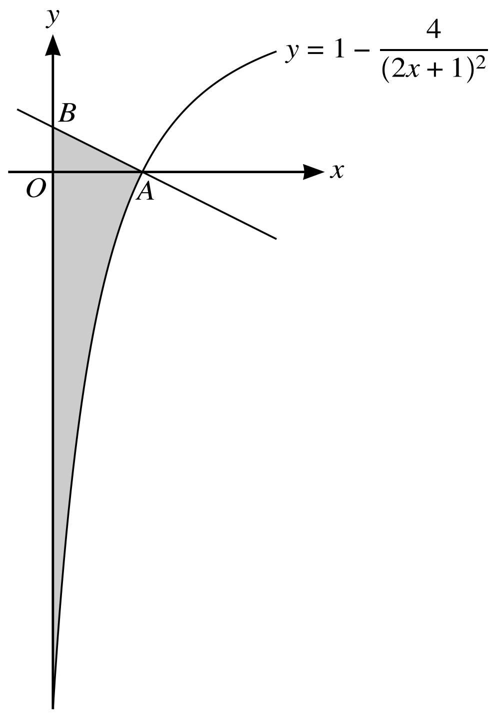
(i) Express $y = x^2 + 4x + 3$ in the form $y = (x + a)^2 + b$, where a and b are constants. Hence, for $x \geqslant -2$, express x in terms of y. [4]

(ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [6]

91. [9709/w19/12/q3]

A curve is such that $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$, where k is a constant. The points $P(1, -1)$ and $Q(4, 4)$ lie on the curve. Find the equation of the curve. [4]

92. [9709/w19/12/q10]



The diagram shows part of the curve $y = 1 - \dfrac{4}{(2x+1)^2}$. The curve intersects the x-axis at A. The normal to the curve at A intersects the y-axis at B.

(i) Obtain expressions for $\dfrac{dy}{dx}$ and $\int y\,dx$. [4]

(ii) Find the coordinates of B. [4]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

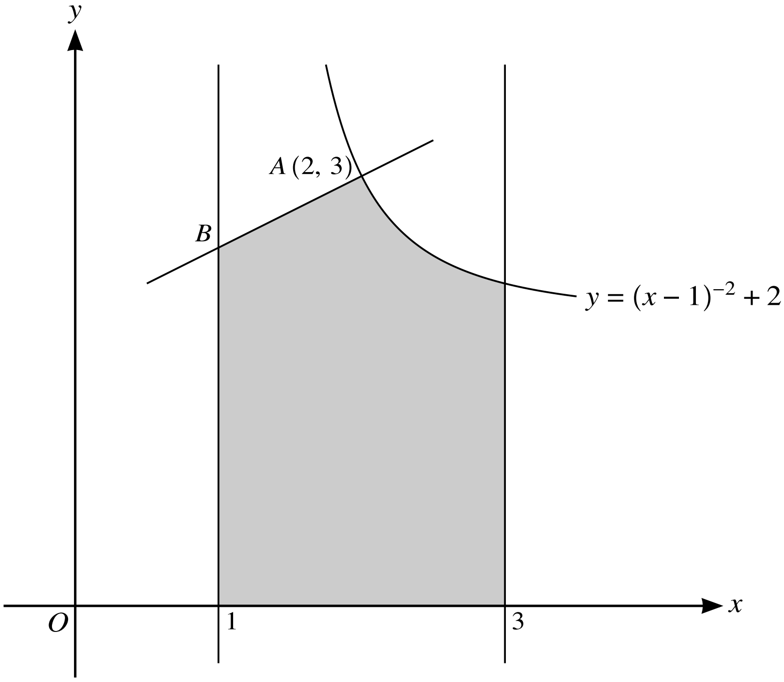
93. [9709/w19/13/q8]

A function f is defined for $x > \frac{1}{2}$ and is such that $f'(x) = 3(2x-1)^{\frac{1}{2}} - 6$.

(i) Find the set of values of x for which f is decreasing. [4]

(ii) It is now given that $f(1) = -3$. Find $f(x)$. [4]

94. [9709/w19/13/q11]



The diagram shows part of the curve $y = (x-1)^{-2} + 2$, and the lines $x = 1$ and $x = 3$. The point A on the curve has coordinates $(2, 3)$. The normal to the curve at A crosses the line $x = 1$ at B.

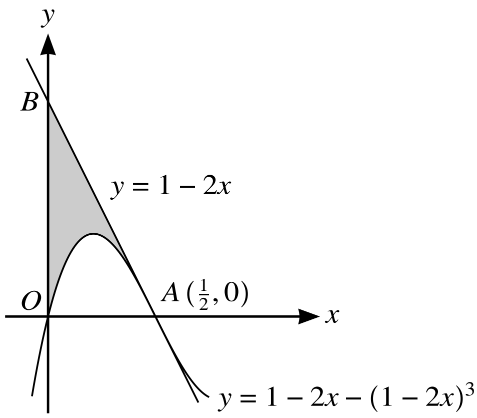
(i) Show that the normal AB has equation $y = \frac{1}{2}x + 2$. [3]

(ii) Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [8]

95. [9709/m18/12/q1]

A curve passes through the point (4, −6) and has an equation for which $\frac{dy}{dx} = x^{-\frac{1}{2}} - 3$. Find the equation of the curve. [4]

96. [9709/m18/12/q11]



The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1 - 2x)^3 \, dx$. [2]

(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

97. [9709/s18/12/q9]

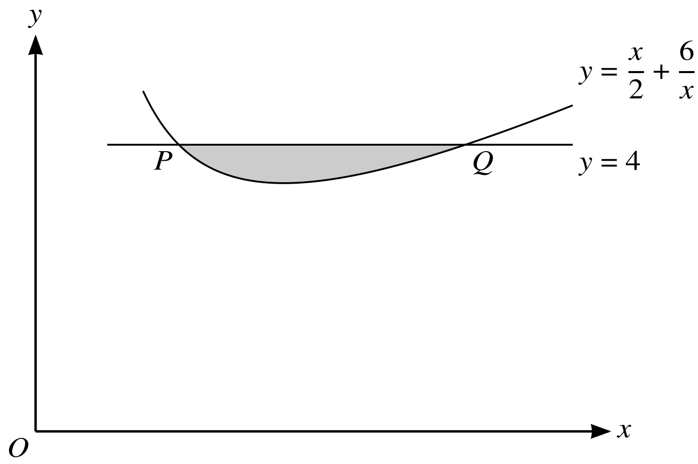
A curve is such that $\frac{dy}{dx} = \sqrt{(4x + 1)}$ and (2, 5) is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

98. [9709/s18/12/q11]

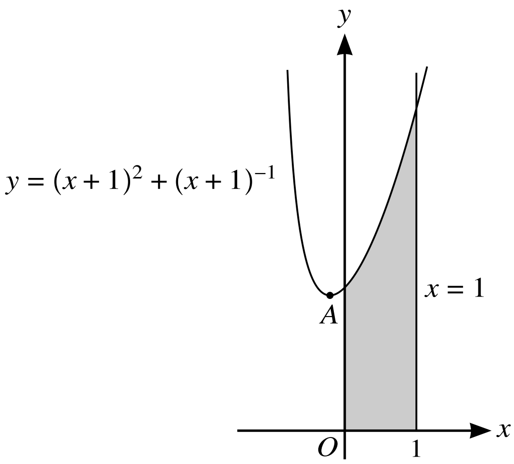


The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

99. [9709/s18/13/q11]



The diagram shows part of the curve $y = (x + 1)^2 + (x + 1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x + 1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

100. [9709/w18/11/q6]

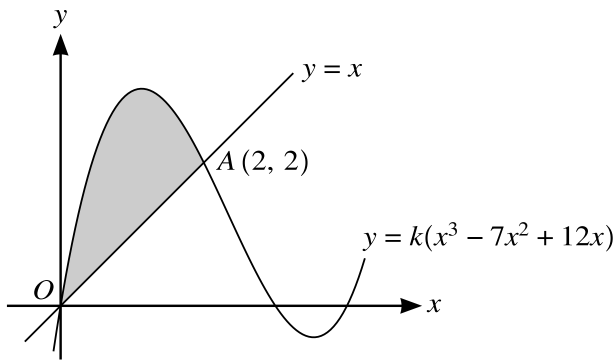
A curve has a stationary point at $(3, 9\frac{1}{2})$ and has an equation for which $\frac{dy}{dx} = ax^2 + a^2x$, where a is a non-zero constant.

(i) Find the value of a. [2]

(ii) Find the equation of the curve. [4]

(iii) Determine, showing all necessary working, the nature of the stationary point. [2]

101. [9709/w18/11/q7]



The diagram shows part of the curve with equation $y = k(x^3 - 7x^2 + 12x)$ for some constant k. The curve intersects the line $y = x$ at the origin O and at the point $A\ (2,\ 2)$.

(i) Find the value of k. [1]

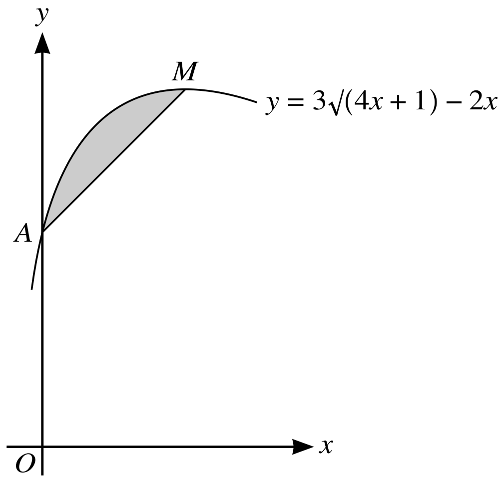
(ii) Verify that the curve meets the line $y = x$ again when $x = 5$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [5]

102. [9709/w18/12/q2]

Showing all necessary working, find $\displaystyle\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) \mathrm{d}x.$ [4]

103. [9709/w18/12/q11]



The diagram shows part of the curve $y = 3\sqrt{(4x + 1)} - 2x$. The curve crosses the y-axis at A and the stationary point on the curve is M.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [5]

(ii) Find the coordinates of M. [3]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

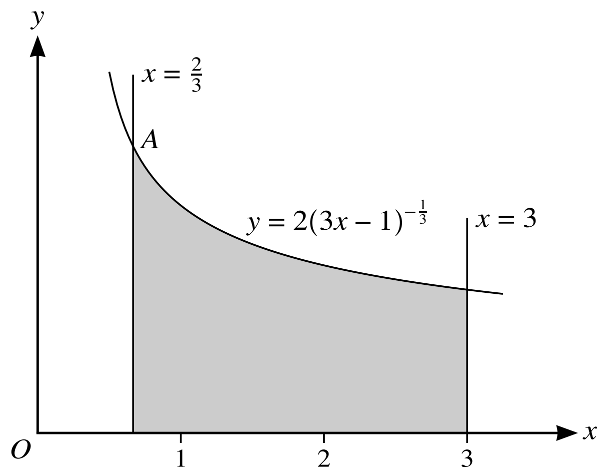
104. [9709/w18/13/q8]

A curve passes through (0, 11) and has an equation for which $\frac{dy}{dx} = ax^2 + bx - 4$, where a and b are constants.

(i) Find the equation of the curve in terms of a and b. [3]

(ii) It is now given that the curve has a stationary point at (2, 3). Find the values of a and b. [5]

105. [9709/w18/13/q10]

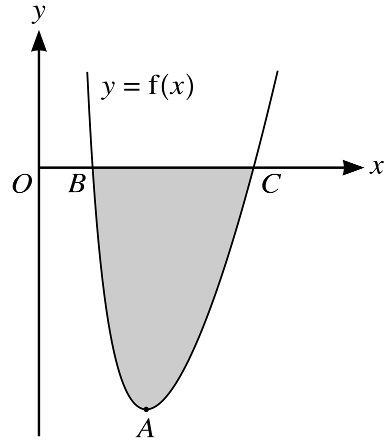


The diagram shows part of the curve $y = 2(3x - 1)^{-\frac{1}{3}}$ and the lines $x = \frac{2}{3}$ and $x = 3$. The curve and the line $x = \frac{2}{3}$ intersect at the point A.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [5]

(ii) Find the equation of the normal to the curve at A, giving your answer in the form $y = mx + c$. [5]

106. [9709/m17/12/q10]



The diagram shows the curve $y = f(x)$ defined for $x > 0$. The curve has a minimum point at A and crosses the x-axis at B and C. It is given that $\frac{dy}{dx} = 2x - \frac{2}{x^3}$ and that the curve passes through the point $\left(4, \frac{189}{16}\right)$.

(i) Find the x-coordinate of A. [2]

(ii) Find $f(x)$. [3]

(iii) Find the x-coordinates of B and C. [4]

(iv) Find, showing all necessary working, the area of the shaded region. [4]

107. [9709/s17/11/q7]

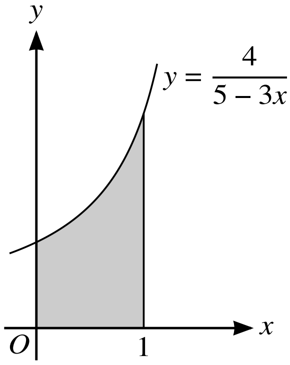
A curve for which $\frac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve. [3]

(ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

(iii) Find the set of values of x for which the gradient of the curve is positive. [3]

108. [9709/s17/11/q10]



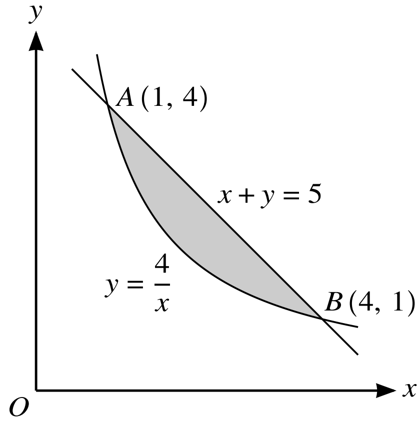
The diagram shows part of the curve $y = \dfrac{4}{5-3x}$.

(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

The shaded region is bounded by the curve, the coordinate axes and the line $x = 1$.

(ii) Find, showing all necessary working, the volume obtained when this shaded region is rotated through 360° about the x-axis. [5]

109. [9709/s17/12/q6]



The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \dfrac{4}{x}$ at the points $A\,(1,\ 4)$ and $B\,(4,\ 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [7]

110. [9709/s17/13/q10]

(a)

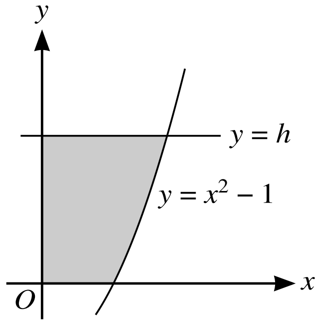


Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

(i) The shaded region is rotated through 360° about the ***y*-axis**. Show that the volume of revolution, V, is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

(ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

(b)

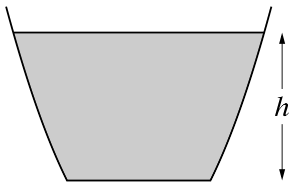


Fig. 2

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, $V\,\text{cm}^3$, of water is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. Water is poured into the bowl at a constant rate of $2\,\text{cm}^3\,\text{s}^{-1}$. Find the rate, in $\text{cm}\,\text{s}^{-1}$, at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

111. [9709/s17/13/q11]

The function f is defined for $x \geqslant 0$. It is given that f has a minimum value when $x = 2$ and that $f''(x) = (4x + 1)^{-\frac{1}{2}}$.

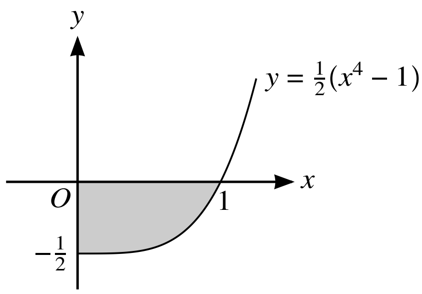
(i) Find $f'(x)$. [3]

It is now given that $f''(0)$, $f'(0)$ and $f(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $f(0)$. [3]

(iii) Find $f(x)$, and hence find the minimum value of f. [5]

112. [9709/w17/11/q10]



The diagram shows part of the curve $y = \frac{1}{2}(x^4 - 1)$, defined for $x \geqslant 0$.

(i) Find, showing all necessary working, the area of the shaded region. [3]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

(iii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the y-axis. [5]

113. [9709/w17/12/q8]

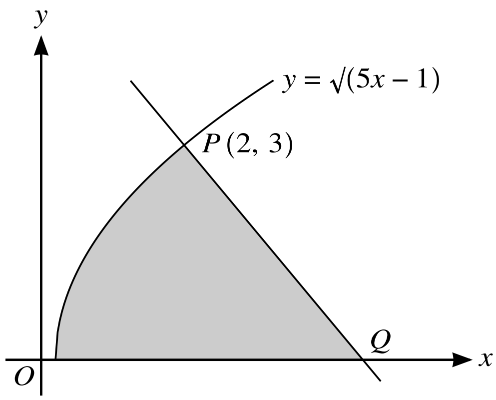
A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

(iii) Given that the curve passes through the point (6, 2), find the equation of the curve. [4]

114. [9709/w17/12/q10]

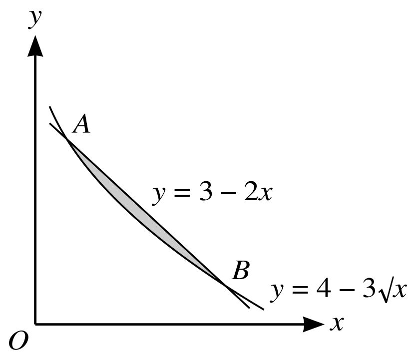


The diagram shows part of the curve $y = \sqrt{(5x - 1)}$ and the normal to the curve at the point $P\,(2, 3)$. This normal meets the x-axis at Q.

(i) Find the equation of the normal at P. [4]

(ii) Find, showing all necessary working, the area of the shaded region. [7]

115. [9709/w17/13/q8]



The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B.

(i) Find by calculation the x-coordinates of A and B. [3]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

116. [9709/w17/13/q10]

A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = ax^2 + bx$, where a and b are positive constants.

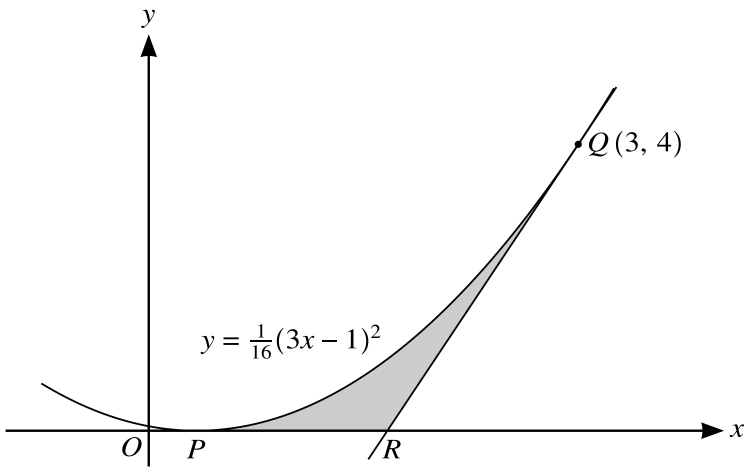
(i) Find, in terms of a and b, the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

(ii) It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $\mathrm{f}(x)$. [6]

117. [9709/m16/12/q2]

A curve for which $\frac{dy}{dx} = 3x^2 - \frac{2}{x^3}$ passes through $(-1, 3)$. Find the equation of the curve. [4]

118. [9709/m16/12/q10.3]



The diagram shows part of the curve $y = \frac{1}{16}(3x-1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

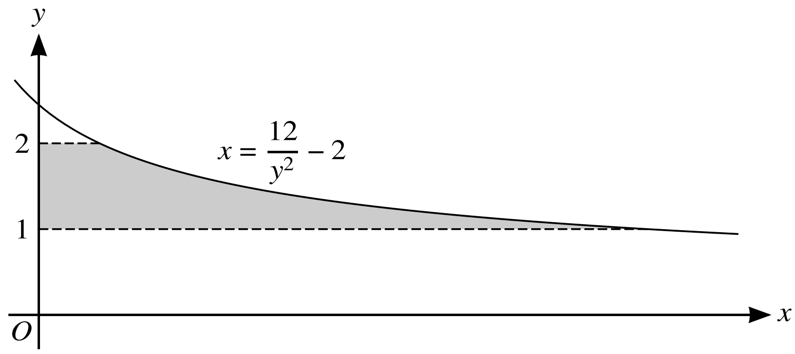
(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

119. [9709/s16/11/q3]



The diagram shows part of the curve $x = \frac{12}{y^2} - 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 2$. Showing all necessary working, find the volume, in terms of π, when this shaded region is rotated through 360° about the y-axis. [5]

120. [9709/s16/11/q4]

A curve is such that $\frac{dy}{dx} = 2 - 8(3x+4)^{-\frac{1}{2}}$.

(i) A point P moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the y-coordinate as P crosses the y-axis. [2]

The curve intersects the y-axis where $y = \frac{4}{3}$.

(ii) Find the equation of the curve. [4]

121. [9709/s16/12/q2]

A curve is such that $\frac{dy}{dx} = \frac{8}{(5-2x)^2}$. Given that the curve passes through $(2, 7)$, find the equation of the curve. [4]

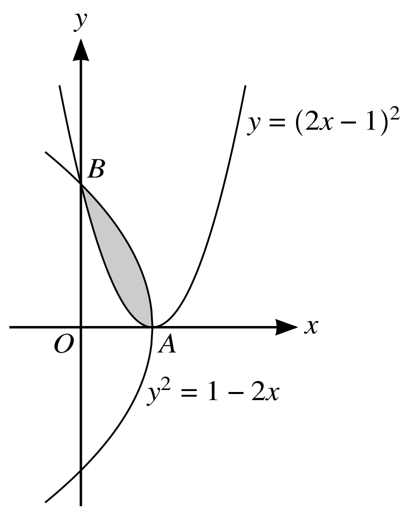
122. [9709/s16/13/q11]

Triangle ABC has vertices at $A\,(-2, -1)$, $B\,(4, 6)$ and $C\,(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

123. [9709/w16/11/q7]



The diagram shows parts of the curves $y = (2x-1)^2$ and $y^2 = 1 - 2x$, intersecting at points A and B.

(i) State the coordinates of A. [1]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

124. [9709/w16/11/q10]

A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$. The point A is the only point on the curve at which the gradient is -1.

(i) Find the x-coordinate of A. [3]

(ii) Given that the curve also passes through the point $(4, 10)$, find the y-coordinate of A, giving your answer as a fraction. [6]

125. [9709/w16/12/q1]

A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{(4x + 1)}}$. The point (2, 5) lies on the curve. Find the equation of the curve. [4]

126. [9709/w16/12/q10]

A function f is defined by $f : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Solve the equation $f(x) = 6$, giving answers in terms of π. [3]

The function g is defined by $g : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $g^{-1}(x)$. [3]

127. [9709/w16/13/q2]

The coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^5$ is 100. Find the value of the constant a. [4]

128. [9709/w16/13/q3]

Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

129. [9709/w16/13/q10]

A curve is such that $\frac{dy}{dx} = \frac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A\,(a^2, 3)$ lies on the curve. Find, in terms of a,

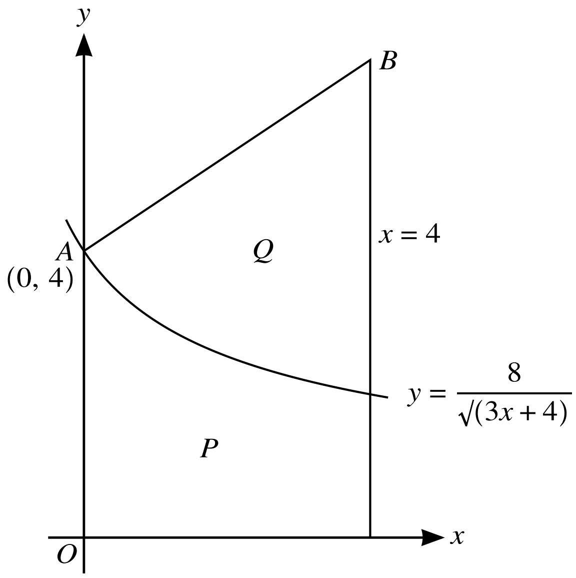
(i) the equation of the tangent to the curve at A, simplifying your answer, [3]

(ii) the equation of the curve. [4]

It is now given that $B\,(16, 8)$ also lies on the curve.

(iii) Find the value of a and, using this value, find the distance AB. [5]

130. [9709/s15/11/q10]



The diagram shows part of the curve $y = \dfrac{8}{\sqrt{(3x+4)}}$. The curve intersects the y-axis at A (0, 4). The normal to the curve at A intersects the line $x = 4$ at the point B.

(i) Find the coordinates of B. [5]

(ii) Show, with all necessary working, that the areas of the regions marked P and Q are equal. [6]

131. [9709/s15/12/q1]

The function f is such that $f'(x) = 5 - 2x^2$ and $(3,\ 5)$ is a point on the curve $y = f(x)$. Find $f(x)$. [3]

132. [9709/s15/12/q10]

The equation of a curve is $y = \dfrac{4}{2x - 1}$.

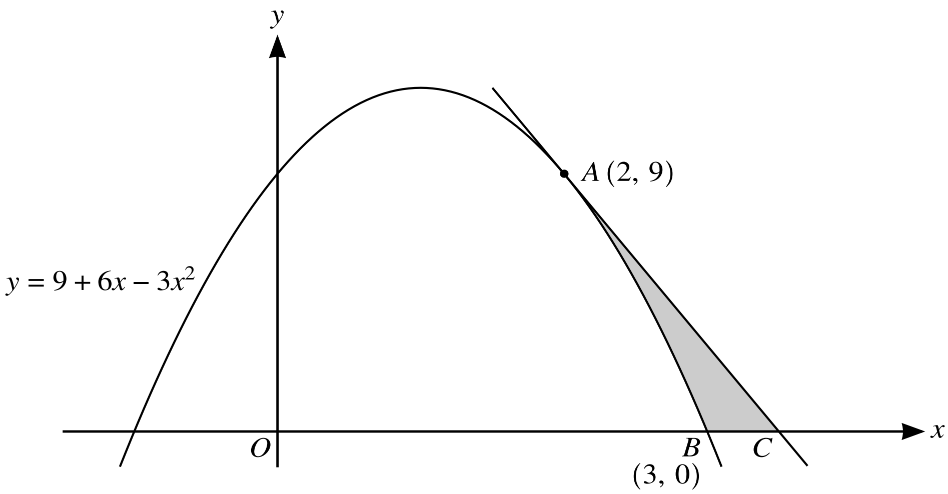
(i) Find, showing all necessary working, the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [4]

(ii) Given that the line $2y = x + c$ is a normal to the curve, find the possible values of the constant c. [6]

133. [9709/s15/13/q2]

A curve is such that $\frac{dy}{dx} = (2x + 1)^{\frac{1}{2}}$ and the point (4, 7) lies on the curve. Find the equation of the curve. [4]

134. [9709/s15/13/q10]



Points $A\,(2,\ 9)$ and $B\,(3,\ 0)$ lie on the curve $y = 9 + 6x - 3x^2$, as shown in the diagram. The tangent at A intersects the x-axis at C. Showing all necessary working,

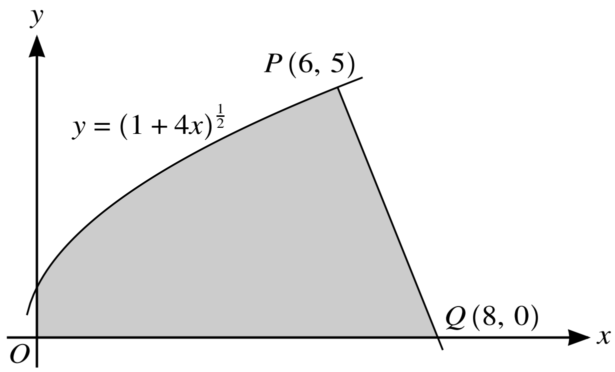
(i) find the equation of the tangent AC and hence find the x-coordinate of C, [4]

(ii) find the area of the shaded region ABC. [5]

135. [9709/w15/11/q2]

The function f is such that $f'(x) = 3x^2 - 7$ and $f(3) = 5$. Find $f(x)$. [3]

136. [9709/w15/11/q11]



The diagram shows part of the curve $y = (1 + 4x)^{\frac{1}{2}}$ and a point $P\,(6,\ 5)$ lying on the curve. The line PQ intersects the x-axis at $Q\,(8,\ 0)$.

(i) Show that PQ is a normal to the curve. [5]

(ii) Find, showing all necessary working, the exact volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [7]

[In part **(ii)** you may find it useful to apply the fact that the volume, V, of a cone of base radius r and vertical height h, is given by $V = \frac{1}{3}\pi r^2 h$.]

137. [9709/w15/12/q9.1]

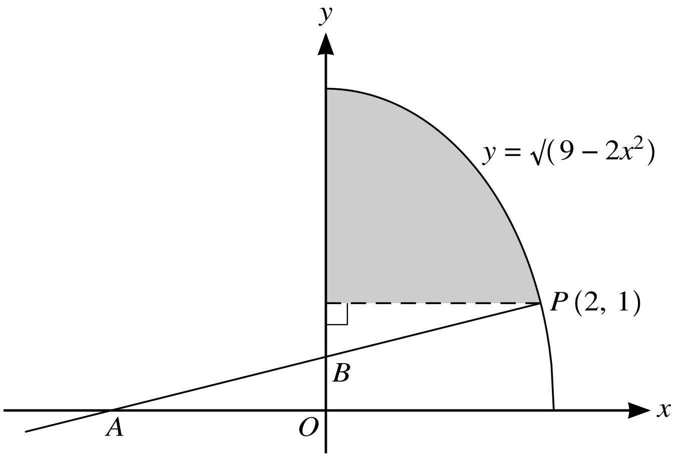
The curve $y = \mathrm{f}(x)$ has a stationary point at $(2,\ 10)$ and it is given that $\mathrm{f}''(x) = \dfrac{12}{x^3}$.

(i) Find $\mathrm{f}(x)$. [6]

(ii) Find the coordinates of the other stationary point. [2]

(iii) Find the nature of each of the stationary points. [2]

138. [9709/w15/12/q10]



The diagram shows part of the curve $y = \sqrt{(9 - 2x^2)}$. The point $P\,(2,\ 1)$ lies on the curve and the normal to the curve at P intersects the x-axis at A and the y-axis at B.

(i) Show that B is the mid-point of AP. [6]

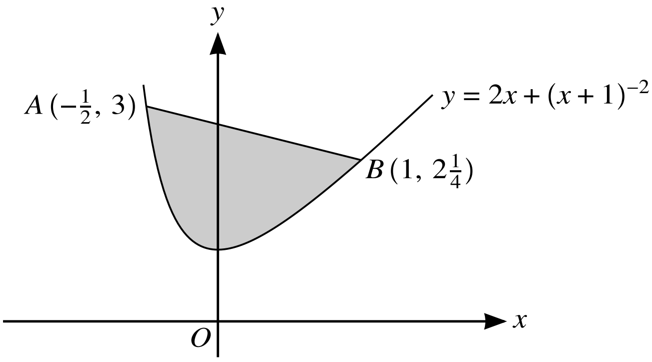
The shaded region is bounded by the curve, the y-axis and the line $y = 1$.

(ii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [5]

139. [9709/w15/13/q10]

The function f is defined by $f(x) = 2x + (x + 1)^{-2}$ for $x > -1$.

(i) Find $f'(x)$ and $f''(x)$ and hence verify that the function f has a minimum value at $x = 0$. [4]



The points $A\left(-\frac{1}{2}, 3\right)$ and $B\left(1, 2\frac{1}{4}\right)$ lie on the curve $y = 2x + (x + 1)^{-2}$, as shown in the diagram.

(ii) Find the distance AB. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [6]

Formula Sheet MF19

Cambridge Assessment
International Education

List MF19

List of formulae and statistical tables

Cambridge International AS & A Level
Mathematics (9709) and Further Mathematics (9231)

For use from 2020 in all papers for the above syllabuses.

CST319

2508709701

Edited by Thoridal

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3} \times$ base area $\times$ height

Area of curved surface of cone = $\pi r \times$ slant height

Arc length of circle $= r\theta$ (θ in radians)

Area of sector of circle $= \frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \qquad S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \qquad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1), \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n$$, where n is a positive integer

and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$, where n is rational and $|x| < 1$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1\,, \qquad 1+\tan^2\theta \equiv \sec^2\theta\,, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A\pm B) \equiv \sin A\cos B \pm \cos A \sin B$$

$$\cos(A\pm B) \equiv \cos A\cos B \mp \sin A \sin B$$

$$\tan(A\pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1-\tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi\,, \qquad 0 \leqslant \cos^{-1} x \leqslant \pi\,, \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$v\frac{du}{dx} + u\frac{dv}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(\boldsymbol{x})$	$\int \mathbf{f}(\boldsymbol{x})\,\mathbf{d}\boldsymbol{x}$	
x^n	$\frac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x \rvert$	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	$(x > a)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert$	$(\lvert x \rvert < a)$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln\lvert \mathrm{f}(x)\rvert$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$$

FURTHER PURE MATHEMATICS

Algebra

Summations:

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1), \qquad \sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1), \qquad \sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Maclaurin's series:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leqslant x \leqslant 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 < x < 1)$$

Trigonometry

If $t = \tan\frac{1}{2}x$ then:

$$\sin x = \frac{2t}{1+t^2} \qquad \text{and} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x \equiv 1, \qquad \sinh 2x \equiv 2\sinh x \cosh x, \qquad \cosh 2x \equiv \cosh^2 x + \sinh^2 x$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \qquad (x \geqslant 1)$$

$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$

Integration

(Arbitrary constants are omitted; *a* denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$	
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\frac{1}{2}x + \frac{1}{4}\pi)\rvert$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\frac{1}{2}x)\rvert$	$(0 < x < \pi)$
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	
$\operatorname{sech}^2 x$	$\tanh x$	
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$	$(x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$	

MECHANICS

Uniformly accelerated motion

$$v = u + at\ , \qquad s = \tfrac{1}{2}(u+v)t\ , \qquad s = ut + \tfrac{1}{2}at^2\ , \qquad v^2 = u^2 + 2as$$

FURTHER MECHANICS

Motion of a projectile

Equation of trajectory is:

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}, \qquad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \qquad \text{or} \qquad \frac{v^2}{r}$$

Centres of mass of uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\frac{3}{8}r$ from centre

Hemispherical shell of radius r: $\frac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{3}{4}h$ from vertex

PROBABILITY & STATISTICS

Summary statistics

For ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad \text{standard deviation } = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}, \qquad \text{standard deviation } = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$$

Discrete random variables

$$\mathrm{E}(X) = \Sigma xp, \qquad \mathrm{Var}(X) = \Sigma x^2 p - \{\mathrm{E}(X)\}^2$$

For the binomial distribution $\mathrm{B}(n, p)$:

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \qquad \mu = np, \qquad \sigma^2 = np(1-p)$$

For the geometric distribution Geo(p):

$$p_r = p(1-p)^{r-1}, \qquad \mu = \frac{1}{p}$$

For the Poisson distribution $\mathrm{Po}(\lambda)$

$$p_r = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}, \qquad \mu = \lambda, \qquad \sigma^2 = \lambda$$

Continuous random variables

$$\mathrm{E}(X) = \int x\,\mathrm{f}(x)\,\mathrm{d}x, \qquad \mathrm{Var}(X) = \int x^2\,\mathrm{f}(x)\,\mathrm{d}x - \{\mathrm{E}(X)\}^2$$

Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Central Limit Theorem:

$$\bar{X} \sim \mathrm{N}\left(\mu, \ \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

$$\mathrm{N}\left(p, \ \frac{p(1-p)}{n}\right)$$

FURTHER PROBABILITY & STATISTICS

Sampling and testing

Two-sample estimate of a common variance:

$$s^2 = \frac{\Sigma(x_1 - \overline{x}_1)^2 + \Sigma(x_2 - \overline{x}_2)^2}{n_1 + n_2 - 2}$$

Probability generating functions

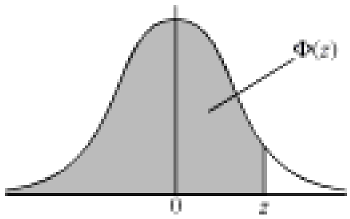
$$\mathrm{G}_X(t) = \mathrm{E}(t^X), \qquad \mathrm{E}(X) = \mathrm{G}'_X(1), \qquad \mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \leqslant z).$$

For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

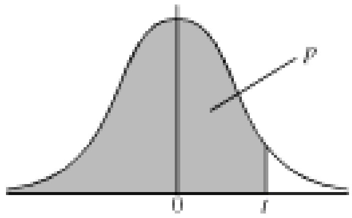
$$\mathrm{P}(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom, then, for each pair of values of p and ν, the table gives the value of t such that:

$P(T \leqslant t) = p$.

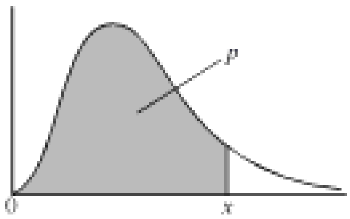


p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2-DISTRIBUTION

If X has a χ^2-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of x such that

$P(X \leqslant x) = p$.



p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu = 1$	0.0^31571	0.0^39821	0.0^23932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

WILCOXON SIGNED-RANK TEST

The sample has size n.

P is the sum of the ranks corresponding to the positive differences.

Q is the sum of the ranks corresponding to the negative differences.

T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *T*

	Level of significance			
One-tailed	0.05	0.025	0.01	0.005
Two-tailed	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n, each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

WILCOXON RANK-SUM TEST

The two samples have sizes m and n, where $m \leqslant n$.

R_m is the sum of the ranks of the items in the sample of size m.

W is the smaller of R_m and $m(n + m + 1) - R_m$.

For each pair of values of m and n, the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of W

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n, the normal distribution with mean $\frac{1}{2}m(m+n+1)$ and variance $\frac{1}{12}mn(m+n+1)$ should be used as an approximation to the distribution of R_m.

BLANK PAGE

BLANK PAGE

Syllabus 26-27 Pure Mathematics 1

1 Pure Mathematics 1 (for Paper 1)

1.1 Quadratics

Candidates should be able to:	Notes and examples
• carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form	e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant	e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term 'repeated root' is included.
• solve quadratic equations, and quadratic inequalities, in one unknown	By factorising, completing the square and using the formula.
• solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic	e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.
• recognise and solve equations in x which are quadratic in some function of x.	e.g. $x^4 - 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$, $\tan^2 x = 1 + \tan x$.

1.2 Functions

Candidates should be able to:	Notes and examples
• understand the terms function, domain, range, one-one function, inverse function and composition of functions	
• identify the range of a given function in simple cases, and find the composition of two given functions	e.g. range of $f : x \mapsto \frac{1}{x}$ for $x \geqslant 1$ and range of $g : x \mapsto x^2 + 1$ for $x \in \mathbb{R}$. Including the condition that a composite function gf can only be formed when the range of f is within the domain of g.
• determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases	e.g. finding the inverse of $h : x \mapsto (2x + 3)^2 - 4$ for $x < -\frac{3}{2}$.
• illustrate in graphical terms the relation between a one-one function and its inverse	Sketches should include an indication of the mirror line $y = x$.
• understand and use the transformations of the graph of $y = f(x)$ given by $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$ and simple combinations of these.	Including use of the terms 'translation', 'reflection' and 'stretch' in describing transformations. Questions may involve algebraic or trigonometric functions, or other graphs with given features.

1 Pure Mathematics 1

1.3 Coordinate geometry

Candidates should be able to:	Notes and examples
• find the equation of a straight line given sufficient information	e.g. given two points, or one point and the gradient.
• interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems	Including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.
• understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the circle with centre (a, b) and radius r	Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.
• use algebraic methods to solve problems involving lines and circles	Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry. Implicit differentiation is not included.
• understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.	e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve.

1.4 Circular measure

Candidates should be able to:	Notes and examples
• understand the definition of a radian, and use the relationship between radians and degrees	
• use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.	Including calculation of lengths and angles in triangles and areas of triangles.

1 Pure Mathematics 1

1.5 Trigonometry

Candidates should be able to:	Notes and examples
• sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)	Including e.g. $y = 3\sin x$, $y = 1 - \cos 2x$, $y = \tan\left(x + \frac{1}{4}\pi\right)$.
• use the exact values of the sine, cosine and tangent of $30°$, $45°$, $60°$, and related angles	e.g. $\cos 150° = -\frac{1}{2}\sqrt{3}$, $\sin \frac{3}{4}\pi = \frac{1}{\sqrt{2}}$.
• use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations	No specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected.
• use the identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$	e.g. in proving identities, simplifying expressions and solving equations.
• find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).	e.g. solve $3\sin 2x + 1 = 0$ for $-\pi < x < \pi$, $3\sin^2\theta - 5\cos\theta - 1 = 0$ for $0° \leqslant \theta \leqslant 360°$.

1.6 Series

Candidates should be able to:	Notes and examples
• use the expansion of $(a + b)^n$, where n is a positive integer	Including the notations $\binom{n}{r}$ and $n!$ Knowledge of the greatest term and properties of the coefficients are not required.
• recognise arithmetic and geometric progressions	
• use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions	Including knowledge that numbers a, b, c are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent). Questions may involve more than one progression.
• use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.	

1 Pure Mathematics 1

1.7 Differentiation

Candidates should be able to:	Notes and examples
• understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for first and second derivatives	Only an informal understanding of the idea of a limit is expected. e.g. includes consideration of the gradient of the chord joining the points with x coordinates 2 and $(2 + h)$ on the curve $y = x^3$. Formal use of the general method of differentiation from first principles is not required.
• use the derivative of xn (for any rational n), together with constant multiples, sums and differences of functions, and of composite functions using the chain rule	e.g. find $\frac{dy}{dx}$, given $y = \sqrt{2x^3 + 5}$.
• apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change	Including connected rates of change, e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables.
• locate stationary points and determine their nature, and use information about stationary points in sketching graphs.	Including use of the second derivative for identifying maxima and minima; alternatives may be used in questions where no method is specified. Knowledge of points of inflexion is not included.

1.8 Integration

Candidates should be able to:	Notes and examples
• understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except –1), together with constant multiples, sums and differences	e.g. $\int (2x^3 - 5x + 1)\,dx$, $\int \frac{1}{(2x+3)^2}\,dx$.
• solve problems involving the evaluation of a constant of integration	e.g. to find the equation of the curve through $(1, -2)$ for which $\frac{dy}{dx} = \sqrt{2x+1}$.
• evaluate definite integrals	Including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}}\,dx$ and $\int_1^\infty x^{-2}\,dx$.
• use definite integration to find – the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves – a volume of revolution about one of the axes.	A volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and $y = 5$ rotated about the x-axis.